A STUDY OF HALF-SIB CORRELATIONS AND PARENT-OFFSPRING CORRELATIONS UNDER HALF-SIB MATING SYSTEM

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DECLARATION

I hereby declare that the present thesis entitled 'a study of half-sib correlations and parent-offspring correlations under half-sib mating system,' is a bonafide record of research work done by me during the course of research and that the thesis had not previously formed the basis for the award to me of any degree, diploma, fellowship, associateship or other similar title, of any other University or Society.

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3rd February, 1983.

CERTIFICATE

Certified that the thesis herewith submitted contains the results of bonafide research work carried out independently by Smt. R. SHAILAJA, under my guidance and supervision. No part of the work embodied in this thesis has been submitted earlier for the award of any degree, fellowship, or associateship to her.

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INTRODUCTION AND REVIEW OF LITERATURE

INTRODUCTION
AND
REVIEW OF LITERATURE

The resemblance between relatives is one of the basic genetic phenomena displayed by metric characters, and the degree of resemblance is a property of the character that can be determined by relatively simple measurements made on the population without special experimental technique. The degree of resemblance provides the means of estimating the amount of additive genetic variance, and it is the proportionate amount of additive variance (i.e., heritability) that chiefly determines the best breeding method to be used for improvement.

Genetic correlation between relatives gives a consistent measure of resemblance between them and hence can find a lot of application in genetic basis of selection. The correlation between common relatives such as full sib or parent-offspring takes a value of half and the correlation between half-sibs takes a value of one fourth under an equilibrium random mating population. But when regular systems of inbreeding is practiced, the correlation coefficient between the relatives increases with the degree of inbreeding.

Different authors have proposed apparently very different methods of obtaining the genotypic correlations between relatives under different inbred systems. One among them is a straightforward but long procedure by which the absolute frequencies of the various combinations of the stated relatives in the general population are first found out and then the correlation is calculated from such a 'correlation table'. The correlation between full-sibs under different generations of full-sib mating system have been obtained by Li ('Population Genetics', Page:119) by such a long process. The procedure of obtaining the frequencies of uncle-nephew or first cousin combinations is entirely too tedious even with the help of matrix notations (Hogben, 1933). Thus the algebraic methods become cumbersome when used for the more complex or irregular inbreeding systems or for more than one pair of genes.

An entirely different technique developed by Wright (1921) utilises the concept of path coefficients. This method gives the required correlation coefficient almost instantly, once the relationship is specified, but does not give us any information about the frequencies of the various combinations of the relatives in the population. Moreover, one has to be familiar with the mathematical therems concerning the path coefficients before he can use them to derive the required correlations. The factors defined by Fisher are precisely the same as the path coefficients defined by Wright.

The full specification of all the properties of populations which have undergone a specified number of generations of inbreeding according to some system of consanguineous mating has not been made by any of the above authors.

Fisher (1949) developed a generation matrix theory by which all the properties can, with sufficient labour, be elucidated. The method provides a simple and flexible method of working out the frequencies of different types of mating under regular systems of inbreeding. The method was first presented in the literature apparently by Bartlett and Haldane. Though the progress towards heterozygosis for sex-linked character was studied by Fisher and Haldane (1937, 1955), they gave only a general treatment of the subject.

Kempthorne (1955) calculated the correlation of parent-offspring pairs and full-sib pairs in generations of full-sib mating by making use of the generation matrix theory. Horner (1956) worked out the correlation of parent-offspring pairs and fullsib pairs in generations of parent-offspring mating. But both these workers delt with autosomal genes only. Korde (1960) worked out the correlation between relatives for a sex-linked character under full-sib mating by making use of the generation matrix theory.

The generation matrix method is based on the primitive concepts of the genotype and the results of Mendelian segregation. The method gives the mating types in an arbitrary generation arising from an arbitrary population by a regular system of inbreeding. If the frequencies of mating types are arranged as a column matrix, say, $\underline{f}$, and generations are denoted by subscripts in parenthesis, then $\underline{f}^{(n)} = \underset{\sim}{A}\,\underline{f}^{(n-1)}$ where $\underset{\sim}{A}$ is the generation matrix. Hence it follows that

$$\underline{f}^{(n)} = \underset{\sim}{A}^{n}\ \underline{f}^{(0)}$$

This shows that the frequencies in the n^{th} generation can be worked out if one knows the matrix $\underset{\sim}{A}$ as well as the initial vector $\underline{f}^{(0)}$. By this method the joint distribution of pairs of relatives at any generation of a specified system of mating is obtained and thus the correlation is worked out directly from the two-way table of the relatives, known as the "correlation table".

The method of stochastic process and its final reduction to some basic matrices, introduced by Li and Sacks (1954), under the name I.T.O. method, provides the frequencies of various relative pairs and their correlation in a very simple manner. This method is restricted to a single locus with two alleles, under random mating.

The stochastic matrices, $\underset{\sim}{I}$, $\underset{\sim}{T}$ and $\underset{\sim}{O}$ are matrices of conditional probabilities. From these three basic matrices the matrix of conditional probabilities for bilineal or unilineal relatives can be worked out. Using them "the correlation table" and the correlation between relatives can be easily worked out.

Eventhough Fisher, Haldane and Li derived the generation matrix for full-sib mating with sex-linked genes, it is in fact Korde and George who made use of this generation matrix technique in studying the inbreeding systems. George (1974) conducted a detailed study of parent-offspring and full-sib correlations separately under full-sib mating and parent-offspring mating systems, both for autosomal as well as sex-linked genes. Two methods viz., the I.T.O method, employing stochastic matrices, as well as generation matrix methodology, have been studied. The I.T.O method applicable to the single locus with two alleles is extended to multiple allele case under random mating. Further, he found that in general, the I.T.O method is not applicable to inbred populations. However, for autosomal genes and in the case of parent-offspring mating system, the joint distribution of the parent-offspring relationship could be expressed in terms of T and F (suitably defined) matrices. In the case of sex-linked genes the I.T.O method was found to be applicable for finding the joint distribution and correlation coefficient for brother - brother and father-son relationships, both for full-sib as well as parent-offspring mating system. Further, a general theory for obtaining the correlation between one parent and k offspring as well as the correlation between both the parents and k offspring under a given system of mating have been developed both for autosomal as well as sex-linked genes.

All these authors confined to the two systems of inbreeding namely full-sib mating and parent-offspring mating, for autosomal as well as sex-linked characters. Inspite of the fact that the rate of increase in homozygosis is faster in full-sib mating than in half-sib mating, full-sibs are not available in large numbers and sometimes may not be available at all. In livestock in which a great number of females may be mated to one sire, half-sib mating is perhaps the most rapid practical method of fixing characters, for the rate of increase in homozygosis is fairly rapid.

Half-sibs are individuals having one parent in common and the other parent different. A group of half-sibs is therefore the progeny of one individual mated to a random group of the other sex and having one offspring by each mate.

As the generation matrix in the case of half-sibs is not easily possible, a detailed study of the derivation of the joint distribution of half-sib pairs and the correlations therefrom has not been conducted so far. However, Li, in his book entitled 'Population Genetics, (1956)' have established a general formula to determine the correlation coefficient between different relative pairs such as full-sibs, parent and offspring, half-sibs in different generations of the specified mating system, from the corresponding inbreeding coefficient. It is as given below:

$$m = \frac{1 + 6F' + F''}{4\,(1 + F')} \qquad (1)$$

where

F = inbreeding coefficient in the n^{th} generation

F' = inbreeding coefficient in the $(n-1)^{th}$ generation

F'' = inbreeding coefficient in the $(n-2)^{th}$ generation

and m = correlation coefficient in the n^{th} generation.

The inbreeding coefficient in the different generations of half-sib mating can be obtained using the recurrence relation

$$F = \frac{1}{8}(1+6F' + F'') \qquad \text{—— (2)}$$

established by Li, by using path method. Thus by using the relationships (1) and (2), Li could obtain the correlation coefficients between half-sib pairs (paternal) in different generations of half-sib mating, directly. As for the numerical values of the correlation coefficient the method is quite simple; but it does not give any information about the absolute frequencies of the half-sib mating types and their joint distribution in the different generations of half-sib mating.

An attempt has been made by George (1979) to study the half-sib and parent-offspring correlations under the first three generations of half-sib mating, in the autosomal gene case, assuming single locus with two alleles. The correlation tables of half-sib pairs have been derived from first principles while the joint distribution of parent-offspring pairs are obtained by the generation matrix technique. Further, he proposed a general theory for obtaining the correlations between both the parents and k offspring as well as the correlations between one of the parent and k offspring, in the autosomal gene case. The corresponding results in the sex-linked gene case have not been delt with.

In this thesis, an attempt is made to extend the results obtained by George (1979) to the fourth generation of half-sib mating and hence generalise the result for the n^{th} generation of half-sib mating. An attempt is also made to extend this theory to the case of sex-linked genes, single locus with two alleles. The study is divided into two chapters, viz., Chapter I and Chapter 2. Chapter 1 concerns with autosomal genes - the study of half-sib correlations, one parent-one offspring correlations, both the parents - several offspring correlations, and one parent - several offspring correlations is made under different sections. In Chapter 2 , a study of correlations of the various types of half-sib pairs and parent-offspring pairs, assuming sex-linked genes, is dealt with.

RESULTS

CHAPTER 1

AUTOSOMAL GENES

1.1 CORRELATION BETWEEN HALF-SIB PAIRS UNDER HALF-SIB MATING

A number of systems of mating between half-sibs may be derived. Here we consider only the simplest system of one male mated with an infinite number of his half-sisters, who are also half-sisters to each other (Fig.1) As the generation matrix for half-sib mating system is not available, the joint distributions between half-sib pairs are derived directly from the first principles.

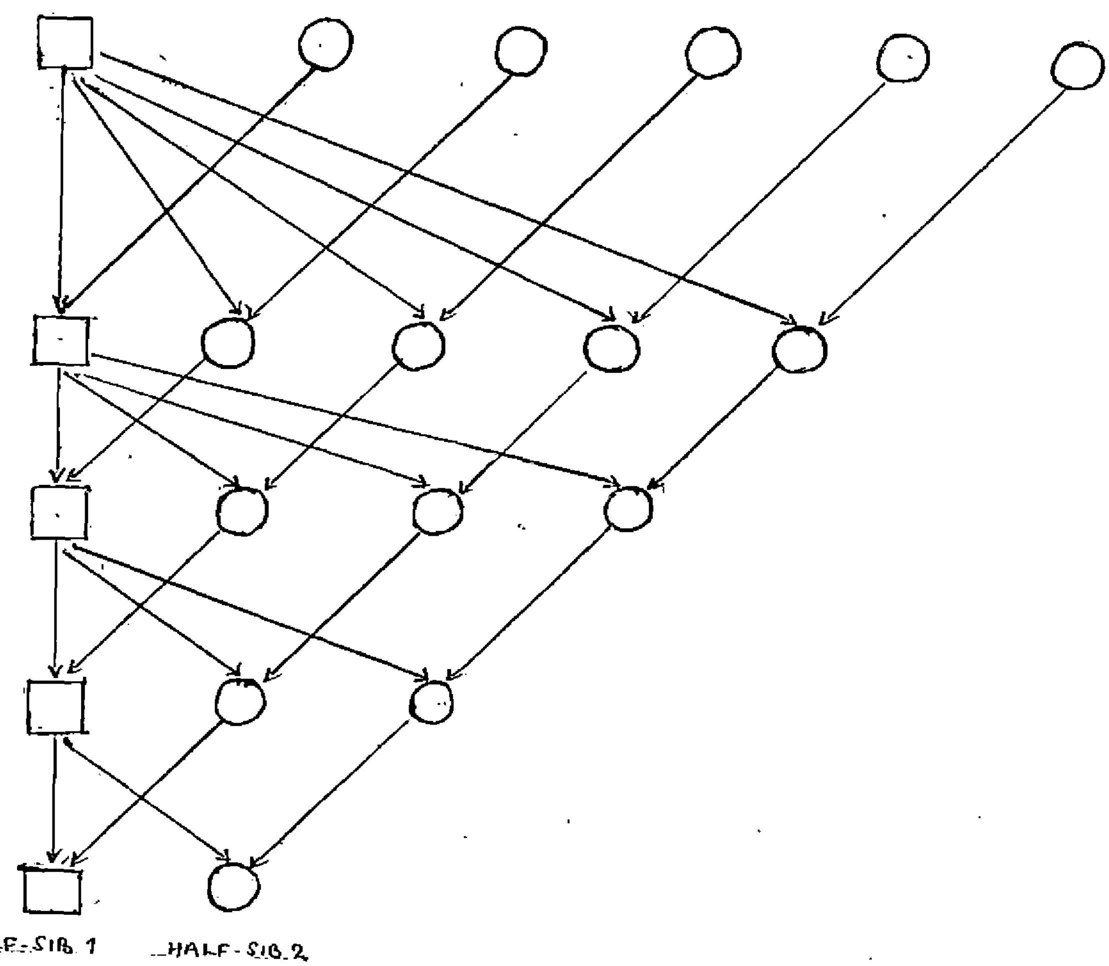

FIG: 1 . MATING BETWEEN ONE MALE AND HIS HALF-SISTERS

The joint distribution of half-sib pairs under random mating can be obtained as follows. Consider a single locus with two alleles A-a with proportion p and q respectively. Here the constitution of the male can be AA, Aa or aa. Let the male be AA. Now the proportion of the half-sib pairs can be obtained considering the mating between a male to his half-sisters, who are also half-sisters to one another as

$$AA\ (p^2AA + 2pqAa + q^2aa) = pAA + qAa$$

Let the male be Aa, then the proportion of the half-sibs can be obtained as

$$Aa\ (p^2AA + 2pqAa + q^2aa) = \tfrac{1}{2}\,pAA + \tfrac{1}{2}Aa + \tfrac{1}{2}qaa$$

Again, let the male be aa, then the proportion of half-sibs can be obtained as

$$aa\ (p^2AA + 2pq\ Aa + q^2aa) = p\ Aa + q\ aa$$

combining all these three cases we get the table of proportion of half-sibs for the different constitutions of the male as given in table 1.1

Table 1.1

Half-sibs

		AA	Aa	aa
Males	AA (p^2)	p	q	0
	Aa (2pq)	$\frac{1}{2}p$	$\frac{1}{2}$	$\frac{1}{2}q$
	aa (q^2)	0	p	q

Now considering all the half-sib pairs under each male we get the following table 1.2

Table 1.2

	(AA,AA)	(AA,Aa)	(AA,aa)	(Aa,AA)	(Aa,Aa)	(Aa,aa)	(aa,AA)	(aa,Aa)	(aa,aa)
AA (p^2)	p^2	pq	0	pq	q^2	0	0	0	0
Aa ($2pq$)	$\frac{1}{4}p^2$	$\frac{1}{4}p$	$\frac{1}{4}pq$	$\frac{1}{4}p$	$\frac{1}{4}$	$\frac{1}{4}q$	$\frac{1}{4}pq$	$\frac{1}{4}q$	$\frac{1}{4}q^2$
aa (q^2)	0	0	0	0	p^2	pq	0	pq	q^2

Hence the joint distribution of half-sib pairs under random mating can be obtained by pooling the corresponding pairs by weighting with p^2, $2pq$ and q^2 according as the male is AA, Aa and aa respectively, as shown in table 1.3

Table 1.3

Correlation table of half-sib pairs under random mating.

