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RELATIONSHIP BETWEEN PLOT SIZE AND PLOT VARIANCE

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Introduction

An empirical relationship between plot size and plot variance was developed by Smith (1938). This law states that,

$$\log V_x = \log V_1 - b.\log x \qquad (1)$$

where V_x is the variance of yield per unit area among plots or experimental units of size x elements or individuals, V_1 is the variance among plots of size unity and b is the regression coefficient indicating the relationship between adjacent individuals, or elements. The limiting values of the regression coefficient are zero and one, unless inter-experimental-unit competition is present. If the experimental unit is composed of a random selection of x individuals, b = 1 and if the x individuals are identical, b 0. When there is correlation between adjacent elements as in the case of field experiments, the value of b will be less than unity. Smith computed the b values for 38 different sets of uniformity trial data and found that most of the values fell within the range of 0.2 to 0.8.

Instead of an empirical approach, a theoretical approach is attempted in this paper, based on certain models.

Development of models

Consider a uniformity trial consisting of N individuals or elements. Assume that the yield obtained from the elements are distributed normally with mean a and variance V_1 The yields may be correlated. Let r_i denote the correlation coefficient between elements which are i elements apart. Thus r_1 denotes the correlation coefficient between adjacent elements, r, the correlation coefficient between individuals which are two elements apart, ie, having one element in between etc,. Then it is obvious that the yield obtained from plots of x elements say y will have mean x. a and variance,

$$V(y_x) = V_1 [x + 2(x-1) r_1 + 2(x-2) r_2 + \dots + 2r_{x-1}] \qquad (2)$$

The coefficient of variation of plot yields will be

$$C.V.(y_x) = \frac{V_1^{\frac{1}{2}} [x + 2(x-1) r_1 + 2(x-2) r_2 + \dots + 2r_{x-1}]^{\frac{1}{2}}}{} \qquad (3)$$

Further if Vx is defined as the variance of yield per unit area among plots of size x.

$$[x + 2(x-1) r_1 + 2(x-2) r_2 + \ldots + 2 r_{x-1}]$$

The following models are considered.

I. $r_i = 0; \quad i = 1, 2, \ldots x-1$

II. $r_i = 1; \quad i = 1, 2, \ldots x-1$

III. $r_i = r \quad i = 1, 2, \ldots x-1$

IV. $r_i = r^i; \quad i = 1, 2, \quad x-1$

V. $r_i = \frac{}{i}; \quad i = 1, 2, \ldots x-1$

VI. $r_i = \frac{r}{i^2}; \quad i = i, 2, \quad x-1$

VII. $r_i = \frac{r}{x}; \quad i = 1, 2, \ldots x-1$

VIII. $r_i = \frac{r}{x^2}; \quad i = 1, 2, . \quad x-1$

IX. $r_i = \frac{}{x-1}; \quad i = 1, 2, \ldots x-1$

X. $r_i = r^x; \quad i = 1, 2, \quad x-1$

XI. [illegible]

XII. [illegible] $(x-1)$

XIII. $r_i = \frac{k^2 \, x^p \, b -}{(x-1)}; \quad i$

XIV. $r_i : \frac{Y \quad e^{-kx} - 1}{-1}; \quad i = 1, 2 \ldots x -$

$$r_i = [x(1 + a \cdot e^{-b.x}) - k]/k(x-1); \quad i = 1, 2, \ldots x-1.$$

Model I When $r_i = 0$,

$$V(y_x) = V_1 . x \tag{5}$$

$$C.V.(y_x) = V_1 / a.x^{\frac{1}{2}} \tag{6}$$

and, $V_x = V_1 / x$

This means that the coefficient of variation decreases as x increases. Thus it is possible to increase accuracy by suitably increasing the plot size. This situation agrees with the Fairfield Smith variance law, that $V_x = V_1 / x^b$ and that $b = 1$ for random distribution of yields.

Model II When $r_i = 1$,

$$V(y_x) = V_1 x^2 \quad (8)$$

$$C.V.(y_x) = V_1^{\frac{1}{2}}/a \quad (9)$$

and $V_x = V_1$.

