OPTIMALITY OF BLOCK DESIGNS USED IN ONE WAY ELIMINATION OF HETEROGENEITY

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THESIS

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I hereby declare that this thesis entitled Optimality of Block Designs Used in One way Elimination of Heterogeneity is a bonafide record of research work done by me during the course of research and that the thesis has not previously formed the basis for the award to me of any degree diploma associateship fellowship or other similar title of any other University or Society

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Introduction

INTRODUCTION

Block designs are usually used for experiments where it is important to eliminate heterogeneity in one direction A design is an arrangement of v treatments in b blocks Being only an arrangement each and every permutation of the v treatments in b blocks leads to a design The prime objective is to obtain the estimates of variance of every treatment contrast among the treatment effects and as such designs to be adopted in a practical situation has to conform to this prime objective Based on the basic statistical tools and the inferential statistics certain principles for co opting a design for the conduct of an experiment has been worked out

The total application of the basic principles of experimentation starts from randomised complete block design (RCBD) only As the name itself implies RCBD requires at least one replication of all the treatments in a block This imposes severe restriction on the number of treatments that could be used simultaneously in an experiment When the number of treatments is large adoption of RBD may lead to increase of error variance due to larger block size This led to the devising of incomplete block designs as also the fractional replication of a set of treatments Added to this the terminologies like variance balancing efficiency balancing equal replication connectedness were also developed Based on these terminologies the various designs were grouped as families of proper designs families of connected block designs etc To account for all the designs that have been developed so far is really a herculian task but the prime objective of a design is co-option for an experiment that would lead to an inference regarding the differential effects of treatments with maximum precision The adoption of a design in a given situation has to depend on the statistical properties of the design In a given class of design one should choose a design which is good according to some well defined statistical criterion

When the number of treatments are not too large the blind recommendation in most of the cases is a complete block design and designs like latin square lattice etc are used when heterogeneity is to be eliminated in two directions Main objection regarding the adoption of incomplete block designs is the difficulty involved in computation But with the advent of high speed computers this has no logical basis The adoption of design other than complete block design to a practical situation is not a straight forward task as the same has to necessarily satisfy various optimality criteria that were developed

Situations arise where a group of treatments are compared with a control or a group of controls The problem is to allocate the experimental units optimally to control so as to maximise the probability associated with the joint confidence statement concerning the many to one comparisions between the mean of the control treatment and the means of the test treatments Usually the design used is RBD with control treatment replicated in all blocks This actually results in unnecessary replication of control treatment When control treatment is replicated in every block of a BIBD the balance of the design itself will be upset For comparing test treatments with a control a new general class of designs called balanced treatment incomplete block designs (BTIBD) can be used It is desired to find out whether BTIBD is more efficient than using RBD with the control treatment replicated in all blocks

In certain agricultural experiments we need the comparison among test treatments as also between the test treatments and control with more precision The design which is optimal for this situation is to be found out

Thus the present study is based on the following objectives

(1) to examine the practical utility of the various optimality criteria

(2) to find optimal designs for comparing test treatments with a control

(3) to examine E optimality of extended E optimal designs

Review of Literature

REVIEW OF LITERATURE

A comprehensive review of the systematic advancement in the field of optimal experimental designs that led to the generation of a broad class of optimal designs is presented below

Bose and Clatworthy (1955) studied some classes of partially balanced designs They showed that the parameters of all partially balanced incomplete block designs (PBIBD) depend upon three integral parameters namely block size replication and number of treatments with a few additional restrictions

Many models used in statistical investigations can be formulated in terms of least square theory The parameters are usually estimated by the theory of least squares Ehrenfeld (1955) opined that the efficiency and sensitivity of a design may be very much affected by the choice of the design matrix The choice of the design matrix is equivalent to that of p vectors in N dimensional Euclidean space

Though in the nearly fifties of this centuary itself certain theoretical foundations of optimality of designs was done a cohesive approach for the practical utility of the same was yet to be worked out It was Kiefer (1959) who surveyed the till recent developments in the theory of determination of optimal experimental designs In due course he developed the idea of optimum experimental design on line with Wald s decision theory He thought of methods to verify whether or not the given design satisfy the optimality criteria and thereafter to construct designs satisfying the optimality criteria The usual Gauss Markoff set up was assumed

Conniffe and Stone (1975) studied some incomplete block designs of maximum efficiency For a given block size replication and number of treatments it is well known that a balanced design if one exists is of maximum efficiency But for all parameter combinations BIBD do not exist In such cases they suggested a certain type of Group Divisible (GD) designs and defined efficiency as the inverse ratio of the average variance of a treatment difference to that of a complete block design with the same replication

Eccleston and Russel (1975) obtained necessary and sufficient conditions for connectedness of a design by restricting the class of designs under consideration through the concept of orthogonality between pairs of factors The degree of restriction imposed was mild A design of n factors is said to be connected if the rank of the information matrix is equal to m n+1

Shah *et al* (1976) showed that in a two factor design which is optimal for one factor is optimal jointly for both the factors with respect to A D and E optimalities and that a linked block design is optimal for the estimation of treatment differences