Half-sib 1

		AA	Aa	aa	Total
Half-sib 2	AA	$\frac{1}{2}p^3(1+p)$	$\frac{1}{2}p^2q(1+2q)$	$\frac{1}{2}p^2q^2$	p^2
	Aa	$\frac{1}{2}p^2q(1+2p)$	$\frac{1}{2}pq(1+4pq)$	$\frac{1}{2}pq^2(1+2q)$	$2pq$
	aa	$\frac{1}{2}p^2q^2$	$\frac{1}{2}pq^2(1+2q)$	$\frac{1}{2}q^3(1+q)$	q^2
Total		p^2	$2pq$	q^2	1

The correlation coefficient between half-sibs under random mating can be obtained directly from the table by using the product moment correlation coefficient formula as

$$r^{(0)}_{H.S} = \frac{1}{4} = 0.25 \qquad \text{—— (1.1)}$$

The joint distribution of half-sib pairs under the first generation of half-sib mating can be obtained directly by considering different lines as follows. There are three lines in this case. They are line (i) corresponding to the male AA with proportion p^2 and with the genotypic array of half-sibs pAA + q Aa; line (ii) corresponding to the male Aa with proportion 2pq and with the genotypic array of the half-sibs $\frac{1}{2}$p AA + $\frac{1}{2}$Aa + $\frac{1}{2}$q aa and line (iii) corresponding to the male aa with the proportion q^2 and with the genotypic array of half-sibs pAa + qaa. The procedure consists of finding a two-way table of the frequencies of half-sib pairs corresponding to each line and then pooling these tables to get the joint distribution of half-sib pairs by weighting them with respect to the proportion of the different lines. In the first generation of half-sib mating, the two-way table of half-sib pairs corresponding th. the line (i) is obtained as given in table (1.4)

Table 1.4

	AA	Aa	aa	Total
AA	$\frac{1}{16}(1+p^2)(1+3p)$	$\frac{1}{8}q(1+p)(1+2p)$	$\frac{1}{16}q^2(1+p)$	$\frac{1}{4}(1+p^2)$
Aa	$\frac{1}{8}q(1+p)(1+2p)$	$\frac{1}{4}q(1+pq)$	$\frac{1}{8}q^2$	$\frac{1}{2}q(1+p)$
aa	$\frac{1}{16}q^2(1+p)$	$\frac{1}{8}q^2$	$\frac{1}{16}a$	$\frac{1}{4}q^2$

Similarly the two-way table of half-sib pairs corresponding to line (ii) and line (iii) can be obtained as given in table 1.5 and table 1.6 respectively.

Table 1.5

	AA	Aa	aa	Total
AA	$\frac{1}{128}(1+2p^2)(1+4p)$	$\frac{1}{32}(1+2p)(1+p+2pq)$	$\frac{1}{128}(1+2p)(1+2q)$	$\frac{1}{16}(1+2p)^2$
Aa	$\frac{1}{32}(1+2p)(1+p+2pq)$	$\frac{1}{32}(5+2pq)$	$\frac{1}{32}(1+2q)(1+q+2pq)$	$\frac{1}{8}(3+4pq)$
aa	$\frac{1}{128}(1+2p)(1+2q)$	$\frac{1}{32}(1+2q)(1+q+2pq)$	$\frac{1}{128}(1+2q^2)(1+4q)$	$\frac{1}{16}(1+2q)^2$

Table 1.6

	AA	Aa	aa	Total
AA	$\frac{1}{16}p^3$	$\frac{1}{8}p^2$	$\frac{1}{16}p^2(1+q)$	$\frac{1}{4}p^2$
Aa	$\frac{1}{8}p^2$	$\frac{1}{4}p(1+pq)$	$\frac{1}{8}p^2(1+q)(1+2q)$	$\frac{1}{2}p(1+p)$
aa	$\frac{1}{16}p^2(1+q)$	$\frac{1}{8}q(1+q)(1+2q)$	$\frac{1}{16}(1+q)^2(1+3q)$	$\frac{1}{4}(1+q)^2$

Now, pooling these three two-way tables corresponding to the lines (i), and (ii) and (iii) by weighting with p^2, $2pq$ and q^2 respectively, we get the joint distribution of half-sib pairs under the first generation of half-sib mating, as given in table 1.7

Table 1.7

Correlation table of half-sib pairs under the first generation of half-sib mating.

Half-sib 1

Half-sib 2	AA	Aa	aa	Total
AA	$\frac{1}{64}p(1+11p+36p^2+16p^3)$	$\frac{1}{16}pq(1+9p+8p^2)$	$\frac{1}{64}pq(3+16pq)$	$\frac{1}{8}p(1+7p)$
Aa	$\frac{1}{16}pq(1+9p+8p^2)$	$\frac{1}{16}pq(9+16pq)$	$\frac{1}{16}pq(1+9q+8q^2)$	$\frac{7}{4}pq$
aa	$\frac{1}{64}pq(3+16pq)$	$\frac{1}{16}pq(1+9q+8q^2)$	$\frac{1}{64}q(1+11q+36q^2+16q^3)$	$\frac{1}{8}q(1+7q)$
Total	$\frac{1}{8}p(1+7p)$	$\frac{7}{4}pq$	$\frac{1}{8}q(1+7q)$	1

The correlation coefficient between half-sib pairs can be obtained directly from this correlation table as

$$r_{H.S}^{(1)} = 0.389 \qquad \text{---} \qquad (1.2)$$

In the same manner the joint distribution of half-sib pairs under the second generation of half-sib mating can be obtained. Here there are seven lines we have to consider. These seven lines have come from the three lines of the previous generation, two from the first, three from the second and two from the third. By pooling all the two-way tables obtained from these lines by weighting properly we can get the final correlation table for the half-sib pairs in the second generation of half-sib mating as given in table 1.8

Table 1.8

Correlation table of half-sib pairs under the second generation of half-sib mating

Half-sib 1

		AA AA	Aa Aa	aa	Total
	AA	$\frac{1}{1024}p(45+315p+560p^2+104p^3+24pq^2)$	$\frac{pq}{512}(59+256p+128p^2)$	$\frac{pq}{1024}(61+128pq)$	$\frac{p}{32}(7+25p)$
Half-sib 2	Aa	$\frac{pq}{512}(59+256p+128p^2)$	$\frac{pq}{256}(149+128pq)$	$\frac{pq}{512}(59+256q+128q^2)$	$\frac{25}{16}pq$
	aa	$\frac{1}{1024}pq(61+128pq)$	$\frac{1}{512}pq(59+256q+128q^2)$	$\frac{1}{1024}q(45+315q+560q^2+104q^3+24p^2q)$	$\frac{1}{32}q(7+25q)$
Total		$\frac{1}{32}p(7+25p)$	$\frac{25}{16}pq$	$\frac{1}{32}q(7+25q)$	1

The correlation coefficient between the half-sib pairs under the second generation of half-sib mating can be obtained directly from the correlation table as

$$r^{(2)}_{H.S} = 0.5 \qquad \text{—— (1.3)}$$

In the same manner the correlation table of half-sib pairs under the third generation of half-sib mating can be constructed by considering the seventeen lines, obtained from the seven lines of the previous generation. Out of these seventeen lines, two lines have come from the first line, three from the second, two from the third, three from the fourth, two from the fifth, three from the sixth, and two from the seventh line of the previous generation. By pooling the two-way tables corresponding two each of the seventeen lines after properly weighting, the correlation table for the half-sib pairs under the third generation of half-sib mating can be obtained as given in table 1.9.

The correlation coefficient of half-sib pairs under the third generation of half-sib mating can be directly obtained from this correlation table as

$$r^{(3)}_{H.S} = 0.584 \qquad \text{—— (1.4)}$$

Table 1.9

Correlation table of half-sib pairs under the third generation of half-sib mating

Half-sib 1

		AA	Aa	aa	
	AA	$\frac{1}{16384}p(1461+7275p+6624p^2+1024p^3)$	$\frac{1}{8192}pq(1251+3312p+1024p^2)$	$\frac{1}{16384}pq(1029+1024pq)$	
Half-sib 2	Aa	$\frac{1}{8192}pq(1251+3312p+1024p^2)$	$\frac{1}{8192}pq(4554+2048pq)$	$\frac{1}{8192}pq(1251+3312q+1024q^2)$	
	aa	$\frac{1}{16384}pq(1029+1024pq)$	$\frac{1}{5192}pq(1251+3312q+1024q^2)$	$\frac{1}{16384}q(1461+7275q+6624q^2+1024q^3)$	
Total		$\frac{1}{128}p(39+89p)$	$\frac{89}{64}pq$	$\frac{1}{128}q(39+89q)$	1

In a similar manner, the correlation table for half-sib pairs under the fourth generation of half-sib mating can be obtained by considering the fourtyone lines obtained from the seventeen lines of the previous generation. Out of these fourty one lines, two lines have come from the first line of the previous generation, three lines have come from the second, two lines from the third, three lines from the fourth, two lines each from the fifth and sixth, three lines from the seventh, two lines from the eight, three lines from the ninth, two lines from the tenth, three lines from the eleventh, two lines each from the twelfth and thirteenth, three lines from the fourteenth, two lines from the fifteenth, three lines from the sixteenth and two lines from the seventeenth line of the previous generation. Corresponding to each of these fourty one lines, a two-way table of frequencies of the half-sib pairs can be obtained. Then the correlation table of half-sib pairs under the fourth generation of half-sib mating can be obtained by pooling these two-way tables after weighting them with appropriate weights. The joint distribution of half-sib pairs in the fourth generation of half-sib mating is as given in table 1.10. The correlation coefficient of half-sib pairs under the fourth generation of half-sib mating can be directly obtained from this correlation table as

$$r^{(4)}_{H.S} = 0.650 \qquad --- \quad (1.5)$$

Table 1.10

Correlation table of half-sib pairs under the fourth generation of half-sib mating

		Half-sib 1			
		AA	Aa	aa	
Half-sib 2	AA	$\frac{1}{262144}p(37989+134427p+81536p^2+8192p^3)$	$\frac{1}{131072}pq(22971+40768p+8192p^2)$	$\frac{1}{262144}pq(15909+8192pq)$	
	Aa	$\frac{1}{131072}pq(22971+40768p+8192p^2)$	$\frac{1}{131072}pq(67402+16384pq)$	$\frac{1}{131072}pq(22971+40768q+8192q^2)$	
	aa	$\frac{1}{262144}pq(15909+8192pq)$	$\frac{1}{131072}pq(22971+40768q+8192q^2)$	$\frac{1}{262144}q(37989+134427q+81536q^2+8192q^3)$	
Total		$\frac{1}{512}p(195+317p)$	$\frac{317}{256}pq$	$\frac{1}{512}q(195+317q)$	1

Proceeding similarly the correlation coefficients of half-sib pairs under the fifth, sixth, etc generations of half-sib mating can be worked out directly from the correlation tables, by assuming additive genetic effects and using the product-moment correlation coefficient formula. The correlation coefficient of half-sib pairs under the first ten generations of half-sib mating is given in table 1.16 and these correlations are exhibited graphically in fig.2 by curve 2.

It is apparent from the figure that, as the number of generation increases, the correlation between half-sib pairs also increases.

1.2 CORRELATION BETWEEN PARENT-OFFSPRING PAIRS UNDER HALF-SIB MATING

The joint distribution of parent and offspring in the n^{th} generation of half-sib mating with single locus with two alleles A and a with proportions 'p' and 'q' (= 1-p) can be obtained by pairing one of the parents with an offspring obtained from the respective mating, out of the nine types of half-sib matings, viz., AA x AA, AA x Aa, AA x aa, Aa x AA, Aa x Aa, Aa x aa, aa x AA, aa x Aa, and aa x aa. Here we consider the mating between a fixed sire and his daughter, grand-daughter, etc. The generation matrix in this case has been obtained by George (3) as

$$\underset{\sim}{B} = \begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 1 \end{bmatrix} \qquad (1.6)$$

Thus the joint distribution of parent-offspring pairs under the n^{th} generation of half-sib mating can be obtained as:

$$\underset{\sim}{Z}^{(n)} = \underset{\sim}{B}\ \underline{U}^{(n)} \qquad \text{---- (1.7)}$$

where

$\underline{Z}^{(n)}$ is the column vector of frequencies of parent-offspring pairs in the n^{th} generation of half-sib mating and $\underline{U}^{(n)}$ is the column vector of frequencies of half-sib mating types in the n^{th} generation of half-sib mating, as already obtained in section 1.1.