Here the coefficient of variation is independent of x and hence there is no way of fixing the optimum plot size or rather all plot sizes are of equal efficiency. Here V_x is in agreement with the Smith's law where b = 0.

Model III When $r_i = r$,

$$V(y_x) = V_1 . x [rx + (1-r)] \quad (11)$$

a .x²

and V_x V, [r... (1 - r)] (13)

A

This model also yields a coefficient of variation with a decreasing trend.

Model IV When $r_i = r^i$,

+ (14)

(1 - r) j=0

$$C.V.(y_x) = V_1^{\frac{1}{2}} [x + 2r A \frac{r^{x-1})}{(1-r)} .x - 2r \sum_{j=0}^{x-2} (j+1) r^j] \quad (15)$$

a.x.

$$\text{and } V_x = V_1 [x + 2r \frac{(1 - r^{x-1})}{(1-r)} x - 2r \sum_{j=0}^{x-} (j+1) r^j] \quad (16)$$

x-

If powers of the order of x - 1 and higher are negligible, the above functions can be simplified as follows.

(1 - r)·

V₁ [x (1 -

The coefficient of variation of yield decreases as x increases.

Model V. When $r_i = r/i$,

1)] (20)

C. V. $(y_x) =$ $V_1^{\frac{1}{2}}$ [x + 2rx (1 + i + I + ——— + $_{x-1}$) -2r(x- 1)]$^{\frac{1}{2}}$ (21)

a. x. (22)

+ * + * + • +)

and V = V_1 [x+ 2rx (1 . $\frac{1}{x-1}$ – 2r (x 1)]

This model also gives a decreasing function of x for the coefficient of variation of plot yields.

Model VI When $r_i = r/i^2$,

x—1 £ x—1 1

x—1 x—1

C. V $(y_x) = V_1^{\frac{1}{2}}$ [x+2r x £ $\frac{1}{\cdots}$ –2r 2 J -] $^{\frac{1}{2}}$

x—1 x—1

and V = V_1 [x+2r x S $\frac{1}{j}$ –2r c -J (25)

Model *VII* When r_i - r/x

$V(y_x) = V_1 [x + (x - 1) r]$ (26)

C. V. $(y_x) = \frac{V_1^{\frac{1}{2}} [x + (x-1) r]^{\frac{1}{2}}}{a. x}$ (27)

and $V_x =$ x + (x, (28)

Model VIII When $r_i = r/x^2$

$V(y_x) = V_1 [x + \frac{x-1}{x} r]$ (29)

C. V. $(y_x) = \frac{V_1^{\frac{1}{2}} [x + \frac{x-1}{x} r]^{\frac{1}{2}}}{a. x}$ (30)

and V_x — V_1 [x + r]

Model IX When $r_i = \frac{}{x-1}$

$V(y_x) = 0$ (32)

$C.V(y_x) = 0$ (33)

and $V_x = 0$ (34)

Model X When $r_i = r^x$

$V(y_x) = V_1 x [1 + (x-1) r^x]$ (35)

(36)

a. $x^{\frac{1}{2}}$

and $V_x = V_1 [1 + (x-1) r^x]$ (37)

XI When $r = \frac{c^2 x^2 -}{x^2 \quad v}$

$V(y_x) = c^2 V_1 x^2$ (38)

$C.V.(y_x) = \frac{c}{a} V^*$ (39)

and $V_x = c^2 V_1$ (40)

Here the coefficient of variation remains constant.