The class of designs conjectured to be optimal among all designs was defined by John and Mitchell (1977) The optimum design was determined for each of the optimality criteria A D and E for the class of designs in which $v \leq 12$ $r<10$ and $v \leq b$ For $v>b$ the duals of optimal designs for $v \leq b$ was presented

Williams *et al* (1977) obtained two replicate resolvable designs satisfying a certain optimality criterion and compared with designs given by previous workers Recommendations were also given on the choice of efficient designs

Cheng (1978) studied optimal designs for the elimination of multi way heterogeneity The C matrix for the n way heterogeneity setting when $n>2$ was derived He showed that a

Youden hyper rectangle was E optimal and a Youden hypercube was A and D optimal

The problem of finding an optimal design for the elimination of one way heterogeneity when a balanced block design does not exist was studied by Cheng (1978) He proved a general result on optimality of certain asymmetrical designs and applied to the block design set up He showed that of a Group Divisible Partially Balanced Block Design (GDPBBD) had two groups and λ_2 λ +1 then it was optimal with respect to a very general class of criteria and if there was a GDPBBD with two groups and λ λ_2+1 then it was optimal with respect to another class of criteria He also obtained uniqueness of optimal designs

Shafiq and Federer (1979) extended the concept of balanced incomplete block designs to generalized N ary balanced block designs They developed some criteria to select designs in the class with smallest variance of a contrast

Some conditions were given by Cheng (1980) under which there was a regular graph design which is E optimal He showed that a GD design with λ_2 λ+1>1 and group size 2 was E optimal and the result did holds for a PBIBD with cyclic scheme λ_2 λ ±1 and v 5 He also found that if d was a BIBD a group divisible design with λ_2 λ +1 a group divisible design with λ λ_2+1>1 and group size 2 or a PBIBD with a cyclic scheme λ_2 λ +1 and v 5 then duals of d was also E optimal

Jacroux (1980) investigated the E optimality of regular graph designs within the various classes of proper block designs He derived several sufficient conditions for the existence of an E optimal regular graph design He showed that when a regular graph design exists whose minimum non zero eigen value was large enough then an E optimal regular graph design existed

The properties of E optimal designs within the various classes of proper block designs in which treatments were not replicated the same number of times was discussed by Jacroux(1980) He derived several sufficient conditions for designs to be E optimal within the classes considered and several methods of constiucting designs which satisfy these conditions

Jacroux and Seely (1980) gave two sufficient conditions for an incomplete block design to be (M S) optimal For binary designs the conditions were

(1) the elements in each row excluding the diagonal element of the association matrix differ by at most one and

(2) the off diagonal elements of the block characteristic matrix differ by at most one

They showed how the conditions can be utilized for non binary designs and that for blocks of size two in terms of the association matrix

Kageyama and Tsuji (1980) gave bounds on the latent root of the C matrix and the number of blocks for a variance balanced block design

Bechhofer and Tamhane (1981) developed the theory of optimal incomplete block designs for comparing several treatments with a control This class of designs was appropriate for comparing simultaneously $p \geq 2$ test treatments with a control treatment when observations were taken in incomplete blocks of common size $k < p+1$ They proposed a new general class of incomplete block designs that were balanced with respect to test treatments and called balanced test treatment incomplete block designs (BTIBD) Some methods of construction using generator designs were also discussed They described a procedure for making exact confidence statements for the multiple comparision problem

Cheng (1981) found that when the number of rows were equal to the number of columns a Generalised Youden Design (GYD) was optimal as long as the rows and columns together formed a balanced block design and is called a Psuedo Youden Design (PYD) A square GYD was also a PYD but not conversly A PYD was easier to construct and had the same efficiency as a GYD if they existed simultaneously He combined patchwork and geometric methods to construct a family of PYD s He constructed a 6x6 PYD with 9 varieties ie number of rows less than number of varieties which is never achieved by a square GYD

Constantine (1981) proved that when a BIBD or a GD design with λ_2 λ+1 extended by certain disjoint and binary blocks and a BIBD abridged by a certain number of such blocks were E optimal

Constantine (1982) showed that several families of PBIB designs with relatively few blocks were E optimal over the collection of all block designs This included the Partial Geometrics with two associate classes PBIB designs with λ 1 λ_2 0 and fewer blocks than varieties PBIB designs with triangular schemes of size n λ 0 λ_2 1 and block size greater than or equal to n/2 PBIB designs with schemes based on v varieties with λ 0 λ_2 1 $k>\sqrt{v}$ The duals of these design were also E optimal He observed that in certain settings Partial Geometrics were the unique E optimal designs

E optimality of several different types of block and row column designs that have unequally replicated treatments was proved by Jacroux (1982) He showed that such unequally replicated designs could maximize the information on treatment effects without wasting units in the case of elimination of heterogeneity in one or two directions

Constantine (1983) showed that in any block design containing a set of treatments and a control the average variance of the best linear unbiased estimates of the elementary contrasts with the control were proportional to the trace of the inverse of a suitable principal minor of the information matrix A BIB design with blocks reinforced by the control was then proved to minimise the average variance over all the designs which had the control replicated once in every block This result was extended to the setting of two way elimination of heterogeneity when the control appeared bk times and any other variety appeard rk times in each row

