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BALANCING OF FIRST ORDER RESIDUAL EFFECT THROUGH ORTHOGONAL LATIN SQUARES

T. V. Sathianandan and K. C. George

College of Veterinary and Animal Sciences, Mannuthy, 680651, Kerala, India

In long term experiments like perennial crop experiments, dairy cattle experiments, bio-assays, clinical trial etc., the effects of the treatments are influenced by residual effects of previous treatments. Residual effects which persist for only one period, called first order residual effects, are more important in long term experiments. Designs in which each treatment follows every other treatments equally frequently are said to be balanced for first order residual effects. Using such designs direct and residual effects can be estimated. Williams (1949) gave a general method of construction of such designs using module differences. Sheehe and Bross (1961), Federer and Atkinson (1964), Nair (1967) and Berenblut (1968) have also given methods of construction of designs balanced for residual effects. A method of construction of such designs using orthogonal latin squares is given in this study.

Materials and Methods

The method of construction is based on orthogonal latin squares. When the number of treatments is a prime number or power of a prime number, there will be (s—1) orthogonal latin squares of order s. If $\propto$ is a primitive element of a Galois Field GN $(p^n = s)$, where p is a prime number, then the element in the xth row and y th column of the i th latin square is given by the expression.

$u_i u_x + u_y$, where i = 1, 2,............(s—1), x, y = 0, 1, 2,.........., (s—1)

$u_0 — 0$, $u_1 = 1$, $u_2 = \propto$, $u, = \propto^2$,............$u_{s-1} = \propto^{s-2}$

Results and Discussion

Theorem In a set of (s—1) orthogonal latin squares of order $s \times s$, each treatment follows each other treatment exactly (s—1) times.

To prove this theorem which states about the method of construction, the following lemma are required.

Lemma 1 Let the first rows of each of the (s—1) orthogonal latin squares of order $(s \times s)$ has numbers 1, 2,........, s in that order in the s columns. Take any (s—1) numbers except j, one in each of the (s—1) columns other than the j th column of one of the orthogonal latin squares P_i, say. If these numbers are distinct, a set of elements which fall on these numbers when the (s—1) orthogonal latin squares are superimposed on P_i will contain each of the (s—1) number (other than j) (s—2) times.

Proof Let a_1, a_2,..........., a_{j-1}. a_{j+1},......., a_s be the numbers in the first, second,...........(j—1)th, (j + 1)th,...........s th columns of P_i. Assume that these numbers are distinct. When (s—1) orthogonal latin squares are superimposed on P_i, the numbers other than j, which fall on.

a_1 are, a, and other (s—2) numbers other than j,

a_2 are, a_2 and other (s—2) numbers other than j, etc.

In the set thus obtained each of the numbers other than j occurs exactly (s—2) times.

Lemma 2 Let there be (s—1) orthogonal latin squares of order $s \times s$ in the numbers 1, 2 . . . . . ., s. Let the first row of, each latin square be 1, 2,. . . . ., s in that order in the s columns. Take any one of the orthogonal latin squares, say P_i and take (s—1) numbers other than j in the (s—1) columns excluding the j th column. If the set of elements which fall on these numbers, when the (s—1) orthogonal latin squares are superimposed on P_i, contain each of the numbers other than j equally frequently, the (s—1) numbers taken in P_i are all distinct.

Proof The result can be established by reaching at a contradiction. If possible let there be two identical numbers say (a_1, a_1) among the (s—1) numbers taken in P_i in (s—1) columns other than the j th. Without loss of generality we can assume that (a_1, a_1) occur in the first two columns. It then follows that when (s—1) orthogonal latin squares are superimposed on P_i, the numbers which fall on a_1.

in the first column are a_i and (s—2) numbers other than j

in the second column are a_i and (s—2) numbers other than j

In the first of these, 1 will not be present and in the second, 2 will not be present. Thus in the overall set of elements a_1 will be present ($s—_1$) times and each other number other than j atmost (s—2) times. Hence if the set is to contain all numbers other than j equally frequently, the original set from P_i should contain (s—1) distinct numbers.

Proof of the theorem stated Take one of the orthogonal latin squares say P_i, and replace all numbers other than those immediately preceding j by zero. Denote the square so obtained by D_i, i=1, 2. . . . . (s—1). Let D = D_1 + D_2 + ...+ D_{s-1}, which will be a square with j th column and last row containing only zeroes, and in other places we get elements which preceed j. Superimpose the (s—1) orthogonal latin squares over D and obtain the set of numbers excluding j< which fall on the non-zero element of D. By lemma 1, each of the numbers other than j will occur equally frequently in this set and therefore by lemma 2 the basic set consisting of the elements of D will contain each of the numbers 1, 2,. . . . ., s other than j equally frequently.

Let α be a primitive element of GF (p^n = s). Then the elements of the Galois Field are given by $u_0 = 0$, $u_1 = 1$, $u_2 = \alpha$, $u_3 = \alpha^2$ ...., $u_{s-1} = \alpha^{s-2}$. Then if we put j, which is defined by $u_i u_x + u_y = u_j$, in the x th row and y th column of i th latin square, i = 1, 2, ..., (s—1), we get a set of (s—1) orthogonal latin squares of order s x s. In all these latin squares j will occur in the (j+1) st column of first row.

Excepting this for any given i,

$u_i u_x + u_y = u_j$ .... (1)

has (s—1) solutions. The element preceeding j in the same column when (1) is true is j^1 given by

$u_i u_{x-1} + u_y = u_{j1}$ .... (2)

Taking the differecne between (1) and (2) we get

$u_i (u_x - u_{x-1}) = u_j - u_{j1}$ ...;.......... (3)

Since $u_x \neq u_{x-1}$ and $u_j \neq u_{j1}$, (3) will have a non-zero solution for a fixed i. Equation (2) has exactly (s—1) solutions for a fixed i. As this is true for all i = 1, 2, ... (s—1) in the set of (s—1) orthogonal latin squares (s—1) (s—1) numbers immediately preceed j. We have shown that among these numbers each of the numbers 1, 2, ... s other than j occurs equally frequently. Thus each number preceeds j exactly (s—1) times in the set of (s—1) orthogonal latin squares of order s.

When the number of treatments s is a prime number or power of a prime number this method of construction could be used. In this type of designs each treatment will be followed by each other treatment (s—1) times exactly. Hence using these designs residual effects could be estimated more efficiently. So when residual effects of treatments are equally important as direct effects this design could be used more appropriately.

Summary

A general method of construction of designs that are balanced for first order residual effects, when the number of treatments is prime or power of a prime number, using orthogonal latin squares has been given. The residual effects are more efficiently estimated in this type of designs and are useful in long term experiments like perennial crop experiments, feeding trials etc.

സംഗ്രഹം

ലാററിൻ സമചതുരങ്ങളെ ഉപയോഗപ്പെടുത്തിക്കൊണ്ട് ദീർഘകാല പരീക്ഷണങ്ങൾ നടത്തുമ്പോൾ ഉപയോഗിക്കുവാൻ തക്ക മാതൃകയുടെ നിർമ്മാണരീതി ro'l«!Kj§i/!>1 ച്ചിരിക്കുന്നു.

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