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ANALYSIS OF VARIANCE WHEN OBSERVATIONS ARE SUBJECT TO ERRORS DUE TO ROUNDING - OFF

In field experiments, even when the observation is on a continuous variate, it is recorded in a discrete fashion; discontinuity being introduced by rounding-off the values correct to the nearest integer, the nearest 100 gms or by some other approximation. This introduces an additional source of variability into the data. If the measurement is relatively accurate, or if the discontinuity is small, this may not very much affect the results of analysis. But if the discontinuity is large, it can vitiate the results. In any statistical model, the assumption is that the yield y, is a function of the effects, with a random error e superimposed thereon. The random errors are assumed to be normally and independently distributed with mean zero and constant variance σ^2. When there are errors due to rounding-off, it introduces another component of error e' which ranges from $-c/2$ to $c/2$, c being the discontinuity, when values are rounded-off correct to the nearest c-unit. It is clear that e' values will follow a rectangular distribution. Hence $E(e') = 0$ and variance $(e') = c^2/12$. Further e and e' are independent.

Consider the data from an experiment laid out in randomised block design with v treatments and r replications. The model for yield y_{ij}, from the i-th block corresponding to the j-th treatment is $y_{ij} = m + b_i + t_j + e_{ij} + e'_{ij}$ allowing for the errors due to rounding-off, where m is the general mean, b_i is the block effect due to the i-th block, t_j is the treatment effect due to the j-th treatment, e_{ij} is the random effect following the normal law and e'_{ij} is the error due to rounding-off following the rectangular distribution. This leads to the following results.

$$E \sum_i \sum_j (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2 = \sigma^2 + c^2/12$$

$$E \sum_i \sum_j (\bar{y}_{i.} - \bar{y}_{..})^2 = \sigma^2 + c^2/12 + v\,\sigma^2_B$$

$$E \sum_i \sum_j (\bar{y}_{.j} - \bar{y}_{..})^2 = \sigma^2 + c^2/12 + r\,\sigma^2_T$$

where $\bar{y}_{i.} = \sum_j y_{ij}/v$, $\bar{y}_{.j} = \sum_i y_{ij}/r$ and $\bar{y}_{..} = \sum_i \sum_j y_{ij}/rv$. Further, σ^2_B is the variance of block effects and σ^2_T is the variance of the treatment effects. Hence the best estimate of σ^2 from the error line is M. S. (Error) — $c^2/12$. Under the hypothesis that the block effects are equal, the estimate of σ^2 from the block totals is M.S. (Blocks) — $c^2/12$ and under the hypothesis of equality of treatment effects, the estimate of σ^2 from the treatment totals is M. S. (Treatments) — $c^2/12$. Thus the F-ratio for testing block effects is

$$\frac{\text{M. S. (Blocks)} - c^2/12}{\text{M. S. (Error)} - c^2/12}$$

and the F-ratio for testing treatment effects is

$$\frac{\text{M. S. (Treatments)} - c^2/12}{\text{M. S. (Error)} - c^2/12}$$

. This shows that the analysis of variance table should be as follows.

Analysis of variance table.

Source	S. S.	d. f.	Uncorrected m. s.	Corrected m. s.	F
Total	$\sum_i \sum_j (y_{ij} - \bar{y}_{..})^2$	rv-1			
Blocks	$\sum_i \sum_j (\bar{y}_{i.} - \bar{y}_{..})^2$	r-1	$M.S_B$	$M.S'_B = M.S_B - c^2/12$	$M.S'_B/M.S'_E$
Treatments	$\sum_i \sum_j (\bar{y}_{.j} - \bar{y}_{..})^2$	v-1	$M.S_T$	$M.S'_T = M.S_T - c^2/12$	$M.S'_T/M.S'_E$
Error	$\sum_i \sum_j (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2$	(r-1) (v-1)	$M.S_E$	$M.S'_E = M.S_E - c^2/12$	

The F-ratio for blocks will be greater than what can be obtained without the correction when $M.S_B$ is greater than $M.S_E$. Similarly when $M.S_T$ is greater than $M.S_E$, the F-ratio for treatments is greater than that obtainable without the correction. In the analysis of data from other designs also, the correction to be made is is in the M. S. corresponding to the various sources and the quantity to be subtracted is $c^2/12$

സംഗ്രഹം

ശാസ്ത്രീയ പരീക്ഷണങ്ങളിൽ ശേഖരിക്കപ്പെടുന്ന നിരീക്ഷണങ്ങൾ ഏറ്റവും സമീപസ്ഥിതമായ ഏകകത്തിലേക്ക് (c) ഏകദേശനം ചെയ്യുന്നതുകൊണ്ട് പിശകുകൾ സംഭവിക്കുന്നു. ഇത്തരം പിശകുകളെ സംശോധനം ചെയ്യുന്നതിന് വ്യതിയാന വിശ്ലേഷണത്തിൽ വിവിധ സ്രോതസ്സുകളുടെ വർഗമാധ്യങ്ങളിൽ rolorra" $c^2/12$ വ്യവകലനം ചെയ്യേണ്ടതാകുന്നു.

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