Jacroux (1983) considered the determination and construction of E optimal block designs within various classes of designs having v treatments arranged in b blocks of size k Several sufficient conditions were derived for a design to be E optimal within these classes He used the sufficient conditions to investigate E optimality of designs in the presence of a control and to establish the E optimality of certain designs which were obtained through augmentation

The problem of finding optimum incomplete block designs for comparing p test treatments with a control was studied by Majumdar and Notz (1983) BIBDs were found to be D optimal A optimal and E optimal designs were also obtained

Hedayat and Majumdar (1985) obtained A optimal designs for comparing v test treatments with a control in b blocks of size k each They gave several series of A optimal designs whose parameters were in the range $2<k<8$ $k<v\leq 30$ $v\leq b\leq 50$ They studied extensively A optimal designs in blocks of size two through a combination of theoritical results and numerical investigations They also reported that several families of BIBD in the test treatments augmented by t replications of a control in each block

were A optimal As a particular case they showed that these designs with t 1 were optimal whenever $(k\ 2)^2 + 1<v<(k\ 1)^2$ irrespective of the number of blocks

Jacroux (1985) reported that under certain conditions the E and MV optimal group divisible designs having parameters $\lambda\ \lambda_2+1$ and whose corresponding C matrix had maximal trace can be used to construct E and MV optimal row column designs where heterogeneity were to be eliminated in two directions and where v treatments were being tested in b columns and k rows

Sathe and Bapat (1985) investigated that if some blocks were deleted from a BIBD then under certain conditions on the parameters the resulting design was E optimal

Several methods of constructing E optimal block designs having blocks of unequal size were suggested by Lee and Jacroux (1987) They extended the results concerning E optimality of designs which could be obtained by augmenting BIBDs and group divisible designs with blocks of equal size to the case of augmenting these designs with blocks of unequal size

Stufken (1987) studied the problem of comparing test treatment with a control in a proper block design He gave conditions on the parameters of both R type and S type designs that guarantee their A optimality and demonstrated how these conditions could be used to obtain families of A optimal designs

Gupta and Singh (1990) studied the E optimality of block designs within sub classes of competing designs with varying replications and unequal block sizes They obtained several sufficient conditions for a design to be E optimal within the classes considered They also got some methods of constructing E optimal designs in these sub classes satisfying the sufficient conditions

Jacroux (1990) studied the problem of optimally comparing a set of test treatments to a set of s controls under a 0 way elimination of heterogeneity model

E optimality of row column designs over a certain class of connected designs were studied by Singh and Gupta (1991) They gave some methods of constructing E optimal row column designs

Bhaumik (1993) showed that in the presence of linear trend an A optimal BIBD was Cheng s type 2 Υ_f optimal He provided an algorithm for the construction of an A optimal BIBD in the presence of a linear trend k is even and r is odd

Das (1993) reported that under certain conditions a group divisible design having parameters λ_2 λ+1 was E optimal and could be used to construct E optimal block and row column designs with unequal replicates to handle experimental situations in which heterogeneity were to be eliminated in either one or two directions

Materials and Methods

MATERIALS AND METHODS

A design is usually associated with experiments conducted to verify the superiority of one treatment over the other and to draw inferences regarding the same This is achieved by defining a contrast of treatment effects

<u>Incidence matrix</u>

Let there be v treatments arranged in b blocks such that the j th block contains k_j experimental units and the i th treatment appears r_i times in the entire design i 1 2 v j 1 2 b Underlying any block design there exists a matrix N of order vxb whose (i j) th element n_{ij} (≥ 0) is the number of times i th treatment appears in the j th block The matrix N is called the incidence matrix of the design and the matrix NN' is called the concurrence matrix

Define $\mathbf{k}$ $(k_1 \;\; k_2 \qquad k_b)$

$\mathbf{r}$ $(r_1 \;\; r_2 \qquad r_v)$

K diag $(k_1 \;\; k_2 \qquad k_b)$

R diag $(r_1 \;\; r_2 \qquad r_v)$

The matrix C R $NK^{1}N$ is called the information matrix or C matrix of the design

Connectedness

A block design is said to be connected if all the elementary treatment contrasts are estimable The property of connectedness is related to the rank of the information matrix C To determine whether a given matrix is connected or not following simple rules are used (Chakrabarti (1963))

(1) If every element of C is non zero the design is connected

(2) If C contains a row (or column) of non zero elements the design is connected

(3) Consider the last row of C and find the non zero elements of this row If at least one element in any row above these elements is non zero the design is connected

Balancing

There are two types of balancing

(1) Variance balanced designs

A block design is said to be variance balanced if it permits the estimation of all estimable normalized treatment contrasts with the same variance A necessary and sufficient condition for this is that all non zero eigen values of the information matrix are equal

(2) Efficiency balanced designs

A design is said to be efficiency balanced if every contrast of treatment effect is estimated with the same efficiency factor

Considering the usual intra block model let v b,k denote respectively the number of treatments number of blocks and block size

Let the inference problem be specified as

$$P \quad \eta \quad L\Upsilon \quad \text{with } LJ \; 0$$

where L is a $p \times v$ matrix of known elements Let D_p denote the class of all connected block designs For any design $d \in D_p$ let V_d denote the dispersion matrix of $\hat{\eta}$ using d The following definitions relate to three important optimality criteria