Thus, $\underline{U}^{(1)}$, the column vector of frequencies of half-sib pairs obtained after random mating (i.e., half-sib mating types in the first generation of half-sib mating) is as given below:

$$\underline{U}^{(1)} = \begin{bmatrix} \frac{1}{2}p^3(1+p) \\ \frac{1}{2}p^2q(1+2p) \\ \frac{1}{2}p^2q^2 \\ \frac{1}{2}p^2q(1+2p) \\ \frac{1}{2}pq(1+4pq) \\ \frac{1}{2}pq^2(1+2q) \\ \frac{1}{2}p^2q^2 \\ \frac{1}{2}pq^2(1+2q) \\ \frac{1}{2}q^3(1+q) \end{bmatrix} \qquad (1.8)$$

Then $\underline{Z}^{(1)}$ the column vector of frequencies of parent-offspring pairs in the 1^{st} generation of half-sib mating can be obtained as

$$\underline{Z}^{(1)} = \underset{\sim}{B}\ \underline{U}^{(1)}$$

Therefore,

$$\underline{Z}^{(1)} = \begin{bmatrix} \frac{1}{4}p^2(1+3p) \\ \frac{3}{4}p^2q \\ 0 \\ \frac{1}{8}pq(1+6p) \\ pq \\ \frac{1}{8}pq(1+6q) \\ 0 \\ \frac{3}{4}pq^2 \\ \frac{1}{4}q^2(1+3q) \end{bmatrix} \qquad (1.9)$$

Hence the joint distribution of parent-offspring pairs in the first generation of half-sib mating can be obtained as given in the table (1.11)

Table 1.11.

Correlation table of parent-offspring pairs in the first generation of half-sib mating.

		Offspring AA	Aa	aa	Total
Parent	AA	$\frac{1}{4}p^2(1+3p)$	$\frac{3}{4}p^2q$	0	p^2
	Aa	$\frac{1}{8}pq(1+6p)$	pq	$\frac{1}{8}pq(1+6q)$	$2pq$
	aa	0	$\frac{3}{4}pq^2$	$\frac{1}{4}q^2(1+3q)$	q^2
Total		$\frac{1}{8}p(1+7p)$	$\frac{7}{4}pq$	$\frac{1}{8}q(1+7q)$	1

The correlation coefficient between the parent and offspring in the first generation of half-sib mating can be obtained from the above table assuming additive genic effect and scoring AA, Aa, aa as 2, 1, 0 respectively, as

$$ {}_{H.S}r^{(1)}_{P.O} = 0.5892 \qquad \text{----} \qquad (1.10) $$

The column vector of frequencies of half-sib mating types in the second generation of half-sib mating is

$$ \underline{U}^{(2)} = \begin{bmatrix} \frac{1}{64}p(1+11p+36p^2+16p^3) \\ \frac{1}{16}pq(1+9p+8p^2) \\ \frac{1}{64}pq(3+16pq) \\ \frac{1}{16}pq(1+9p+8p^2) \\ \frac{1}{16}pq(9+16pq) \\ \frac{1}{16}pq(1+9q+8q^2) \\ \frac{1}{64}pq(3+16pq) \\ \frac{1}{16}pq(1+9q+8q^2) \\ \frac{1}{64}pq(1+11q+36q^2+16q^3) \end{bmatrix} \qquad (1.11) $$

Hence $\underline{Z}^{(2)} = \underset{\sim}{B}\,\underline{U}^{(2)}$ and the correlation table of parent-offspring pairs in the second generation of half-sib mating can be formed from $\underline{Z}^{(2)}$ as, Table (1.12)

Table 1.12

Correlation table of parent-offspring pairs in the second generation of half-sib mating

Offspring

		AA	Aa	aa	Total
	AA	$\frac{1}{64}p(3+27p+34p^2)$	$\frac{1}{64}pq(5+34p)$	0	$\frac{1}{8}p(1+7p)$
Parent	Aa	$\frac{1}{64}pq(11+34p)$	$\frac{7}{8}pq$	$\frac{1}{64}pq(11+34q)$	$\frac{7}{4}pq$
	aa	0	$\frac{1}{64}pq(5+34q)$	$\frac{1}{64}q(3+27q+34q^2)$	$\frac{1}{8}q(1+7q)$
Total		$\frac{1}{32}p(7+25p)$	$\frac{25}{16}pq$	$\frac{1}{32}q(7+25q)$	1

Hence the correlation coefficient between parent-offspring pairs in the second generation of half-sib mating can be directly obtained from the above table as,

$$ {}_{H.S}r^{(2)}_{P.O} = 0.6672 \qquad --- (1.12) $$

The column vector of frequencies of half-sib mating types in the third generation of half-sib mating is

$$\underline{U}^{(3)} = \begin{bmatrix} \frac{1}{1024}p(45+315p+560p^2+104p^3+24p^2q) \\ \frac{1}{512}pq(59+256p+128p^2) \\ \frac{1}{1024}pq(61+128pq) \\ \frac{1}{512}pq(59+256p+128p^2) \\ \frac{1}{256}pq(149+128pq) \\ \frac{1}{512}pq(59+256q+128q^2) \\ \frac{1}{1024}pq(61+128pq) \\ \frac{1}{512}pq(50+256q+128q^2) \\ \frac{1}{1024}q(45+315q+560q^2+104q^3+24pq^2) \end{bmatrix} \qquad (1.13)$$

So the column vector of the joint distribution of parent-offspring pairs in the third generation of half-sib mating can be obtained as,

$$\underline{Z}^{(3)} = \underset{\sim}{B} \; \underline{U}^{(3)}$$

and the corresponding correlation table is as given in table (1.13).

The correlation coefficient, ${}_{H.S}r^{(3)}_{P.O}$, of parent-offspring pairs in the third generation of half-sib mating can be obtained from this table as,

$${}_{H.S}r^{(3)}_{P.O} = 0.7248 \qquad --- \quad (1.14)$$

Table 1.13

Correlation table of parent-offspring pairs in the third generation of half-sib mating

Offspring

		AA	Aa	aa	Total
	AA	$\frac{1}{128}p(13+67p+48p^2)$	$\frac{1}{128}pq(15+48p)$	0	$\frac{1}{32}p(7+25p)$
Parent	Aa	$\frac{1}{64}pq(13+24p)$	$\frac{25}{32}pq$	$\frac{1}{64}pq(13+24q)$	$\frac{25}{16}pq$
	aa	0	$\frac{1}{128}pq(15+48q)$	$\frac{1}{128}q(13+67q+48q^2)$	$\frac{1}{32}q(7+25q)$
Total		$\frac{1}{128}p(39+89p)$	$\frac{89}{64}pq$	$\frac{1}{128}q(39+89q)$	1

The column vector of frequencies of half-sib mating types in the fourth generation of half-sib mating is,

$$\underset{\sim}{U}^{(4)} = \begin{bmatrix} \frac{1}{16384} p(1461+7275p+6624p^2+1024p^3) \\ \frac{1}{8192} pq(1251+3312p+1024p^2) \\ \frac{1}{16384} pq(1029+1024pq) \\ \frac{1}{8192} pq(1251+3312p+1024p^2) \\ \frac{1}{8192} pq(4554+2048pq) \\ \frac{1}{8192} pq(1251+3312q+1024q^2) \\ \frac{1}{16384} pq(1029+1024pq) \\ \frac{1}{8192} pq(1251+3312q+1024q^2) \\ \frac{1}{16384} pq(1461+7275q+6624q^2+1024q^3) \end{bmatrix} \quad (1.15)$$

Hence $\underset{\sim}{Z}^{(4)} = \underset{\sim}{B}\ \underset{\sim}{U}^{(4)}$

and the correlation table of parent-offspring pairs in the fourth generation of half-sib mating can be formed from $\underset{\sim}{Z}^{(4)}$ as given in table (1.14)

Table 1.14

Correlation table of parent-offspring pairs in the fourth generation of half-sib mating.

		Offspring			
		AA	Aa	aa	Total
Parent	AA	$\frac{1}{2048}p(339+1167p+542p^2)$	$\frac{1}{2048}pq(285+542p)$	0	$\frac{1}{128}p(89+39p)$
	Aa	$\frac{1}{2048}pq(441+542p)$	$\frac{356}{512}pq$	$\frac{1}{2048}pq(441+542q)$	$\frac{89}{64}pq$
	aa	0	$\frac{1}{2048}pq(285+542q)$	$\frac{1}{2048}q(339+1167q+542q^2)$	$\frac{1}{128}q(89+39q)$
Total		$\frac{1}{512}p(195+317p)$	$\frac{317}{256}pq$	$\frac{1}{512}q(195+317q)$	1

Hence the correlation coefficient between parent-offspring pairs in the fourth generation of half-sib mating can be directly obtained from the above table as,

$$_{H.S}r^{(4)}_{P.O} = 0.7697 \qquad \text{----} \quad (1.16)$$

The column vector of frequencies of half-sib mating types in the fifth generation of half-sib mating is,

$$\underline{U}^{(5)} = \begin{bmatrix} \frac{1}{262144}p(37989+134427p+81536p^2+8192p^3) \\ \frac{1}{131072}pq(22971+40768p+8192p^2) \\ \frac{1}{262144}pq(15909+8192pq) \\ \frac{1}{131072}pq(22971+40768p+8192p^2) \\ \frac{1}{131072}pq(67402+16384pq) \\ \frac{1}{131072}pq(22971+40768q+8192q^2) \\ \frac{1}{262144}pq(15909+8192pq) \\ \frac{1}{131072}pq(22971+40768q+8192q^2) \\ \frac{1}{262144}q(37989+134427q+81536q^2+8192q^3) \end{bmatrix} \qquad (1.17)$$

Hence the column vector of frequencies of parent-offspring pairs in the fifth generation of half-sib mating can be obtained as

$$\underline{Z}^{(5)} = \underset{\sim}{B}\ \underline{U}^{(5)}$$

and the corresponding correlation table can be formed from $\underline{Z}^{(5)}$ as table 1.15.

Table 1.15

Correlation table of parent-offspring pairs in the fifth generation of half-sib mating.

		Offspring AA	Offspring Aa	Offspring aa	Total
	AA	$\frac{1}{8192}p(1905+4757p+1530p^2)$	$\frac{1}{8192}pq(1215+1530p)$	0	$\frac{1}{512}p(195+317p)$
Parent	Aa	$\frac{1}{8192}pq(1215+1530p)$	$\frac{317}{512}pq$	$\frac{1}{8192}pq(1215+1530q)$	$\frac{317}{256}pq$
	aa	0	$\frac{1}{8192}pq(1215+1530q)$	$\frac{1}{8192}q(1905+4757q+1530q^2)$	$\frac{1}{512}q(195+317q)$
Total		$\frac{1}{2048}p(919+1129p)$	$\frac{1129}{1024}pq$	$\frac{1}{2048}q(919+1129q)$	1

The correlation coefficient, ${}_{H.S}r^{(5)}_{P.O}$, of parent-offspring pairs in the fifth generation of half-sib mating can be obtained directly from the correlation table as,

$$ {}_{H.S}r^{(5)}_{P.O} = 0.8054 \quad \text{----} \quad (1.18) $$

In a similar manner, the joint distribution and the correlation coefficient for parent and offspring for the sixth, seventh, etc. generations of half-sib mating can be worked out. The correlation coefficients between parent and offspring in the first ten generations of half-sib mating are given in the table (1.16). These correlations are also shown graphically in Fig.1 by curve 1. From the figure it is clear that as the number of generation increases, the parent offspring correlation also increases. While comparing with the half-sib correlation coefficients it is found that the parent-offspring correlation is of higher order than the half-sib correlation.

Table 1.16

Correlations of half-sib pairs and parent-offspring pairs in the first ten generations of half-sib mating system.

Generation (n)	0	1	2	3	4	5	6	7	8	9	10
Half-sib correlation ${}_{H.S}r^{(n)}_{H.S}$	0.250	0.389	0.500	0.584	0.650	0.703	0.746	0.782	0.811	0.836	0.858
Parent-offspring correlation ${}_{H.S}r^{(n)}_{P.O}$	0.500	0.589	0.667	0.725	0.770	0.805	0.834	0.858	0.878	0.894	0.908

Note: 0^{th} generation stands for random mating.