Model XII When $r_i = \frac{(\quad - b)}{(x \quad 1 y}.\log_e x$

$V(y_x) = V_1 .x. [1 + (1-b) \log_e x]$ (41)

Taking $1 + (1-b) \log_e x$ as approximately equal to $e^{(1-b)\log_e x} = x^{1-}$ we get,

$V(y_x) = V_1\, x a-b$ (42)

Further based on the same approximation,

$C.V(y_x) =$ (43)

a.x^2

and $V_x = \frac{V}{A}$ (44)

This is the Fairfield Smith variance law. Here $b = 1 - \frac{}{\log_e x}$ (45)

In this case, when r = 0, b = 1 and when r = 1, $b = 1 - \frac{(x-1)}{\log_e x}$. Further, when r is negative, b > 1. The value of r can be negative only when there is interunit competition. Due to the assumption that $r_i = 1 - \frac{\log \ldots b}{(x-1)}$ is independent of x. Thus the regression coefficient b in the Smith's law can be made use of in comparing the efficiencies of different plot sizes. The relative efficiency due to plots of size k. x. compared to those of size k is k^b which is independent of x. and the increase in efficiency due to a small increase in plot size is (b/x). E_x where $E_x = \frac{x^b}{*_1}$ is the efficiency of plots of size x.

Model XIII . When $r_i = \frac{K^{1-2} x^p b^{-x} \; 1}{x-1}$ (46)

$V(y_x) = k^2 V_1 x^{p-1} b^{-x}$

$C.V(y_x) = \frac{k V_1^{\frac{1}{2}} x^{(p-1)/2} b^{-x/2}}{a}$ (47)

$V_x = k^2 V_1 x^{p-1} b^{-x}$ (48)

Model XIV When $r_i = \frac{x.e}{x-1}$

$V(y_x) = V_1 x^2 e^{-kx}$ (49)

$C.V.(y_x) = \frac{V_1^{\frac{1}{2}} e^{-kx/2}}{a}$ (50)

and $V_x = V_1 e^{-kx}$

Model XV When $r_i = \frac{x(1 + a \cdot e^{-bx}) - k}{k(x-1)}$

$V(y_x) = \frac{V_1^2 x^2 (1 + a.e^{-bx})}{k}$ (52)

$C.V.(y_x) = \frac{V_1^{\frac{1}{2}}(1 + a.e^{-bx})^{\frac{1}{2}}}{k.a.}$ (53)

and $V_x = \frac{V_1}{k}(1 + a\, e^{-bx})$ (54)

Estimation of optimum plot size

Five different approaches to the estimation of optimum plot size are attempted.

(i) Maximising the per unit information. The per unit information is defined as $1/V_x$ and is denoted by Iu (x).

(ii) Minimising the cost per unit of information. A linear cost function C = p + q. x is assumed. Thus the cost per unit of information is,

$$\frac{\text{cost per plot of size x}}{\text{information per unit}} = \frac{(p+q.x)}{1/V_x} = (p+q.x).\,V_x$$

This is denoted by $C_1(x)$.

(x) $(p + q.\,x)\,V_x$ (55)

(iii) Maximising the curvature of the function Vx . For the curve y =Vx , the radius of curvature is,

Thus maximising the curvature means minimising R. It is easier to minimise

log R = (3/2) log [1 + $(y')^2$] - log y" (57)

The optimum plot size is the integer next higher to the value of x which minimises log R.

(iv) Maximising the curvature of the function C. V (y_x). = W (say). Here also only a lower bound to the optimum plot size can be estimated.

(v) Prescribing the value of coefficient of variation for the required plot and then finding the plot size which will give this coefficient of variation per plot. Then the optimum plot size to give W_0, the prescribed value of coefficient of variation, can be estimated.

(i) Maximising the per unit information.

In model I, Iu (x) = x/V_1 and hence Iu (x) increases with x. Thus there is no maximum value for per unit information. In model II, Iu (x) = $1/V_1$, a constant value. Model III gives

Iu (x) =

This is an increasing function of x and hence there is no maximum value.

In model IV,

Iu (x) =

$$- r^{x-1} \backslash \cdot x - 2r \sum_{j=0}^{x-2} (j + 1) r^j]$$

$$\bar{\ } V_1 \ [x (1-r^2) - 2r]$$

These functions also do not have maximum values. In model V,

$$Iu (x) = \frac{x^2}{V_1 [x + 2rx(1 + \frac{1}{2} + .. + \frac{1}{x-1}) - 2r (x - 1)]} \quad (60)$$

This does not lead to an optimum plot size. Model VI has,

$$[V_1 x + 2xr \sum_{j=1}^{x-1} \frac{1}{j^2} - 2r \sum_{j=1}^{x-1} \frac{1}{j}] \quad (61$$