<u>A optimality</u>

A design $d^* \subset D_p$ is said to be A optimal in D_p if

$$\text{trace}(V_d{}^*) < \text{trace}(V_d)$$

for any other design $d \in \mathcal{D}_p$

The A optimality criterion chooses that design for which the average variance of the estimates of all normalized treatment contrasts is the least

D optimality

A design $d^* \in \mathcal{D}_p$ is said to be D optimal in $\mathcal{D}_p$ if

$$\det (V_d{}^*) < \det (V_d)$$

for any other design $d \in \mathcal{D}_p$ D optimality criterion chooses the design for which the generalized variance of the estimated parameter vector η is minimum

E optimality

A design $d^* \in \mathcal{D}_p$ is said to be E optimal in $\mathcal{D}_p$ if

$$\max \lambda d^* \leq \max \lambda d$$

where $\max \lambda d^*$ ($\max \lambda d$) is the largest eigen value of $V_d{}^*(V_d)$ for any other design $d \in \mathcal{D}_p$

E optimality criterion relates to the minimization of the maximum variance of estimates of all normalized treatment contrasts, $\hat{\eta}$

Let r and λ denote the greatest integers not exceeding bk/v and r(k 1)/v 1 respectively Let d $\subset D$(v b k) have C matrix C_d and incidence matrix N_d with row sums r_{d1} r_{d2} r_{d3} r_{dv} Then $z_{d1} \leq$ r(k 1)/{(v 1)k} If the entries of $N_d N_d$ (λ_{dij}) are such that λ_{dij} > r(k 1)/(v 1) for all $i \neq j$ then

z_{d1} r(k 1)v/(v 1)k then d is E optimal in D (v b k) Let m< λ be a nonnegative integer and let s be the smallest positive integer such that (r s)(k 1)/(v 1) < m+1 If λ_{dij} >m+1 for all $i \neq j$ then an E optimal design d $\in D$(v b k) must be such that $r_{di} \geq$ r s+1 for i 1 2 v (Jacroux 1980)

In any block design d $\subset D$(v b k) the following inequalities hold

(1) $z_{d1} \leq$ v/(v 1) min $_{(1<i<v)}$ (r_{d1} 1/k Σn_{dij}^2)

(2) $z_{d1} \leq$ {(k 1)vr_{d1}}/{k(v 1)}

(3) $z_{d1} \leq$ min $_{2<n<v}$[(k 1)/kn r_{di}+2/nk(n r) λ_{dij}]

<u>Balanced</u> <u>Treatment</u> <u>Incomplete</u> <u>Block</u> <u>Designs</u>

Let the treatments be indexed by 0 1 p with 0 denoting the control treatment and 1 2 p denoting the p $\geq$ 2 test treatments Let k < p+1 denote the common size of each block and b the number of blocks N kb is the total number of experimental units If treatment i is assigned to the h th plot of the j th block ($0 \leq i < p$ $1 < h < k$ $1 < j < b$) let y_{ijh} denote the corresponding random variable We assume the usual additive linear model assuming there is no treatment block interaction

$$Y_{ijh} \quad \mu + \alpha_i + \beta_j + e_{ijk}$$

with $\sum_{i=0}^{p} \alpha \;\; 0$ $\sum_{j}^{b} \beta_j \;\; 0$ e_{ijh} are assumed to be $N(0 \;\; \sigma^2)$ random variables

Consider a class of designs for which

$\mathrm{var}(\alpha_0 \;\; \alpha_i) \qquad t^2 \sigma^2$ $(1 < i < p)$ and

$\mathrm{corr}(\hat{\alpha}_0 - \hat{\alpha} \;\; \hat{\alpha}_0 - \hat{\alpha}_{i_2}) \qquad \rho$ $(1 < i_1 < i_2)$ the parameters t and ρ depend on the design employed

For given (p k b) consider a design with the incidence matrix $\{r_{ij}\}$ where r_{ij} is the number replication of the i th treatment in the j th block

Let $\lambda_{i_1 i_2} \quad \sum_{j=1}^{b} r_{i_1 j} r_{i_2 j}$ denote the total numberr of times that the i_1 th treatment appears with the i_2 th trxeatment in the same block over the whole design ($i_1 \neq i_2$ $0 < i_1$ $i_2 < p$) Then the necessary and sufficient conditions for a design to be balanced treatment incomplete block design are

$$\lambda_0 \quad \lambda_{02} \qquad \lambda_{0p} \quad \lambda_0 \qquad \text{and}$$

$$\lambda_{12} \quad \lambda_{13} \qquad \lambda_p \;\; p = \lambda_1$$

That is each test treatment must appear with the control treatment in the same block λ_0 times over the design and each test treatment must appear with every other test treatments the same ~~total~~ number of times

Let T_i be the sum of all observations of the i th treatment $(0<i<p)$ and B_j be the sum of all observations in the j th block $(1<j\leq b)$