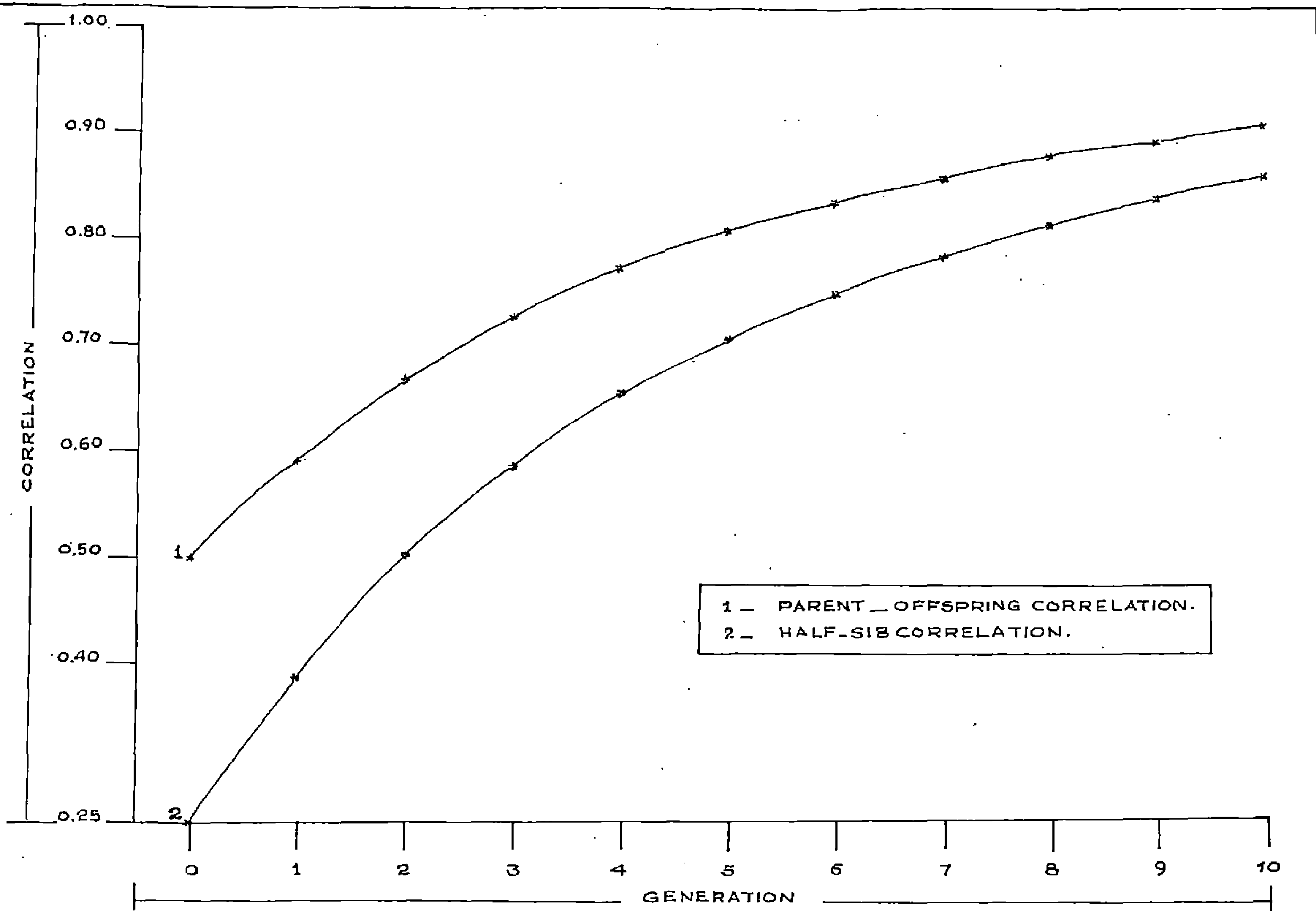

FIG: 2. HALF-SIB CORRELATIONS AND PARENT-OFFSPRING CORRELATIONS IN THE AUTOSOMAL GENE CASE, UNDER HALF-SIB MATING SYSTEM

1.3 CORRELATION BETWEEN PARENT AND K OFFSPRING UNDER HALF-SIB MATING SYSTEM

In quantitative genetic studies, the correlation between one given relative and a number of individuals of the other relative is used for predicting the breeding value of the given relative. For instance, the correlation between one parent and several of its offsprings provides with the accuracy in predicting the breeding value of the parent with the help of its offspring. It is a major determinant in increasing the response to selection. The behaviour of such correlations, especially in inbred populations is not fully known. In this chapter, the correlations between one parent and several of its offspring, as well as, both the parents and several of its offspring have been worked out in the lines of George and Narain (4), for the first ten generations of half-sib mating.

Theoretical Procedure

The theoretical procedure for finding the correlations between both the parents and K offspring and the correlation between one parent and K offspring are exactly same as that of the autosomal gene case for parent-offspring mating and full-sib mating explained by George and Narain (4).

Consider a single locus with two alleles A-a, so that with additive genetic affect we score the values of the genotypes AA, Aa, aa as 2,1,0 respectively. Again consider that there are K offspring produced from the matings of these genotypes. The scores of these resulting offspring is then obtained by adding the scores of the individual offspring. It is then obvious that the maximum score for K offspring is 2K, when all the offspring are of the type AA. Similarly the minimum score of ten offspring will be '0', when all the offspring are of the type aa. So far as the scores of parents are concerned, we have two distinct cases. In one we consider that the sum of scores of two parents which will range between 0 and 4 and in the second case we consider only the score of one parent, ranging between 0 and 2.

1.31 CORRELATION BETWEEN BOTH THE PARENTS AND K OFFSPRING.

Let the frequency of the nine mating types, viz., AA x AA, AA x Aa, AA x aa, Aa x AA, Aa x Aa, Aa x aa, aa x AA, aa x Aa, a x aa be $u_{22}^{(n)}$, $u_{21}^{(n)}$, $u_{20}^{(n)}$, $u_{12}^{(n)}$, $u_{11}^{(n)}$, $u_{10}^{(n)}$, $u_{02}^{(n)}$, $u_{01}^{(n)}$, $u_{00}^{(n)}$ respectively in the n^{th} generation of the given system of mating. In order to determine the probabilities of K offspring in each of these mating types we have to obtain the conditional probabilities of K offspring, given the particular type of mating between two similar or two dissimilar homozygotes, viz., AA x AA, aa x aa, AA x aa, all the resulting offspring will be of uniform types with probability unity. In the case of mating between a homozygote and heterozygote viz., AA x Aa or aa x Aa, the resulting offspring consists of homozygotes and heterozygotes in the ratio 1:1. The probability generating function of the sum of the scores, $s_k (= z_1+z_2+. .. z_k)$, will be $\left[\frac{1}{2}(1+z)\right]^k$, similarly in the case of mating between two heterozygotes, the probability generating function of s_k will be $\left[\frac{1}{4}(1+z)\right]^k$.

Let $P^{(1)}_{x,y}$ represent the joint probability of the sum of scores of both the parents, as x, and score of offspring, as y, where $0 \leq x \leq 4$ and $0 \leq y \leq 2k$. It then follows,

$$P^{(1)}_{0,y} = u^{(n)}_{00} \qquad \text{if } y = 0$$

$$= 0 \qquad \text{if } 1 \leq y \leq 2k$$

$$P^{(1)}_{1,y} = \left[u^{(n)}_{10}+u^{(n)}_{01}\right]\binom{k}{y}\left(\tfrac{1}{2}\right)^k \qquad \text{if } 0 \leq y \leq k$$

$$= 0 \qquad \text{if } k+1 \leq y \leq 2k$$

$$P^{(1)}_{2,y} = u^{(n)}_{11}\binom{2k}{y}\left(\tfrac{1}{4}\right)^k \qquad \text{if } 0 \leq y \leq k-1$$

$$= u^{(n)}_{20}+u^{(n)}_{02} + u^{(n)}_{11}\binom{2k}{k}\left(\tfrac{1}{4}\right)^k \qquad \text{if } y = k$$

$$= u^{(n)}_{11}\binom{2k}{y}\left(\tfrac{1}{4}\right)^k \qquad \text{if } k+1 \leq y \leq 2k$$

$$P^{(1)}_{3,y} = 0 \qquad \text{if } 0 \leq y \leq k-1$$

$$= \left[u^{(n)}_{21} + u^{(n)}_{12}\right]\binom{k}{2k-y}\left(\tfrac{1}{2}\right)^k \qquad \text{if } k \leq y \leq 2k$$

$$P^{(1)}_{4,y} = 0 \qquad \text{if } 0 \leq y \leq 2k-1$$

$$= u^{(n)}_{22} \qquad \text{if } y = 2k$$

These probabilities give the bivariate frequency table for obtaining the correlation between x and y as given in table (1.17).

Table 1.17

Bivariate table for the absolute probabilities of the scores of both the parents and k offspring

Score of the parents	Score of the offspring: 0	1 ...	k ...	2k-1	2k	Total
4	0	0 ...	0 ...	0 ...	u_{22}	u_{22}
3	0	0 ...	$(\frac{1}{2})^k(u_{21}+u_{12})$ ...	$(\frac{1}{2})^k\binom{k}{1}(u_{22}+u_{12})$	$(\frac{1}{2})^k(u_{21}+u_{12})$	$u_{21}+u_{12}$
2	$(\frac{1}{2})^{2k}u_{11}$	$(\frac{1}{2})^{2k}\binom{2k}{1}u_{11}$ ...	$(\frac{1}{2})^{2k}\binom{2k}{k}u_{11}$ ...	$(\frac{1}{2})^{2k}\binom{2k}{2k-1}u_{11}$	$(\frac{1}{2})^{2k}u_{11}$	$u_{20}+u_{11}+u_{02}$
1	$(\frac{1}{2})^k(u_{10}+u_{01})$	$(\frac{1}{2})^k\binom{k}{1}(u_{10}+u_{01})$ ...	$(\frac{1}{2})^k\binom{k}{k}(u_{10}+u_{01})$ ...	0	0	$u_{10}+u_{01}$
0	u_{00}	0	0	0	0	u_{00}
Total						1

The correlation coefficient between both the parents and k offspring can be calculated using the product moment formula for correlation coefficient by calculating $\sum fx$, $\sum fx^2$, $\sum fy$, $\sum fy^2$ and $\sum fxy$ from this table as,

$$\sum fx = 4u_{22}+3u_{21}+2u_{20}+3u_{12}+2u_{11}+u_{10}+2u_{02}+u_{01} \quad \ldots \quad (1.19)$$

$$\sum fx^2 = 16u_{22}+9u_{21}+4u_{20}+9u_{12}+4u_{11}+u_{10}+4u_{02}+u_{01} \quad \ldots. \quad (1.20)$$

$$\sum fy = k\left[2u_{22}+\frac{3}{2}u_{21}+u_{20}+\frac{3}{2}u_{12}+u_{11}+\frac{1}{2}u_{10}+u_{02}+\frac{1}{2}u_{01}\right] \quad \ldots \quad (1.21)$$

$$\sum fy^2 = 4k^2u_{22}+k\left(\frac{9k+1}{4}\right)u_{21}+k^2u_{20}+k\left(\frac{9k+1}{4}\right)u_{12}+k\left(\frac{2k+1}{2}\right)u_{11}+ k\left(\frac{k+1}{4}\right)u_{10}+k^2u_{02}+k\left(\frac{k+1}{4}\right)u_{01} \quad \ldots \quad (1.22)$$

$$\sum fxy = k \quad 8u_{22}+\frac{9}{2}u_{21}+2u_{20}+\frac{9}{2}u_{12}+2u_{11}+\frac{1}{2}u_{10}+2u_{02}+\frac{1}{2}u_{01} \ldots \quad (1.23)$$

From these the variance of x, variance of y and the covariance of x and y can be worked out, and thus the correlation coefficient between the scores x and y in both the parents and k offspring can be obtained as,

$$R^{(1)}_{xy} = \frac{cov(x,y)}{v(x)xv(y)} \quad (1.24)$$

The column vectors of frequencies of nine mating types in the first, second, third etc. generations of half-sib mating have already been derived from first principles, under section (1.1) Now the bivariate table of absolute probabilities of the scores of both the parents and k offspring under the first generation of half-sib mating can be obtained by substituting

The $\underline{U}^{(1)}$ values from (1.8) in table (1.18). Hence the values of $\sum fx$, $\sum fx^2$, $\sum fy$, $\sum fy^2$, $\sum fxy$ and the correlation coefficient between the scores of both the parents and k offspring, $\underset{H.S\ \ P.O}{r^{(1)}}(2,k)$, in the first generation of half-sib mating can be worked out using the formulae given in (1.19), (1.20), (1.21), (1.22), (1.23) and (1.24) respectively.

Thus,

$$\underset{H.S\ \ P.O}{r^{(1)}}(2,k) = \sqrt{\frac{5k}{4+5k}} \qquad \ldots \quad (1.25)$$

Similarly, by repeating the above process we can obtain ${}_{H.S}r^{(n)}_{P.O}(2,k)$ for n = 2, 3, 4 et. by substituting the vectors $\underline{U}^{(2)}$, $\underline{U}^{(3)}$, $\underline{U}^{(4)}$ etc. given in (1.11), (1.13), (1.15) respectively in the formulae (1.19), (1.20), (1.21),(1.22), (1.23), and (1.24).

The correlation coefficient between both the parents and k offspring in the first ten generations of half-sib mating is given in table (1.18).

Table 1.18

Correlation between both the parents and k offspring in the first ten generations of half-sib mating.

Generation n	0	1	2	3	4
Correlation coefficient $_{H.S}r^{(n)}_{P.O}(2,k)$	$\sqrt{\frac{k}{1+k}}$	$\sqrt{\frac{5k}{4+5k}}$	$\sqrt{\frac{25k}{14+25k}}$	$\sqrt{\frac{117k}{50+117k}}$	$\sqrt{\frac{529k}{178+529k}}$

	5	6	7
	$\sqrt{\frac{2333k}{634+2333k}}$	$\sqrt{\frac{10105k}{2258+10105k}}$	$\sqrt{\frac{43173k}{8042+43173k}}$

	8	9	10
	$\sqrt{\frac{182497k}{28642+182497k}}$	$\sqrt{\frac{764909k}{102010+764909k}}$	$\sqrt{\frac{3184009k}{363314+3184009k}}$

Note: 0^{th} generation stands for random mating.

It is interesting to note that the correlation coefficients are of the form $\frac{1}{\sqrt{1+\frac{a}{k}}}$ where, a is a fraction changing with n, in the form of the following series

$$a : \frac{4}{5}, \frac{14}{25}, \frac{50}{117}, \frac{178}{529}, \frac{634}{2333}, \ldots$$

By giving different values to k from 1 to 10, the correlation coefficients between both the parents and k offspring have been worked out as given in table (1.19). These correlations are graphically exhibited in figure 3, which gives the trend of the correlation when the number of offspring varies for a particular generation. The figure indicates that the correlation increases very rapidly for smaller values of k, ie., when k = 1 to 4, and for further values of k, the increase in correlation is very negligible. Further, there is no significant increase in correlation beyond the 7^{th} generation of half-sib mating, for all values of k.

Table 1.19

Correlations between both the parents and k offspring when k = 1 to 10, for the first 10 generations of half-sib mating.