In model VII,

$$Iu (x) = V_1 \quad (62)$$

This also does not lead to an optimum plot size. Model VIII gives,

$$Iu (x) = \frac{x^2}{V_1 [x + r(x - 1)/x]}$$

In model IX, Iu (x) = α . Model X gives,

$$Iu (x) = [1 + (x - 1)$$

$$Iu (x) = \frac{}{C^2V_1}$$ in model XI. Fairfield

$$Iu (x) = \frac{x^{\frac{1}{b}}}{V_1} \quad \textbf{(65)}$$

This is an increasing function of x for all values of x. In model XIII,

$$Iu (x) = \frac{}{k^2 V_1 x^p} \frac{b}{-x}, \quad (66)$$

which gives an optimum plot size of x — (p - 1)/ log b. Model XIV gives, Iu (x) – e^{kx}/V which does not possess a maximum value. In model XV,

$$Iu\ (x) = \frac{k}{V_1 (1 + a.e. \quad)} \qquad (67)$$

This also does not lead to a maximum value.

(ii) Minimising the cost per unit of information.

In model I,

$$C_1\ (x) = (p + qx)\ V_1/x \qquad (68)$$

This function does not have a minimum value in the finite range of x.

Model II gives C_1 (x) = (p + qx) V_1 which is an increasing function of x. This leads to the conclusion that plot size should be as small as possible.

In model III,

$$C_1\ (x) = (p + qx)\ V_1\ [r x + (1 - r)]/x \qquad (69)$$

This takes a minimum value when x = $\frac{\quad}{q.\ r}$

Thus the optimum plot size is $\left[\frac{p\ (1 - r)}{q.\ r}\right]^{\frac{1}{2}}$ In model IV,

$$, [x (1 - r) - 2r] \qquad (70)$$

This does not lead to an optimum plot size. Maximisation of C_1 x) under models V and VI lead to complications. Model VII gives,

$$C_1\ (x) \quad \frac{(P + qx)\ V_1\ [x + r\ (x - 1)]}{\quad} \qquad (71)$$

In model VIII, C_1 (x) = $\frac{(p + qx)\ V_n\ fy \perp x \sim [\ r\ ]}{x^2}$ (72)

These also do not yield any useful results. Under model IX, C_1 (x) = 0 In model X, C_1 (x) = $\frac{(p + qx) V_1 [1 + (x - D\ r^x]}{\quad}$

Model XI gives C_1 (x) = (p + qx) c^3 V_1. Thus optimum plot size is not estimable in this case also. Model XII gives,

$$C_1\ (x) = (p + qx)\ V_1/x^b \qquad (74)$$

This leads to an optimum plot size of $\frac{b\ \ p}{-}$ - as given by Smith (1933)

In model XIII, $C_1(x) = k^2 V_1 (p + qx) x^{p-1} b^{-x}$ (75)

Model XIV gives $C_1(x) = (p + qx) V_1 e^{-kx}$ (76)

In model XV, $C_1(x) = \frac{V_1}{k}(p + qx)(1 + a.\ e^{-bx})$ (77)

There ate no maxima for these functions.

(iii) Maximising the curvature of V .

Model I shows that optimum plot size $x_0 > V_1^{\frac{1}{2}}$ But this result is useful only when $V_1 > 1$. In model II V_x has no curvature. In model III the maximum curvature is when $x = V_1^{\frac{1}{2}}(1-r)^{\frac{1}{2}}$. Thus optimum plot size $x_0 > V_1(1-r)^{\frac{1}{2}}$ • Here the result will be useful only if $V_1(1-r) > 1$. Models IV to XI do not yield optimum plot sizes by this method. In model XII the curvature* is maximum at,

$$\left\{\frac{b\ V_1^{2}(2b+1)}{b+2}\right\}^{\frac{1}{2(b+1)}}$$

But invariably this has numerical value less than 1. Model XIII does not give optimum plot size by maximising the curvature of V_x. Model XIV gives maximum curvature for Vx at $x = \frac{1}{k}\log(2k V_1^2)$ (79)