Define $B_i^{*} \quad \sum_{j}^{b} r_{ij} B_j$ and

$$Q_i \quad kT_i \quad B_i^{*} \quad (0\leq i\leq b)$$

Total SS $\sum_{0}^{p}\sum_{j}^{b} Y_{ij}^{2} \quad G^2/n$

Block SS $\sum_{j}^{b} B_j^{2}/p \quad G^2$

where G $\sum_{0}^{p}\sum_{j}^{b} y_{ij}$

$$\text{Treat SS (adj)} \quad \frac{Q_0^{2}\{(p\ 1)(\lambda_0^{2}+\lambda^{2})+(p^{2}+3)\lambda_0\lambda\}}{k(p+1)^{2}\lambda_0^{2}k(\lambda_0+p\lambda)} + \frac{\sum^{p} Q_i^{2}}{k(\lambda_0+p\lambda)}$$

Sum of squares due to error is obtained by subtraction

Standard error for testing treatment differences $\sqrt{t^{2}\sigma^{2}}$

where $t^2 \quad k(\lambda_0+\lambda_1)/\{\lambda_0+p\lambda\}$

The data for numerical illustration are taken from the All India Co ordinated Vegetable Improvement Project (AICVIP) on watermelon cucumber etc

Results and Discussion

RESULTS AND DISCUSSION

The results of the study are discussed in the following heads

4 1 Practical utility of optimality criteria

4 2 Balanced treatment incomplete block designs (BTIBD)

4 2 1 Optimality of BTIB designs

4 1 Practical utility of optimality criteria

A direct application of the optimality criteria is based on the dispersion matrix of all possible elementary contrasts This is difficult due to the fact that the comparison is only a post evaluation of the adoption of the design after the experiment has been completed It could be better if the optimality criteria based on the co efficient matrix (C matrix) is used as the experiment need be conducted only after sorting out a design from a family based on some optimality criteria that are proposed to be used for a just discrimination The success of the optimality criteria are its power of dicrimination

The practical utility and relevance of the various optimality criteria were evaluated Usually the efficiency of a design is compared with any other design based on the computation of relative efficiency In the comparison of a BIBD relative to

RBD the efficiency factor E $\lambda v/rk$ is considered to be the decisive factor Though this might be the situation the average variance when computed as detailed below revealed that the eigen values of the C matrix was most determinantal as an optimality criterion

Let D(v b k) be a class of connected block designs and d D Let z_1 z_2 $z_{v\ 1}$ be the non zero eigen values of the C matrix of the design and x_1 x_2 $x_{v\ 1}$ the corresponding eigen vectors Let

$$P \quad [x_1 \quad x_2 \qquad x_{v\ 1}]$$

Then P t denotes a set of (v 1) orthonormal treatment contrasts each one of which is estimable Let P t be the best linear unbiased estimates of P t where t C—Q is a solution of the normal equations Ct Q and Q is the vector of adjusted treatment totals The dispersion matrix of P t is given by

$$D\ (P\ t) \qquad P\ C^{-}P$$

$$P[\quad z_1 \qquad x_1 x_1\ ]P$$

$$\sigma^{2} A^{\ 1}$$

where $A^{\ 1}$ diag $(z_1^{\ 1}\ z_2^{\ 1} \qquad z_{v\ 1}^{\ 1})$

The average variance of the (v 1) orthonormal contrasts x_1 t 1 1 2 v 1 is given by

A V $\sigma^2(\sum^{v} z_i^{-1})/(v-1)$

But the estimate of variance (σ^2) is not the same in RBD and BIBD As the number of plots within a block is reduced the natural phenomenon that error variance gets reduced with decreased number of plots within a block holds good

The above proposition was examined through a numerical illustration

Consider an RBD for comparing 5 treatments in 5 blocks The incidence matrix of the design is a 5 x 5 matrix with all its entries unity

R diag (5 5 5 5 5)

The information matrix is given by

$$C_d = \begin{bmatrix} 4 & 1 & 1 & 1 & 1 \\ 1 & 4 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 4 & 1 \\ 1 & 1 & 1 & 1 & 4 \end{bmatrix}$$

If the experiment was laid out in BIBD using 5 blocks containing 4 plots each the incidence matrix is given by

$$N = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$R = \text{diag}\ (4\ 4\ 4\ 4\ 4)$$

The information matrix of this design is given by

$$\begin{bmatrix} 3 & 3/4 & 3/4 & 3/4 & 3/4 \\ 3/4 & 3 & 3/4 & 3/4 & 3/4 \\ 3/4 & 3/4 & 3 & 3/4 & 3/4 \\ 3/4 & 3/4 & 3/4 & 3 & 3/4 \\ 3/4 & 3/4 & 3/4 & 3/4 & 3 \end{bmatrix}$$

The non zero eigen values of the C matrix in case of RBD is 5 with multiplicity 4 and the average variance of the 4 orthonormal contrasts is $\sigma^2_R/5$ where σ^2_R is the estimate of of variance in RBD In BIBD the non zero eigen values are 15/4 repeated four times and in this case the average variance was $4/15\ \sigma^2_B$ where σ^2_B is the estimate of variance in BIBD These are given in table 1

Table 1 Average variance of RBD and BIBD for comparing 5 treatments on 5 blocks

	RBD	BIBD
v	5	5
b	5	5
r	5	4
k	5	4
z_1	0 5 5 5 5	0 15/4 15/4 15/4 15/4
A V	$\sigma^2_R/5$	$4\sigma^2_B/15$