Generation (n)	1	2	3	4	5	6	7	8	9	10
0	0.707	0.817	0.866	0.894	0.913	0.926	0.935	0.946	0.949	0.953
1	0.745	0.845	0.889	0.913	0.928	0.939	0.947	0.953	0.958	0.962
2	0.801	0.884	0.918	0.937	0.948	0.956	0.962	0.967	0.970	0.973
3	0.837	0.908	0.936	0.951	0.960	0.966	0.971	0.974	0.977	0.979
4	0.865	0.925	0.948	0.960	0.9678	0.973	0.977	0.980	0.982	0.984
5	0.887	0.938	0.958	0.968	0.964	0.978	0.981	0.983	0.985	0.987
6	0.904	0.948	0.965	0.973	0.978	0.982	0.984	0.986	0.988	0.989
7	0.918	0.957	0.970	0.978	0.982	0.985	0.987	0.989	0.990	0.991
8	0.930	0.963	0.975	0.981	0.985	0.987	0.989	0.990	0.991	0.992
9	0.939	0.968	0.979	0.984	0.987	0.989	0.991	0.992	0.993	0.993
10	0.947	0.973	0.982	0.986	0.989	0.991	0.992	0.993	0.994	0.994

Note: 0^{th} generation stands for random mating.

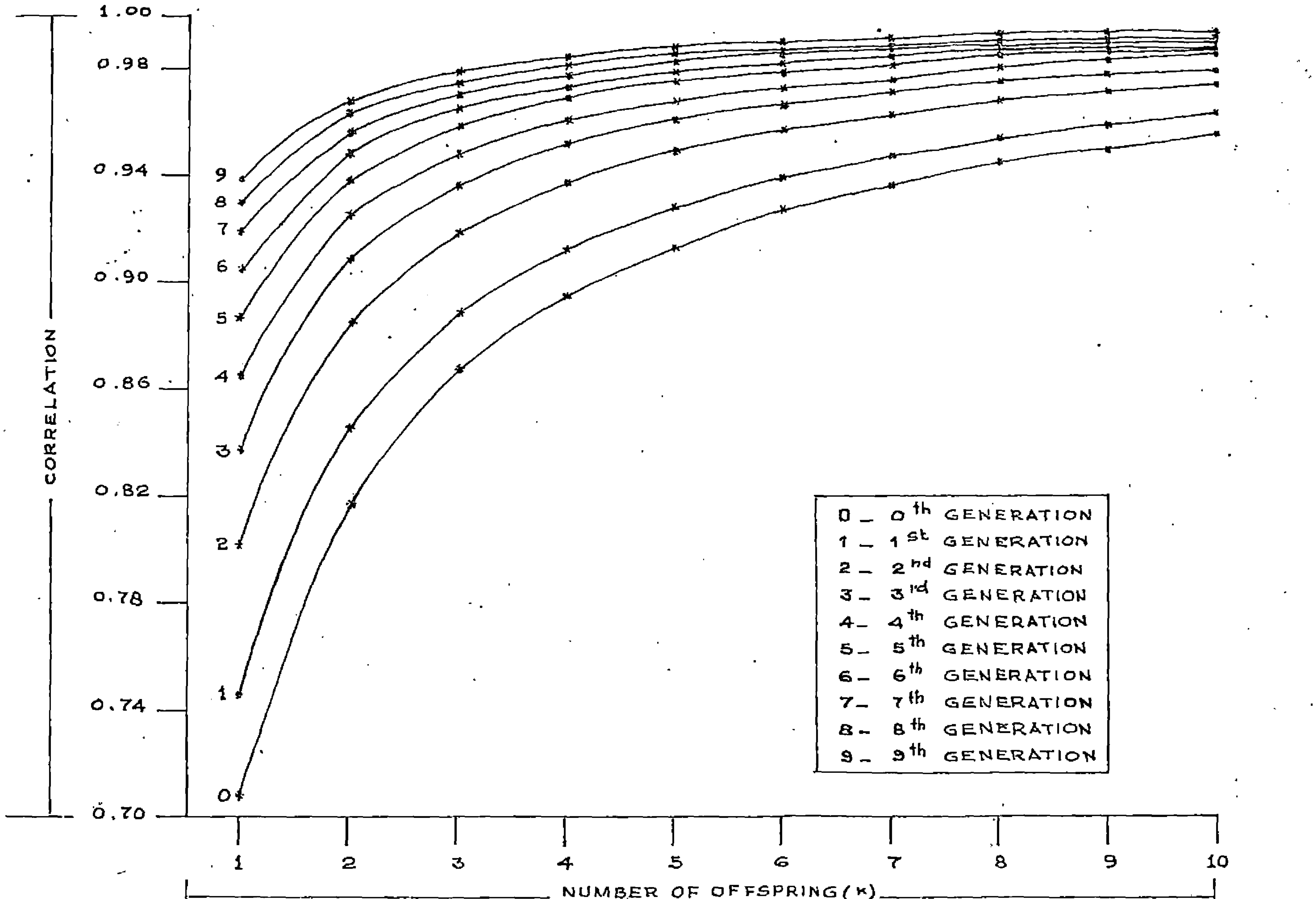

FIG: 3 BOTH THE PARENTS AND SEVERAL OFFSPRING CORRELATIONS IN THE AUTOSOMAL GENE CASE, UNDER HALF-SIB MATING SYSTEM

1.32 CORRELATION BETWEEN ONE OF THE PARENTS AND K OFFSPRING

In order to work out the correlation between one of the parents and k offspring for which this parent is common, we have to pool the probabilities for three types of mating involving a common parent. Let $P^{(2)}_{xy}$ represent the joint distribution of the score of one parent as x and score of offspring as y, where $0 \leq x \leq 2$ and $0 \leq y \leq 2k$. It then follows,

$$
\begin{aligned}
P^{(2)}_{0,y} &= u^{(n)}_{01}\,(\tfrac{1}{2})^k + u^{(n)}_{00} && \text{if } y = 0 \\
&= \binom{k}{y}(\tfrac{1}{2})^k\, u^{(n)}_{01} && \text{if } 1 \leq y \leq k-1 \\
&= (\tfrac{1}{2})^k\, u^{(n)}_{01} + u^{(n)}_{02} && \text{if } y = k \\
&= 0 && \text{if } k+1 \leq y \leq 2k \\
P^{(2)}_{1,y} &= \binom{2k}{y}(\tfrac{1}{2})^{2k}\, u^{(n)}_{11} + \binom{k}{y}(\tfrac{1}{2})^k u^{(n)}_{10} && \text{if } 0 \leq y \leq k-1 \\
&= \binom{2k}{k}(\tfrac{1}{2})^{2k}\, u^{(n)}_{11} + (\tfrac{1}{2})^k u^{(n)}_{10} + (\tfrac{1}{2})^k\, u^{(n)}_{12} && \text{if } y = k \\
&= \binom{2k}{y}(\tfrac{1}{2})^{2k}\, u^{(n)}_{11} + \binom{k}{2k-y}(\tfrac{1}{2})^k\, u^{(n)}_{12} && \text{if } k+1 \leq y \leq 2k \\
P^{(2)}_{2,y} &= 0 && \text{if } 0 \leq y \leq k-1 \\
&= u^{(n)}_{21}(\tfrac{1}{2})^k + u^{(n)}_{20} && \text{if } y = k \\
&= \binom{k}{2k-y} u^{(n)}_{21}\,(\tfrac{1}{2})^k && \text{if } k+1 \leq y \leq 2k-1 \\
&= (\tfrac{1}{2})^k\, u^{(n)}_{21} + u^{(n)}_{22} && \text{if } y = 2k
\end{aligned}
$$

These pwobabilities give the bivariate frequency table for obtaining the correlations between x and y as given in table (1.20).

Table 1.20

Bivariate table for the absolute probabilities of the scores of one of the parents and k offspring

Score of the parent	Score of the offspring 0	1 ...	k ...	2k-1	2k	Total
2	0	0 ...	$(\frac{1}{2})^k \binom{k}{k} u_{21} + u_{20}$ ...	$(\frac{1}{2})^k \binom{k}{1} u_{21}$	$(\frac{1}{2})^k\ u_{21} + u_{22}$	$u_{22} + u_{21} + u_{20}$
1	$(\frac{1}{2})^k u_{11} + (\frac{1}{2})^k u_{10}$	$(\frac{1}{2})^k \binom{2k}{1} u_{11}$ + $(\frac{1}{2})^k \binom{k}{1} u_{10}$ ...	$(\frac{1}{2})^k \binom{2k}{k} u_{11}\frac{1}{2}+$ $(\frac{1}{2})^k\ u_{12} + u_{10}$ ...	$(\frac{1}{2})^{2k} \binom{2k}{2k-1}+$ $(\frac{1}{2})^k \binom{k}{1} u_{12}$	$(\frac{1}{2})^{2k}\ u_{11}+$ $(\frac{1}{2})^k\ u_{12}$	$u_{12} + u_{11} + u_{10}$
0	$(\frac{1}{2})^{2k}\ u_{01} + u_{00}$	$(\frac{1}{2})^k \binom{k}{1} u_{01}$ ...	$(\frac{1}{2})^k u_{01} + u_{10}$ ...	0	0	$u_{02} + u_{01} + u_{00}$
Total						1

The correlation coefficients between one of the parents and k offspring can be calculated in the same way as in section 1.31 by calculating Σfx, Σfx^2, Σfy, Σfy^2 and Σfxy from this table as,

$$\Sigma fx = 2(u_{22}+u_{21}+u_{20}) + u_{12}+u_{11}+u_{10} \tag{1.26}$$

$$\Sigma fx^2 = 4(u_{22}+u_{21}+u_{20}) + u_{12}+u_{11}+u_{10} \tag{1.27}$$

$$\Sigma fy = 2ku_{22}+ \frac{3k}{2} u_{21}+ku_{20}+\frac{3k}{2} u_{12} +ku_{11}+\frac{k}{2} u_{10}+ ku_{02}+\frac{k}{2}u_{01} \tag{1.28}$$

$$\Sigma fy^2 = 4k^2u_{22}+k\left(\frac{9k+1}{4}\right) u_{21}+k^2u_{21}+k\left(\frac{9k+1}{4}\right)u_{12}+ k\left(\frac{2k+1}{2}\right) u_{11}+k\left(\frac{k+1}{4}\right)u_{10}+k^2u_{02}+k\left(\frac{k+1}{4}\right)u_{02} \tag{1.29}$$

$$\Sigma fxy = 4ku_{22}+3ku_{21}+2ku_{20}+\frac{3}{2}u_{12}+ku_{11}+\frac{1}{2}ku_{12} \tag{1.30}$$

From these the variance of x and variance of y and the covariance between x and y can be worked out, and then the correlation between the scores of x and y in one parent and k offspring case can be obtained as,

$$R^{(2)}_{xy} = \frac{\text{cov }(x,y)}{v(x)xv(y)} \tag{1.31}$$

Now the correlation coefficient ${}_{H.S}r^{(n)}_{P.O}(1,k)$, between one of the parents and k offspring in the n^{th} generation of half-sib mating can be obtained by substituting the $U^{(n)}$ values (already obtained in section 1.1) in table (1.21).

The values of $\Sigma fx, \Sigma fx^2, \Sigma fxy, \Sigma fy^2$ and fxy are found out in each generation by using the equations (1.26), (1.27), (1.28), (1.29) and (1.30) respectively and correlation coefficient is calculated using the formula (1.31).

The correlation coefficient between one of the parents and k offspring worked out in this way, for the first five ten generations of half-sib mating are given in table (1.21)

It may be noted that the correlations when k = 1, for different generations are exactly the same as that of one parent one offspring correlations under half-sib mating already obtained in section (1.2) Further, the correlation coefficient in different generations are of the form $\frac{1}{\sqrt{b(1+\frac{c}{k})}}$,

where b and c are fractions changing with 'n' in the form of the following series.

$$b : \frac{8}{5}, \frac{36}{25}, \frac{156}{117}, \frac{668}{529}, \frac{2828}{2333}, \ldots$$

$$c : \frac{4}{5}, \frac{14}{25}, \frac{50}{117}, \frac{178}{529}, \frac{634}{2333}, \ldots$$

Giving different values to k, from 1 to 10, the correlation coefficients between one of the parents and offspring have been worked out as given in table (1.22). These correlations are exhibited graphically in Figure 4. Here also, as in the previous case, the correlation increases as n, the number of generations and k, the number of offspring, increases.

Table 1.21

Correlation between one parent and k offspring in the first ten generations of half-sib mating.

Generation (n)	0	1	2	3
Correlation coefficient $_{H.S}r^{(n)}_{P.O}(1,k)$	$\frac{k}{\sqrt{2k(1+k)}}$	$\frac{5k}{\sqrt{8k(4+5k)}}$	$\frac{25k}{\sqrt{36k(14+25k}}$	$\frac{117k}{\sqrt{156k(50+117k)}}$

4	5	6
$\frac{529k}{\sqrt{668k(178+529k)}}$	$\frac{2333k}{\sqrt{2828k(634+2333k)}}$	$\frac{10105k}{\sqrt{11868k(2258+10105k)}}$

7	8
$\frac{43173k}{\sqrt{49452k(8042+43173k)}}$	$\frac{182497k}{\sqrt{204860k(28642+182497k)}}$

9	10
$\frac{764909k}{\sqrt{844556k(102010+764909k)}}$	$\frac{3184009k}{\sqrt{3467676k(363314+3184009k)}}$

Note: 0^{th} generation stands for random mating.

Table 1.22

Correlations between one parent and k offspring when k = 1 to 10, for the first ten generations of half-sib mating.

Generation (n)	Number of offspring (k) 1	2	3	4	5	6	7	8	9	10
0	0.500	0.577	0.612	0.632	0.646	0.655	0.661	0.667	0.671	0.674
1	0.589	0.668	0.702	0.722	0.734	0.743	0.749	0.754	0.758	0.760
2	0.667	0.736	0.765	0.780	0.790	0.797	0.802	0.806	0.809	0.811
3	0.725	0.786	0.810	0.823	0.831	0.837	0.841	0.844	0.846	0.848
4	0.770	0.823	0.844	0.855	0.861	0.866	0.869	0.872	0.874	0.875
5	0.805	0.852	0.870	0.879	0.885	0.888	0.891	0.893	0.895	0.896
6	0.834	0.881	0.890	0.898	0.903	0.906	0.908	0.910	0.912	0.913
7	0.858	0.894	0.907	0.913	0.917	0.921	0.922	0.924	0.925	0.926
8	0.878	0.909	0.920	0.926	0.929	0.932	0.934	0.935	0.936	0.937
9	0.894	0.921	0.931	0.936	0.939	0.941	0.943	0.944	0.945	0.945
10	0.908	0.932	0.941	0.945	0.948	0.949	0.951	0.952	0.952	0.953

Note: 0^{th} generation stands for random mating.