In model XV the optimum plot size is,

$$x_0 > \frac{2}{b}\log\frac{(2V_1^2 a^2 b^2)}{k^2}$$ (80)

(iv) Maximising the curvature of C. V. (y_x)

Curvature of C. V. (y_x) has a maximum value only in models XII and XV. In model XII the maximum is at

$$\left\{\frac{b^2(b+1)V_1}{2(b+4)a^2}\right\}^{\frac{1}{(b+2)}}$$ (81)

But invariably its value is less than unity. In model XV the maximum curvature is attained at

$$x = \frac{-1}{k}\log(2a^2/kV_1)$$ (82)

which is always negative.

(v) Estimating the plot size to give prefixed value of coefficient of variation per plot.

The prefixed value of coefficient of variation per plot is w_0. Let the coefficient of variation for plots of size unity be w_1.

In model I, $x = (w_1/w_0)^2$ will give the required coefficient of variation per plot, ie, $x_0 = (w_1/w_0)^2$. In model II optimum plot size is not estimable, since the coefficient of variation is constant. In model HI,

$$= \frac{(1-r)\, w_1{}^2}{w_0{}^2 - r w_1} \tag{83}$$

Model IV gives,

w_1 2 (1 r

In models V and VI the estimation of x_0 is difficult. In model VII,

$$x_0 = \frac{(1+r)\, w_1{}^2 \pm [(1+r)^2\, w_1{}^4 - 4\, w_1{}^2\, w_0{}^2]^{\frac{1}{2}}}{2\, w_0{}^2} \tag{85}$$

In model VIII, x, is obtained by solving the equation, (86)

$w_0{}^2\, X^s$ $w_1{}^2$ x^2 $w_1{}^2$ w_1 r x + 2 r = 0

Model IX gives C. V. $(y_x) = 0$. In model X solution for x_0 is difficult. Model XI gives constant coefficient of variation. Model XII gives,

$x_0 = (w_1/w_0)$

In model XIII x_0 is obtained by solving the equation,

$$\log w_0 = \log k + \log w_1 + \frac{}{2} \log x - \log b. \tag{88}$$

$$\text{Model XIV gives } x_0 = \frac{-2}{\cdot} \log (w_0/w_1) \tag{89}$$

In model XV, $x_0 = \frac{}{u} - \log[(k^2 w_0{}^2 \text{ " } w_1{}^2)/w_1{}^2$

Summary

Fifteen models have been considered for studying the plot variances in relation to plot size. The functions for variance of yield per plot V (y), coefficient of variation of yield per plot C. V (y_x) and the variance of yield per unit area V_x have been derived. Estimation of optimum plot size, based on five different criteria have been attempted in each of these situations.

സംഗ്രഹം

പ്ലോട്ടുകൾ തോറുമുള്ള വിളവിന്റെ വ്യതിയാനവും, പ്ലോട്ടിന്റെ വലിപ്പവുമായുള്ള ബന്ധം അപഗ്രഥിക്കുന്നതിന് പതിനഞ്ച് ഗണിതീയരൂപങ്ങൾ പരിഗണിക്കപ്പെട്ടു. C. V. പ്രതി), പ്ലോട്ട് വ്യതിയാനം V(y), പ്രതിപ്ലോട്ട് ഉല്പാദത്തിന്റെ വിചരണ ഗുണാങ്കം (y_x വിളവിന്റെ ഏകകക്ഷേത്ര വ്യതിയാനം V_x , എന്നിവയുള്ള ഫലനങ്ങൾ വ്യുൽപ്പാദിപ്പിക്കപ്പെട്ടു. ഓരോ അവസരത്തിലും പ്ലോട്ടിന്റെ അനുകൂലതമവിസ്തൃതിയുടെ, അഞ്ചു വ്യത്യസ്ത മാനദണ്ഡങ്ങളെ അടിസ്ഥാനമാക്കിയുള്ള ആകലനവും നിർവ്വഹിച്ചിട്ടുണ്ട്.

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