If σ_B^2 is less when compared with σ_R^2 then the average variance of a full set of orthonormal treatment contrasts from a BIBD will not exceed that from a RBD The most determinental factor that are contributing to the average variance of the treatment contrasts is the non zero eigen value of the C matrix As the average variance is proportional to the harmonic mean (HM) of non zero eigen values the minimum non zero eigen value plays the crucial role To put in otherwords the minimal non zero eigen value contributes to the maximum variance The E optimality criterion is exactly based on this aspect__ minimisation of maximum variance

The A optimality criterion is based on the principle of minimisation of average variance where as the D optimality criterion is based on the principle of minimisation of generalised variance Of these three optimality criteria the E optimality criterion may be regarded as of more practical value as it is basedon the minimax principle

As the order of the C matrix increases the extraction of the eigen values is diffiult A more simpler procedure was derived and has been presented as a lemma

Lemma 4 1

Let d $\subset D(v\ b\ k)$ and C_d be the information matrix A design $d^* \in D(v\ b\ k)$ is said to be A optimal over d if

$$\text{Trace } C_d^* \geq \text{Trace } C_d$$

Proof

Let $z_{d1} < z_{d2} \leq \quad < z_{d(v\ 1)}$ be the non zero eigen values of the C_d and $z_{d1}^* < z_{d2}^* < \quad < z_{d(v\ 1)}^*$ be the non zero eigen values of C_d^* The average variance of (v 1) orthonormal contrasts is given by

$$A\ V \qquad \sigma^{2}\sum^{v} \theta/(v\ 1)$$

where θs i 1 2 v 1 are the eigen values of the relevant information matrix The average variance is proportional to the harmonic mean (H M) of the eigen values Trace of a matrix is the sum of its diagonal elements which is again equal to the sum of eigen values of the matrix

$$\text{Trace } C_d \qquad \sum^{v} z_{di}$$

$$\text{Trace } C_d^* \quad \sum^{v} z_{di}^*$$

We have

$$A\ M \geq G\ M \geq H\ M$$

As A M > H M the same relation with respect to H M holds with respect to A M The only difference is that the comparisonis based on the reduction in average variance of all elementary contrasts rather than on the comparison of the minimisation of maximum variance among the treatment contrasts When the arithmetic mean of the eigen values is used the average variance gets reduced and hence it can be taken as an optimality criterion

For illustration consider the class of designs $D(4\ 4\ 3)$ Let $d_1\ d_2 \subset D(4\ 4\ 3)$ The incidence matrix of d_1 and d_2 are given below

$$N_{d1} \quad \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \qquad N_{d2} \quad \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

The two designs are connected

$$R_1 \quad \text{diag} \quad (3\ 3\ 3\ 3)$$

$$R_2 \quad \text{diag} \quad (3\ 3\ 3\ 3)$$

$$C_{d1} \quad \begin{bmatrix} 2 & 2/3 & 2/3 & 2/3 \\ 2/3 & 2 & 2/3 & 2/3 \\ 2/3 & 2/3 & 2 & 2/3 \\ 2/3 & 2/3 & 2/3 & 2 \end{bmatrix}$$

$$C_{d2} = \begin{bmatrix} 4/3 & 1 & 1/3 & 0 \\ 1 & 2 & 2/3 & 1/3 \\ 1/3 & 2/3 & 2 & 1 \\ 0 & 1/3 & 1 & 4/3 \end{bmatrix}$$

Trace C_{d1} 8

Trace C_{d2} 20/3

As trace C_{d1} > trace C_{d2} d_1 is A optimal over d_2

4 2 Balanced Treatment Incomplete Block Designs

In agricultural experiments there may be situations where a set of treatments are to be compared with a standard treatment called control treatment The usual recommendation in such cases is an RBD with the control treatment replicated in all blocks The primary objective could be the comparison of the test treatments with the control the comparison among the test treatments being only secondary Practically if an RBD is laid out it amounts extra usage of plots the block size thus being in an increasing trend The eficiency of BTIB designs compared to RBD for comparing test treatments with a control was examined with the help of numerical data

For comparing 4 treatments with a control the incidence matrix of BTIBD is given by

1	1	1	1
1	0	1	1
1	1	0	1
1	1	1	0
0	1	1	1

Here control is replicated in every block and each test treatment appear with the control treatment 3 times ie $_0$ 3 Also the test treatments is forming a BIBD with λ - 2 The standard error for comparing test treatments with a control was found to be 4 8153 When the data was analysed as RBD the standard error was 5 8652 The advantage here was that the block size was reduced from 5 to 4 and 16 plots in total were required Total number of plots in case of RBD is 20 Thus we can have more efficient estimates of treatment effects with lesser number of plots (Table 2 1) Tables 2 2 and 2 3 are reassertion of the above results The analysis of variance tables are given in appendix 1 a 1 b 1 c

With a careful permutation of treatments in the blocks even for a design obtained in such a way that the balance of a design is not upset with regards to the comparison of test