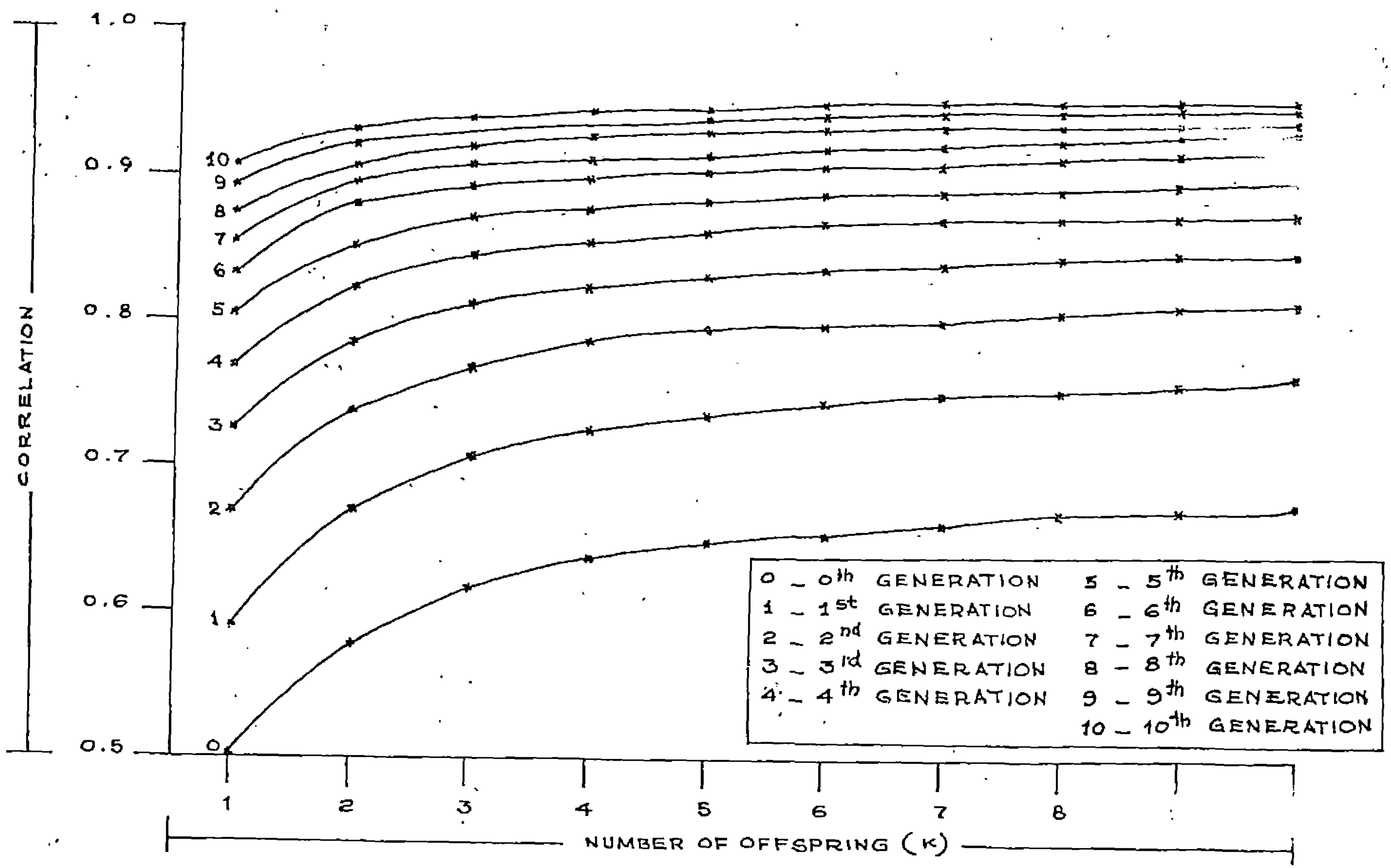

FIG: 4. ONE PARENT AND SEVERAL OFFSPRING CORRELATIONS IN THE AUTOSOMAL GENE CASE, UNDER HALF-SIB MATING SYSTEM

Chapter 2

SEX-LINKED GENES

For autosomal genes the correlations of the different parent-offspring pairs are the same, all being half, and the correlations of different half-sib pairs also, are the same, all being one fourth. For sex-linked genes, however, the asymmetrical chromosomal complement of males and females makes it necessary to distinguish the sexes of the relatives. Thus, in the sex-linked gene case, there are three kinds of half-sib pairs, viz., brother-sister, brother-brother and sister-sister. Likewise there are four kinds of parent-offspring pairs viz., mother-daughter, mother-son, father-daughter and father-son. The correlations of the different half-sib pairs and those of different parent-offspring pairs are considered here seperately.

2.1 CORRELATIONS OF DIFFERENT HALF-SIB PAIRS UNDER HALF-SIB MATING SYSTEM

In the case of the sex-linked genes too, the generation matrix for half-sib pairs cannot be easily obtained. So the method adopted in the autosomal gene case has to be extended to the cases of sex-linked genes with slight modifications characteristic of a sex-linked character.

Sex-linked genes are those located on sex-chromosomes. We shall consider the case of a single locus with two alleles, say, A and a. Let the homogametic type (AA, Aa or aa) be females and the hetrogametic type (A or a) be males. Now, there will be six mating types, viz. AA x A, AA x a, Aa x A, Aa x a, aa x A and aa x a. As before these six types of mating with appropriate frequencies are listed. The probabilities of the various combinations are grouped together. Obviously, there is no correlation between father and son in the case of a sex-linked character, since the son receives his father's Y-chromosome. The correlation for mother-daughter pair is the same as that of parent-offspring pair in the case of autosomal genes except for a shift by one generation. The two correlations for father-daughter pair and mother-son pair are the same.

We shall be dealing with the correlations of different types of half-sib pairs seperately.

2.11 BROTHER - SISTER CORRELATION

The joint distribution of brother-sister pairs under random mating can be obtained as follows. Consider a single locus with two alleles A and a with proportions p and q, respectively. Then it may be verified that the population

$$\begin{pmatrix} A & a \\ p & q \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} AA & Aa & aa \\ p^2 & 2pq & q^2 \end{pmatrix} \text{ is}$$

in equilibrium under panmixia. Thus in a random mating population the constitution of the male can be A or a. Let this male be A. Now the proportion of the half-sibs obtained after random mating can be obtained by considering the mating between A (p) and each of the females AA (p^2), Aa (2pq) and aa (q^2) in the population, as

$$A\ (p^2\ AA + 2pq\ Aa + q^2\ aa) = \tfrac{1}{2}p\ AA + \tfrac{1}{2}q\ Aa + \tfrac{1}{2}p\ A + \tfrac{1}{2}q\ a$$

Now let the male be a, then the proportion of half-sibs corresponding to the male a can be obtained as,

$$a\ (p^2\ AA + 2pq\ Aa + q^2\ aa) = \tfrac{1}{2}p\ Aa + \tfrac{1}{2}q\ aa + \tfrac{1}{2}p\ A + \tfrac{1}{2}q\ a$$

combining the above two cases, we get the table of the proportions of half-sibs for the different constitutions of the male as given in table (2.1).

Table 2.1

Half-sibs

		AA	Aa	aa	A	a
	A	$\frac{1}{2}p$	$\frac{1}{2}q$	0	$\frac{1}{2}p$	$\frac{1}{2}q$
Males	a	0	$\frac{1}{2}p$	$\frac{1}{2}q$	$\frac{1}{2}p$	$\frac{1}{2}q$

Now considering all the brother-sister (half-sibs) pairs under each male, we get the proportions of brother-sister pairs under each male as given in table (2.2)

Table 2.2

Brother-sister pairs

		(A, AA)	(A, Aa)	(A, aa)	(a, AA)	(A, Aa)	(a,aa)
	A(p)	$\frac{1}{4}p^2$	$\frac{1}{4}pq$	0	$\frac{1}{4}pq$	$\frac{1}{4}q^2$	0
Males	a(q)	0	$\frac{1}{4}p^2$	$\frac{1}{4}pq$	0	$\frac{1}{4}pq$	$\frac{1}{4}q^2$

Hence the joint distribution of brother-sister pairs under random mating can be obtained by pooling the corresponding pairs by weighting with p and q respectively, as given in table 2.3

Table 2.3

		Sister			
		AA	Aa	aa	Total
Brother	A	$\frac{1}{4}p^3$	$\frac{1}{2}p^2q$	$\frac{1}{4}pq^2$	$\frac{1}{4}p$
	a	$\frac{1}{4}p^2q$	$\frac{1}{2}pq^2$	$\frac{1}{4}q^3$	$\frac{1}{4}q$
Total		$\frac{1}{4}p^2$	$\frac{1}{2}pq$	$\frac{1}{4}q^2$	$\frac{1}{4}$

Standardising the above table so that the column totals as well as row totals add upto unity, we can write down the correlation table as table 2.4.

Table 2.4

Standarfiised correlation table of brother-sister pairs under random mating

		Sister			
		AA	Aa	aa	Total
Brother	A	p^3	$2p^2q$	pq^2	p
	a	p^2q	$2pq^2$	q^3	q
Total		p^2	$2pq$	q^2	1

The correlation coefficient, ${}_{H.S}r^{(0)}_{B.S}$, of brother-sister pairs obtained after random mating can be calculated directly from the above correlation table, assuming additive genetic effects and by using the product moment formula for correlation coefficients as,

$$ {}_{H.S}r^{(0)}_{B.S} = 0 \qquad \text{--- (2.1)} $$

This zero correlation is quite logical since, in the case of sex-linked genes the son receives his father's y chromosome and the different female parents in the random mating population are non-correlated.

The joint distribution of brother-sister pairs under the first generation of half-sib mating can be obtained directly by considering the different lines as follows. There are two lines in this case; line (i) corresponding to the male A with proportion p and with the genotypic array of half-sibs as

$$\tfrac{1}{2}p\ AA + \tfrac{1}{2}q\ Aa + \tfrac{1}{2}p\ A + \tfrac{1}{2}qa$$

and line (ii) corresponding to the male a with proportion q and with the genotypic array of half-sibs as

$$\tfrac{1}{2}p\ Aa + \tfrac{1}{2}q\ aa + \tfrac{1}{2}p\ A + \tfrac{1}{2}q\ a.$$

The procedure consists of finding a two-way table of the frequencies of brother-sister pairs corresponding to each line and then pooling these tables to get the joint distribution of brother-sister pairs by weighting them with the proportions of the corresponding lines. In the first generation of brother-sister mating, the standardised two-way tables of brother-sister pairs corresponding to line (i) and line (ii) can be obtained as given in table (2.5) and table (2.6) respectively.

Table 2.5

		Sister			
		AA	Aa	aa	Total
	A	$\frac{1}{4}p(1+p)^2$	$\frac{1}{4}q(1+p)(1+2p)$	$\frac{1}{4}q^2(1+p)$	p
Brother	a	$\frac{1}{4}pq(1+p)$	$\frac{1}{4}q^2(1+2p)$	$\frac{1}{4}q^3$	q
Total		$\frac{1}{2}p(1+p)$	$\frac{1}{2}q(1+2p)$	$\frac{1}{2}q^2$	1

Table 2.6

		Sister			
		AA	Aa	aa	Total
	A	$\frac{1}{4}p^3$	$\frac{1}{4}p^2(1+2q)$	$\frac{1}{4}pq(1+q)$	p
Brother	a	$\frac{1}{4}p^2(1+q)$	$\frac{1}{4}p(1+q)(1+2q)$	$\frac{1}{4}q(1+q)^2$	q
Total		$\frac{1}{2}p^2$	$\frac{1}{2}p(1+2q)$	$\frac{1}{2}q(1+q)$	1

Now the joint distribution of the brother-sister pairs under the first generation of brother-sister mating can be obtained by pooling these two-way tables after weighting them with p and q respectively, as given in table (2.7).

Table 2.7

Correlation table of brother-sister pairs under the first generation of half-sib mating.

		Sister AA	Sister Aa	Sister aa	Total
Brother	A	$\frac{1}{4}p^2(1+3p)$	$\frac{1}{4}pq(1+6p)$	$\frac{3}{4}pq^2$	p
Brother	a	$\frac{3}{4}p^2q$	$\frac{1}{4}pq(1+6q)$	$\frac{1}{4}q^2(1+3q)$	q
Total		p^2	2pq	q^2	1

The correlation coefficient, $r^{(1)}_{H.S\ B.S}$, between brother-sister pairs under the first generation of brother-sister mating can be directly obtained from the above correlation table as

$$ {}_{H.S}r^{(1)}_{B.S} = 0.1768 \quad \text{---- (2.2)} $$

In a similar manner, the joint distribution of brother-sister pairs under the second generation of half-sib mating can be obtained. Here there are four lines to be considered. These four lines have come from the two lines of the previous generation, ie., two from each. By pooling all the two-way tables obtained from these lines by weighting them with appropriate weights we can get the final standardised correlation table for the brother-sister pairs under the second generation of half-sib mating as given in table (2.8).

Table 2.8

Correlation table of brother-sister pairs under the second generation of half-sib mating.