Table 2 1 Comparison of BTIBD with RBD

	Design	
	RBD	BTIBD
p	4	4
b	4	4
k	5	4
Number of plots	20	16
S E	5 8652	4 8153

Table 2 2 Comparison of BTIBD with RBD

	Design	
	RBD	BTIBD
p	4	4
b	4	4
k	5	4
Number of plots	?0	16
S E	1 9397	1 832

Table 2 3 Comparison of BTIBD with RBD

	Design	
	RBD	BTIBD
p	4	4
b	4	4
k	5	4
Number of plots	20	16
S E	1 8136	1 5928

treatments with control treeatments the same objective could be achieved

For comparing 3 treatments with a control treatment in 4 blocks of size 3 each the incidence matrix of the BTIBD is as follows

$$N = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Here the control treatment is not replicated in every block The design is balanced with respect to every treatments and thus forming a BIBD Even then the standard error for comparing test treatments with control was 11 82 where as the same using RBD was 16 183 Here also a reduction of 4 plots is obtained with an increase in precision in estimating the treatment effects (Table 3) Analysis of variance table is given in appendix 2

Thus BTIB designs can be used to compare $p > 2$ test treatments with a control more precisely As it requires lesser number of plots than RBD there is saving of experimental

Table 3 Comparison of BTIBD with RBD

	Design	
	RBD	BTIBD
p	3	3
b	4	4
k	4	3
Number of plots	16	12
S E	16 18	11 82

material and resources Thus it is more advantageous to use BTIBD for comparing test treatments with a control

4 2 1 Optimality of BTIB designs

As a BTIBD is balanced more or less the same way as a BIBD all properties of optimality holds good in the case of BTIBD also

Consider the class of designs ?(5 4 4) Let d_1 be a BTIBD and d_2 any other design in this class The incidence matrices of d_1 and d_2 are

$$N_{d1} \quad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \qquad N_{d2} \quad \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

The information matrices obtained are as follows

$$C_{d1} \quad \begin{bmatrix} 3 & 3/4 & 3/4 & 3/4 & 3/4 \\ 3/4 & 9/4 & 1/2 & 1/2 & 1/2 \\ 3/4 & 1/2 & 9/4 & 1/2 & 1/2 \\ 3/4 & 1/2 & 1/2 & 9/4 & 1/2 \\ 3/4 & 1/2 & 1/2 & 1/2 & 9/4 \end{bmatrix}$$

$$C_{d2} \quad \begin{bmatrix} 9/4 & 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 9/4 & 1/2 & 3/4 & 1/2 \\ 1/2 & 1/2 & 9/4 & 3/4 & 1/2 \\ 1/2 & 3/4 & 3/4 & 5/2 & 1/2 \\ 3/4 & 1/2 & 1/2 & 1/2 & 9/4 \end{bmatrix}$$

The eigen values of C_{d1} and C_{d2} and trace are given in table 4 The minimum non zero eigen value of both the C matrices are equal Therefore both the designs are equally optimal considering E optimality As trace C_{d1} > trace C_{d2} BTIBD is A optimal over d_2

4 3 E optimality of extended E optimal designs

In certain experiments comparisons among test treatments are needed with more precision along with the comparison of test treatments with a control This necessitates in additional replication of test treatments The following lemma shows that whenever certain blocks which containing only test treatments are added to BTIBD then under certain conditions it is E optimal

Table 4 Optimality of BTIB Designs

	C_{d1}	C_{d2}
Eigen values	0 11/4 11/4 11/4 15/4	0 3 3 11/4 11/4
Trace	12	23/2

Lemma 4 3 1

When a BTIBD is augmented with m $(0 < m < v/k)$ blocks consiting of only test treatments then such an augmented design continues to be optimal in the family of designs D (v b+m k)

Proof

Let d be an arbitarary design in D (v b+m k) and d^* be a BTIBD extended by m blocks which consists of only test treatments Without loss of generality we can say that the control treatment is replicated the least number of times say r_1 Also only the test treatments are in the m added blocks and $mk < v$ Using the known result that the upper bound of the least non zero eigen value of the information matrix of any design $d \subset D(v\ b\ k)$ is $\{(k\ 1)vr_1\}/\{k(v\ 1)\}$ we have

$$z_{d1} < \{(k\ 1)vr_1\}/\{k(v\ 1)\}$$

But $C_{d1}{}^*$ has the least eigen value which is given by

$$z_{d1}{}^* \quad \{(k\ 1)vr_1\}/\{k(v\ 1)\}$$

Thus $z_{d1} \leq z_{d1}{}^*$ and hence d^* is E optimal over d Therefore BTIBD augmented with certain number of blocks is E optimal

We may conclude that incomplete balanced designs in general are also very much viable for practical experimentation

Summary

SUMMARY

Block designs are used for experiments where it is necessary to eliminate heterogeneity at least in one direction Let D (v b k) denote the class of connected block designs From among the class of design it is desirable to have a design which will estimate the elementary treatment contrasts with maximum precision The criteria for judgement are the optimality criteria

The optimality criteria are based on the dispersion matrix of all possible elementary contiasts As this is a post adoption evaluation technique it is better to have a technique that will sort out a design before the experiment is conducted

The practical utility and relevance of various optimality criteria were evaluated Comparative evaluation of two designs is based on efficiency But a better judgement factor was found to be the non zero eigen values of the C matrix as the average variance of possible elementary contrasts is related with the eigen values of the C matrix For numerical illustration a BIBD was compared with RBD for comparing 5 treatments in 5 blocks The average variance of the 4

orthonormal contrasts was $\sigma^2_R/5$ in the case of RBD and $4/15\ \sigma^2_B$ in BIBD where σ^2_R and σ^2_B are estimates of variance of RBD and BIBD respectively Usually σ^2_B is less comapared to σ^2_R Then BIBD may be preferred due to the lesser number of plots within a block as the number of plots within a block reduces precision increses

As the order of the C matrix increases the extraction of eigen values becomes difficult So an optimality criteria based on the trace of the information matrix was developed The design $d^* \in \mathcal{D}$ (v b k) is A optimal over d $\mathcal{D}$ (v b k) if

$$\text{trace } C_d^* \geq \text{trace } C_d$$

The average variance of (v 1) orthonormal contrasts is given by

$$A \quad V \qquad \sigma^2 \sum^{v} \theta \ /(v \ 1)$$

where θ_i s i 1 2 v 1 are the eigen values of the corresponding C matrix The average variance is inversly propoitional to the harmonic mean (H M) of the eigen values As arithmetic mean (A M) $\geq$ harmonic mean the average variance gets reduced when the arithmetic mean of eigen values is used Trace is equal to the sum of the eigen values of the C Matrix Thus the above condition can be used as an A optimality criterion because the average variance gets reduced which is the condition for A optimality

Numerical illustration shows that BIBD is A optimal over the class of connected designs in $D(4\ 4\ 3)$

When the tratments are to be compared to a control or a set of controls the primary objective could be the comparison of the test treatments with the control In most of these cases RBD is used with the control replicated in all the blocks This can also be done using balanced treatment incomplete block designs (BTIBD) When BTIBD was compared with RBD BTIBD was found to be more efficient Yield data collected from All India Co ordinated Vegetable Improvement Project (AICVIP) were used for illustration

When the comparison between test treatments are also needed with maximum precision along with the comparison of test treatments with the control then also BTIBD can be used This is done by augmenting certain blocks containing only the test treatments to the given BTIBD The augmented design is then E optimal BTIBD was also found to be A optimal

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Appendices

APPENDIX 1 a

Analysis of variance table for RBD

Source	d f	S S	M S	F
Total	19	2360 152		
Blocks	3	170 716	56 905	< 1
Treatments	4	1363 8133	340 9533	4 956
Error	12	825 6228	68 802	

Analysis of variance table for BTIBD

Source	d f	S S	M S	F
Total	15	1229 3073		
Blocks	3	273 7481	91 2494	2 385
Treatments	4	649 4877	162 3719	4 24
Error	8	306 0715	38 259	

APPENDIX 1 b

Analysis of variance table for RBD

Source	d f	S S	M S	F
Total	19	3024 55		
Blocks	3	50 95	16 98	2 256
Treatments	4	2883 30	720 825	95 791
Error	12	90 30	7 525	

Analysis of variance table for BTIBD

Source	d f	S S	M S	F
Total	15	2649 937		
Blocks	3	188 187	62 729	11 32
Treatments	4	2417 411	604 353	109 041
Error	8	44 338	5 542	

APPENDIX 1 c

Analysis of variance table for RBD

Source	d f	S S	M S	F
Total	19	307 53		
Blocks	3	29 515	9 838	1 495
Treatments	4	199 07	49 767	7 565
Error	12	78 94	6 578	

Analysis of variance table for BTIBD

Source	d f	S S	M S	F
Total	15	212 58		
Blocks	3	23 11	7 703	1 840
Treatments	4	155 98	38 995	9 315
Error	8	33 488	4 186	

APPENDIX 2

Analysis of variance table for RBD

Source	d f	S S	M S	F
Total	15	54282 438		
Blocks	3	967 19	322 397	< 1
Treatments	3	48601 190	16200 400	30 93
Error	9	4714 058	523 784	

Analysis of variance table for BTIBD

Source	d f	S S	M S	F
Total	11	28695 667		
Blocks	3	322 336	1074 112	5 76
Treatments	3	24540 917	8180 310	43 87
Error	5	932 414	186 480	

OPTIMALITY OF BLOCK DESIGNS USED IN ONE WAY ELIMINATION OF HETEROGENEITY

By

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ABSTRACT OF A THESIS

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ABSTRACT

Block designs are usually used in experiments where it is important to eliminate heterogeneity at least in one direction From the class of designs it is desired to choose a design which will estimate the elementary treatment contrasts with maximum precision

The optimality criteria are based on the dispersion matrix of all possible elementary contrasts The A optimality criterion based on the information matrix was derived

Usually for comparing test treatments with a control RBD is used with the control trreatment replicated in all blocks The same objective could be achieved by using Balanced Treatment Incomplete Block Designs (BTIBD) BTIBD was found to be more efficient than RBD with the control treatment replicated in all blocks Optimalities of BTIBD were also examined

When a BTIBD was augmented with certain number of blocks such that the augmented blocks contains only the test treatments the resulting design was found to be E optimal