		Sister AA	Aa	aa	Total
	A	$\frac{1}{32}p(1+15p+16p^2)$	$\frac{1}{8}pq(3+8p)$	$\frac{1}{32}pq(3+16q)$	p
Brother	a	$\frac{1}{32}pq(3+16p)$	$\frac{1}{8}pq(3+8q)$	$\frac{1}{32}q(1+15q+16q^2)$	q
Total		$\frac{1}{8}p(1+7p)$	$\frac{7}{4}pq$	$\frac{1}{8}q(1+7q)$	1

The correlation coefficient, $r^{(2)}_{H.S\ B.S}$, between brother-sister pairs under the second generation of half-sib mating can be directly obtained from the correlation table as,

$$r^{(2)}_{H.S\ B.S} = 0.2917 \quad \text{---(2.3)}$$

In the same manner, the corelation table for brother-sister pairs under the third generation of half-sib mating can be obtained by considering the eight lines, obtained, from the four lines of the previous generation, two from each. By pooling the two-way tables corresponding to the different lines by weighting them properly, the correlation table for the brother-sister pairs under the third generation of half-sib mating can be obtained as in table (2.9).

Table 2.9

Correlation table of brother-sister pairs under the third generation of half-sib mating.

		Sister			
		AA	Aa	aa	Total
	A	$\frac{1}{256}p(21+149p+86p^2)$	$\frac{1}{128}pq(57+86p)$	$\frac{1}{256}pq(35+86q)$	p
Brother	a	$\frac{1}{256}pq(35+86p)$	$\frac{1}{128}pq(57+86q)$	$\frac{q}{256}(21+149q+86q^2)$	q
Total		$\frac{1}{32}p(7+25p)$	$\frac{25}{16}pq$	$\frac{1}{32}q(7+25q)$	1

The correlation coefficient; $_{H.S}r^{(3)}_{B.S}$, of brother-sister pairs under the third generation of half-sib mating can be obtained as,

$$_{H.S}r^{(3)}_{B.S} = 0.3903 \qquad --- \qquad (2.4)$$

Proceeding in a similar manner, the correlation coefficient between brother-sister pairs under the fourth, fifth, sixth etc., generations can be found out. The correlation coefficient of brother-sister pairs for the first ten generations of half-sib mating is given in table (2.27).

2.12 BROTHER-BROTHER CORRELATION

The procedure of obtaining the joint distribution of brother-brother pairs under different generations of half-sib mating is similar to that we followed in the case of brother-sister pairs in section 2.11.

The table of proportions of half-sibs for the different constitutions the male under random mating has already been obtained in section 2.11 (Table 2.1). From this table, the proportions of the four types of brother-brother pairs under each male under random mating can be obtained as given in table (2.10).

Table 2.10

		Brother-Brother pairs			
		(A,A)	(A,a)	(a,A)	(a,a)
	A(p)	$\frac{1}{4}p^2$	$\frac{1}{4}pq$	$\frac{1}{4}pq$	$\frac{1}{4}q^2$
Males	a(q)	$\frac{1}{4}p^2$	$\frac{1}{4}pq$	$\frac{1}{4}pq$	$\frac{1}{4}q^2$

Hence the joint distribution of brother-brother pairs under random mating can be obtained by pooling the corresponding pairs by weighting with p or q according as the male is A or a, as given in table (2.11).

Table 2.11

		Brother 2		
		A	a	Total
Brother 1	A	$\frac{1}{4}p^2$	$\frac{1}{4}pq$	$\frac{1}{4}p$
	a	$\frac{1}{4}pq$	$\frac{1}{4}q^2$	$\frac{1}{4}q$
Total		$\frac{1}{4}p$	$\frac{1}{4}q$	$\frac{1}{4}$

Standardising the above table so that the row totals as well as the column totals add upto unity, we can write down the correlation table as in table (2.12).

Table 2.12

Standardised correlation table of brother-brother pairs under random mating

		Brother 2		
		A	a	Total
Brother 1	A	p^2	pq	p
	a	pq	q^2	q
Total		p	q	1

Now the matrix, ${}_{H.S}\underset{\sim}{M}^{(0)}_{B.B}$, of conditional probabilities of brother-brother pairs under random mating can be written down as

$${}_{H.S}\underset{\sim}{M}^{(0)}_{B.B} = \begin{pmatrix} p & q \\ p & q \end{pmatrix}$$ and this can be split as a function of $\underset{\sim}{I}_{22}$ and $\underset{\sim}{Q}_{22}$ (as defined by Li) as

$${}_{H.S}\underset{\sim}{M}^{(0)}_{B.B} = C_0 \times \underset{\sim}{I}_{22} + (1-C_0)\underset{\sim}{Q}_{22}$$

where, $\underset{\sim}{I}_{22} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\underset{\sim}{Q}_{22} = \begin{pmatrix} p & q \\ p & q \end{pmatrix}$ and $C_0 = 0$

Now the correlation between brother-brother pairs under random mating can be calculated by using the technique developed by Li and Sacks (1954) as

$${}_{H.S}r^{(0)}_{B.B} = C_0\, r_I + (1-C_0)\, r_0$$

where $r_I = 1$ and $r_0 = 0$ (r_I and r_0 are the correlation coefficient calculated from the two-way tables obtained by multiplying the rows of $\underset{\sim}{I}_{22}$ and $\underset{\sim}{Q}_{22}$ by p and q respectively.)

Hence, we get

$${}_{H.S}r^{(0)}_{B.B} = C_0 = 0 \qquad --- \quad (2.5)$$

The zero correlation of brother-brother pairs under random mating is because of the fact that the son receives his fathers y-chromosome.

The joint distribution of brother-brother pairs under the first generation of half-sib mating can be obtained by considering the two lines in this generation - line (i) corresponding to the male A and line (ii) corresponding to the male a. Just as in the case of brother-sister pairs, the procedure consists of finding a two-way table of frequencies of brother-brother pairs corresponding to each line and then pooling these tables by weighting them with the corresponding proportions (p or q) of the different lines, to get the joint distribution of brother-brother pairs.

The standardised two-way tables of frequencies of brother-brother pairs corresponding to line (i) and line (ii) are obtained as given in tables (2.13) and (2.14) respectively.

Table 2.13

		Brother 1	
		A	a
Brother 2	A	$\frac{1}{4}(1+p)^2$	$\frac{1}{4}(1+p)q$
	a	$\frac{1}{4}q(1+p)$	$\frac{1}{4}q^2$

Table 2.14

		Brother 2	
		A	a
Brother 1	A	$\frac{1}{4}p^2$	$\frac{1}{4}p(1+q)$
	a	$\frac{1}{4}p(1+q)$	$\frac{1}{4}(1+q)^2$

Now the joint distribution of the brother-brother pairs under the first generation of half-sib mating can be obtained by pooling these two two-way tables by weighting them with p and q respectively, as given in table (2.15).

Table 2.15

Correlation table of brother-brother pairs under the first generation of half-sib mating

		Brother 2		
		A	a	Total
Brother 1	A	$\frac{1}{4}p(1+3p)$	$\frac{3}{4}qp$	p
	a	$\frac{3}{4}pq$	$\frac{1}{4}q(1+3q)$	q
Total		p	q	1

The matrix, ${}_{H.S}\underset{\sim}{M}^{(1)}_{B.B}$, of conditional probabilities of brother-brotherpairs under the first generations of half-sib mating can be written down as

$$_{H.S}\underset{\sim}{M}^{(1)}_{B.B.} = \begin{bmatrix} \frac{1}{4}(1+3p) & \frac{3}{4}q \\ \frac{3}{4}p & \frac{1}{4}(1+3q) \end{bmatrix}$$

and this can be expressed as

$$_{H.S}\underset{\sim}{M}^{(1)}_{B.B} = \frac{1}{4}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{3}{4}\begin{bmatrix} p & q \\ p & q \end{bmatrix}$$

$$= C_1\underset{\sim}{I}_{22} + (1-C_1)\underset{\sim}{Q}_{22}$$

where, $C_1 = \frac{1}{4}$

Hence,

$$_{H.S}r^{(1)}_{B.B} = C_1 r_I + (1-C_1)r_0$$

$$= C_1$$

$$= \frac{1}{4} = 0.25 \qquad \text{---} \qquad (2.6)$$

In the second generation of half-sib mating there are four lines to be considered. By pooling all the two-way tables obtained from these lines by weighting properly, we can get the final correlation table of brother-brother pairs in the second generation of half-sib mating as given in table (2.16).

Table 2.16

Correlation table of brother-brother pairs under the second generation of half-sib mating

		Brother 2 A	Brother 2 a	Total
Brother 1	A	$\frac{1}{16}p(5+11p)$	$\frac{11}{16}pq$	p
	a	$\frac{11}{16}pq$	$\frac{1}{16}q(5+11q)$	q
Total	p		q	1

Then the matrix, ${}_{H.S}\underset{\sim}{M}^{(2)}_{B.B}$, of conditional probabilities of brother-brother pairs under the second generation of half-sib mating can be obtained as

$$ {}_{H.S}\underset{\sim}{M}^{(2)}_{B.B.} = \begin{pmatrix} \frac{1}{16}(5+11p) & \frac{11}{16}q \\ \frac{11p}{16} & \frac{1}{16}(5+11q) \end{pmatrix} $$

and this can be expressed as

$$ {}_{H.S}\underset{\sim}{M}^{(2)}_{B.B} = C_2\underset{\sim}{I}_{22} + (1-C_2)\underset{\sim}{Q}_{22} $$

where, $C_2 = \frac{5}{16}$

Hence $${}_{H.S}r^{(2)}_{B.B} = C_2$$
$$= \frac{5}{16} = 0.313 \qquad --- \quad (2.7)$$

Now, after finding out the two-way tables of frequencies of brother-brother pairs corresponding to the eight lines in the third generation of brother-sister mating, and pooling them by weighting with the coressponding proportions, the correlation table of brother-brother pairs under the third generation of half-sib mating can be obtained as given in table (2.17).

Table 2.17

Correlation table of brother-brother pairs under the third generation of half-sib mating

		Brother 2 A	Brother 2 a	Total
Brother 1	A	$\frac{1}{64}p(25+39p)$	$\frac{39}{64}pq$	p
	a	$\frac{39}{64}pq$	$\frac{1}{64}q(25+39q)$	q
Total		p	q	1

Hence,

$$_{H.S}\underset{\sim}{M}^{(3)}_{B.B} = \begin{bmatrix} \frac{1}{64}(25+39p) & \frac{39}{64}q \\ \frac{39}{64}p & \frac{1}{64}q(25+39q) \end{bmatrix}$$

which can be expressed as

$$_{H.S}\underset{\sim}{M}^{(3)}_{B.B.} = C_3 \underset{\sim}{I}_{22} + (1-C_3)\underset{\sim}{Q}_{22}$$

where $C_3 = \frac{25}{64}$

Therefore,

$$ {}_{H.S}r^{(3)}_{B.B} = C_3 $$

$$ = \frac{25}{64} = 0.391 \quad --- \quad (2.8) $$

Proceeding similarly, the correlation table of brother-brother pairs under different generations of half-sib mating and corresponding conditional probability matrices therefrom can be obtained.

In general,

$$ {}_{H.S}M^{(n)}_{B.B} \doteq C_n I_{22} + (1-C_n)\, O_{22} $$

and hence

$$ {}_{H.S}r^{(n)}_{B.B} = C_n $$

where

$$ C_n = 0,\ \tfrac{1}{4},\ 5/16,\ 25/64,\ 117/256,\ 529/1024,\ 2333/4096,\ \ldots $$

$$ \text{for } n = 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6, \text{ etc.} $$

The correlation coefficients of brother-brother pairs under the first ten generations of half-sib mating is as given in table (2.27). These correlations are represented graphically in Fig.5 by curve (2).

2.13 SISTER-SISTER CORRELATION

The method of construction of the correlation tables of sister-sister pairs under the different generations of half-sib mating is exactly the same as in the case of brother-sister and brother-brother pairs, with a slight difference, that, here we pair each female with each of her half-sisters instead of forming brother-sister pairs or brother-brother pairs as was done in the former two sections (3.11) and (2.12).

The proportions of the nine types of sister-sister pairs under each male under random mating can be obtained as given in table (2.18).

Table 2.18

Sister-sister pairs

		(AA, AA)	(AA, Aa)	(AA, aa)	(Aa, AA)	(Aa, Aa)	(Aa, aa)	(aa, AA)	(aa, Aa)	(aa, aa)
Males	A (p)	$\frac{1}{4}p^2$	$\frac{1}{4}pq$	0	$\frac{1}{4}pq$	$\frac{1}{4}q^2$	0	0	0	0
	a (q)	0	0	0	0	$\frac{1}{4}p^2$	$\frac{1}{4}pq$	0	$\frac{1}{4}pq$	$\frac{1}{4}q^2$

Hence the joint distribution of sister-sister pairs under random mating can be obtained by pooling the corresponding pairs by weighting with p or q according as the male is A or a, as given in table (2.19).

Table 2.19

		Sister 2			
		AA	Aa	aa	Total
Sister 1	AA	$\frac{1}{4}p^3$	$\frac{1}{4}p^2q$	0	$\frac{1}{4}p^2$
	Aa	$\frac{1}{4}p^2q$	$\frac{1}{4}pq$	$\frac{1}{4}pq^2$	$\frac{1}{4}pq$
	aa	0	$\frac{1}{4}pq^2$	$\frac{1}{4}q^3$	$\frac{1}{4}q^2$
Total		$\frac{1}{4}p^2$	$\frac{1}{2}pq$	$\frac{1}{4}q^2$	$\frac{1}{4}$

Standardising the above table so that the row totals as well as the column totals add upto unity, we can write down the correlation table as given in (2.20).

Table 2.20

Correlation table of sister-sister pairs under random mating

		Sister 2			
		AA	Aa	aa	Total
Sister 1	AA	p^3	p^2q	0	p^2
	Aa	p^2q	pq	pq^2	$2pq$
	aa	0	pq^2	q^3	q^2
Total		p^2	$2pq$	q^2	1

The correlation coefficient, $r^{(0)}_{H.S \; S.S}$, of sister-sister pairs under random mating can be calculated from the above table by assuming additive genetic effects and using the product moment correlation coefficient formula as

$$r^{(0)}_{H.S \quad S.S} = \tfrac{1}{4} \quad \text{-- (2.9)}$$

The paternal half-sisters under random mating possess a common gene received from their common male parent, hence a correlation of half.

Now, the standardised two-way table of frequencies of sister-sister pairs corresponding to the two lines (i) and (ii) in the first generation of half-sib mating can be obtained as given in tables (2.21) and (2.22) respectively.

Table 2.21

		Sister 2		
		AA	Aa	aa
	AA	$\frac{1}{4}p(1+p)^2$	$\frac{1}{4}pq(1+p)$	0
Sister 1	Aa	$\frac{1}{4}pq(1+p)$	$\frac{1}{4}q(1+3p)$	$\frac{1}{4}q^2(1+p)$
	aa	0	$\frac{1}{4}q^2(1+p)$	$\frac{1}{4}q^3$

Table 2.22

		Sister 2 AA	Aa	aa
	AA	$\frac{1}{4}p^3$	$\frac{1}{4}p^2(1+q)$	0
Sister 1	Aa	$\frac{1}{4}p^2(1+q)$	$\frac{1}{4}p(1+3q)$	$\frac{1}{4}pq(1+q)$
	aa	0	$\frac{1}{4}$ $pq(1+q)$	$\frac{1}{4}q(1+q)^2$

The joint distribution of sister-sister pairs under the first generation of half-sib mating can be obtained by pooling these two two-way tables by weighting them with p and q respectively, as given in table (2.23).

Table 2.23

Correlation table of sister-sister pairs under the first generation of half-sib mating

		Sister 2 AA	Aa	aa	Total
	AA	$\frac{1}{4}p^2(1+3p)$	$\frac{3}{4}p^2q$	0	p^2
Sister 1	Aa	$\frac{3}{4}p^2q$	$\frac{5pq}{4}$	$\frac{3}{4}pq^2$	$2pq$
	aa	0	$\frac{3}{4}pq^2$	$\frac{1}{4}q^2(1+3q)$	q^2
Total		p^2	$2pq$	q^2	1

The correlation coefficient, ${}_{H.S}r^{(1)}_{S.S}$, of sister-sister pairs under the first generation of half-sib mating can be calculated from the above correlation table, using product moment correlation coefficient formula as,

$$ {}_{H.S}r^{(1)}_{S.S} = \frac{5}{8} = 0.625 \quad \text{--} \quad (2.10)$$

The correlation table of sister-sister pairs under the second generation of half-sib mating can be obtained by pooling the two-way tables corresponding to the four lines after weighting them properly. It is as given in table (2.24).

Table 2.24

Correlation table of sister-sister pairs under the second generation of half-sib mating

Sister 2

		AA	Aa	aa	Total
	AA	$\frac{1}{32}p(1+15p+16p^2)$	$\frac{1}{32}pq(3+16p)$	0	$\frac{1}{8}p(1+7p)$
Sister 1	Aa	$\frac{1}{32}pq(3+16p)$	$\frac{17}{16}pq$	$\frac{1}{32}pq(3+16p)$	$\frac{7}{4}pq$
	aa	0	$\frac{1}{32}pq(3+16q)$	$\frac{1}{32}q(1+15q+16q^2)$	$\frac{1}{8}q(1+7q)$
Total		$\frac{1}{8}p(1+7p)$	$\frac{7}{4}pq$	$\frac{1}{8}q(1+7q)$	1

Hence the correlation coefficient, $r^{(2)}_{H.S\;\;S.S}$, of sister-sister pairs under the second generation of half-sib mating can be obtained from the above correlation table, using product moment correlation coefficient formula as

$$r^{(2)}_{H.S\;\;S.S} = 0.694 \qquad ---- \qquad (2.11)$$

The correlation table of sister-sister pairs under the third generation of half-sib mating can be obtained by pooling the two-way tables of frequencies of sister-sister pairs corresponding to each of the eight lines in the third generation, after weighting them properly, as given in table (2.25)

Table 2.25

Correlation table of sister-sister pairs under the third generation of half-sib mating

		Sister 2			
		AA	Aa	aa	Total
Sister 1	AA	$\frac{1}{256}p(21+149p+86p^2)$	$\frac{1}{256}pq(35+86p)$	0	$\frac{1}{32}p(7+25p)$
	Aa	$\frac{1}{256}pq(35+86p)$	$\frac{61}{64}pq$	$\frac{1}{256}pq(35+86q)$	$\frac{25}{16}pq$
	aa	0	$\frac{1}{256}pq(35+86q)$	$\frac{1}{256}q(21+149q+86q^2)$	$\frac{1}{32}q(7+25q)$
Total		$\frac{1}{32}p(7+25p)$	$\frac{25}{16}pq$	$\frac{1}{32}q(7+25q)$	1

The correlation coefficient, ${}_{H.S}r^{(3)}_{S.S}$, of sister-sister pairs under the third generation of half-sib mating can be obtained directly from this correlation table, using product moment correlation coefficient formula as

$${}_{H.S}r^{(3)}_{S.S} = 0.75 \qquad --- \quad (2.12)$$

Proceeding similarly, the correlations coefficients of sister-sister pairs in the fourth, fifth, sixth, ... generations of half-sib mating can be obtained. The correlation coefficients in the first ten generations of half-sib mating are as given in table (2.27). These correlations are exhibited graphically in Fig.5 by curve (3).

2.2 CORRELATION BETWEEN PARENT AND OFFSPRING UNDER HALF-SIB MATING SYSTEM

George (1974) has derived the generation matrices for the various parent-offspring pairs in the sex-linked gene case viz., mother-daughter, mother-son, father-daughter and father-son and has obtained the joint distribution and correlation coefficients under the differnt generations of full-sib mating and parent-offspring mating system. Here an attempt is made to extend the use of these generation matrices for obtaining the correlation coefficients of the various parent-offspring pairs in the sex-linked gene case under the half-sib mating system of inbreeding.

The different types of parent-offspring pairs are considered seperately under the following sections.

2.21 MOTHER - DAUGHTER CORRELATION

The generation matrix, $\underset{\sim}{A}_{M.D}$, of mother-daughter pairs, obtained by George (1974) is as given below.

$$\underset{\sim}{A}_{M.D} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2.13)$$

Hence the column vector, $\underline{V}^{(n)}_{M-D}$, of joint distribution of mother-daughter pairs under the n^{th} generation of half-sib mating can be obtained as,

$$\underline{V}^{(n)}_{M.D} = \underset{\sim}{A}_{M.D} \; \underline{v}^{(n)} \qquad \text{---} \quad (2.14)$$

where $\underline{v}^{(n)}$ is the column vector of frequencies of half-sib mating types in the n^{th} generation of half-sib mating, as already obtained in section 2.11. The correlation coefficient, ${}_{H.S}r^{(n)}_{M.D}$, of mother-daughter pairs in the n^{th} generation of half-sib mating can be calculated from the correlation table formed from $\underline{V}^{(n)}_{M.D}$, by assuming additive genic effect and scoring AA, Aa, aa as 2, 1, 0 respectively.

Now $\underline{v}^{(1)}$, the row vector of frequencies of brother-sister mating types in the first generation of half-sib mating (ie.) brother-sister pairs obtained after random mating is given by

$$v^{(1)'} = \left[p^3 \quad p^2q \quad 2p^2q \quad 2pq^2 \quad pq^2 \quad q^3 \right] (2.5)$$

Hence the row vector, $\underline{V}^{(1)'}_{M.D}$, of frequencies of mother-daughter pairs in the first generation of half-sib mating can be obtained as

$$\underline{V}^{(1)'}_{M.D} = \underline{v}^{(1)'} \underset{\sim}{A}'_{M.D}$$

ie. $\underline{V}^{(1)}_{M.D} = \left[p^3 \quad p^2q \quad 0 \quad p^2q \quad pq \quad pq^2 \quad 0 \quad pq^2 \quad q^3 \right]$ (2.16)

and the corresponding correlation table can be formed from $\underline{V}^{(1)}_{M.D}$ as given in table (2.26).

Table 2.26

Correlation table of mother-daughter pairs in the first generation of half-sib mating

		Daughter			
		AA	Aa	aa	Total
	AA	p^3	p^2q	0	p^2
Mother	Aa	p^2q	pq	pq^2	$2pq$
	aa	0	pq^2	q^3	q^2
Total		p^2	$2pq$	q^2	1

The correlation coefficient, $_{H.S}r^{(1)}_{M.D}$, of mother-daughter pairs in the first generation of half-sib mating can be obtained from the above table as

$$_{H.S}r^{(1)}_{M.D} = 0.5 \quad --- \quad (2.17)$$

In the same manner, the column vectors of the frequencies of the brother-sister mating types in the second, third and fourth generations of half-sib mating can be obtained as

$$\underline{v}^{(2)}_{M.D} = \underset{\sim}{A}_{M.D}\ \underline{v}^{(2)}$$

$$\underline{v}^{(2)}_{M.D} = \underset{\sim}{A}_{M.D}\ \underline{v}^{(3)}$$

$$\underline{v}^{(4)}_{M.D} = \underset{\sim}{A}_{M.D}\ \underline{v}^{(4)}$$

where $\underline{v}^{(2)}$, $\underline{v}^{(3)}$ and $\underline{v}^{(4)}$, already obtained in section 1.1, are as given in (2.21), (2.22), and (2.23) respectively.

$$\underline{v}^{(2)} = \begin{bmatrix} \frac{1}{4}p^2(1+3p) \\ \frac{3}{4}p^2q \\ \frac{1}{4}pq(1+6p) \\ \frac{1}{4}pq(1+6q) \\ \frac{3}{4}pq^2 \\ \frac{1}{4}q^2(1+3q) \end{bmatrix} \qquad (2.21)$$

$$\underline{v}^{(3)} = \begin{bmatrix} \frac{1}{32}p(1+15p+16p^2) \\ \frac{1}{32}pq(3+16p) \\ \frac{1}{8}pq(3+8p) \\ \frac{1}{8}pq(3+8q) \\ \frac{1}{32}pq(3+16q) \\ \frac{1}{32}q(1+15q+16q^2) \end{bmatrix} \qquad (2.22)$$

$$\underline{v}^{(4)} = \begin{bmatrix} \frac{1}{256}p(21+149p+86p^2) \\ \frac{1}{256}pq(35+86p) \\ \frac{1}{128}pq(57+86p) \\ \frac{1}{128}pq(57+86q) \\ \frac{1}{256}pq(35+86q) \\ \frac{1}{256}q(21+149q+86q^2) \end{bmatrix} \qquad (2.23)$$

Hence the column vectors $\underline{V}^{(2)}_{M.D}$, $\underline{V}^{(3)}_{M.D}$ and $\underline{V}^{(4)}_{M.D}$ of frequencies of mother-daughter pairs in the second, third and fourth generations of brother-sister mating can be obtained as given in (2.24), (2.25) and (2.26) respectively.

$$\underline{V}^{(2)}_{M.D} = \begin{bmatrix} \frac{1}{4}p^2(1+3p) \\ \frac{3}{4}p^2q \\ 0 \\ \frac{1}{8}pq(1+6p) \\ pq \\ \frac{1}{8}pq\ (1+6q) \\ 0 \\ \frac{3}{4}pq^2 \\ \frac{1}{4}q^2(1+3q) \end{bmatrix} \qquad (2.24)$$

$$\underline{V}^{(3)}_{M.D.} = \begin{bmatrix} \frac{1}{32}p(1+15p+16p^2) \\ \frac{1}{32}pq(3+16p) \\ 0 \\ \frac{1}{16}pq(3+8p) \\ \frac{7}{8}pq \\ \frac{1}{16}pq(3+8q) \\ 0 \\ \frac{1}{32}pq(3+16q) \\ \frac{1}{32}q(1+15q+16q^2) \end{bmatrix} \qquad (2.25)$$

$$\underline{V}^{(3)}_{M.D} = \begin{bmatrix} \frac{1}{256}p(21+149p+86p^2) \\ \frac{1}{256}pq(35+86p) \\ 0 \\ \frac{1}{256}pq(57+86p) \\ \frac{25}{32}pq \\ \frac{1}{256}pq(57+86p) \\ 0 \\ \frac{1}{256}pq(35+86q) \\ \frac{1}{256}q(21+149q+86q^2) \end{bmatrix} \qquad (2\text{-}26)$$

The correlations between the mother and daughter in the second, third and fourth generations of half-sib mating are calculated from the correlation tables formed from

$\underline{V}^{(2)}_{M.D}$, $\underline{V}^{(3)}_{M.D}$, $\underline{V}^{(4)}_{M.D}$ as

$$_{H.S}r^{(2)}_{M.D} = 0.5893 \qquad (2.18)$$

$$_{H.S}r^{(3)}_{M.D} = 0.6672 \qquad (2.19)$$

$$_{H.S}r^{(4)}_{M.D} = 0.7249 \qquad (2.20)$$

Proceeding similarly, the correlation coefficients of mother-daughter pairs in the fifth, sixth, seventh etc. generations of half-sib mating can be calculated. The correlations of mother-daughter pairs in the first ten generations of half-sib mating is given in table (2.27). These correlations are exhibited graphically in Fig.5 by curve (4).

2.2 MOTHER-SON CORRELATION

The joint distribution of mother-son pairs in the n^{th} generation of half-sib mating can be obtained as

$$\underline{v}^{(n)}_{M.S} = \underset{\sim}{A}_{M.S} \; \underline{v}^{(n)} \qquad (2.27)$$

where

$\underline{v}^{(n)}_{M.S}$ is the column vector of frequencies of mother-son pairs in the n^{th} generation of half-sib mating, $\underline{v}^{(n)}$ is the column vector of frequencies of brother-sister mating types in the n^{th} generation of half-sib mating and $\underset{\sim}{A}_{M.S}$ is the generation matrix of mother-son pairs as derived by George (1974) as given below.

$$\underset{\sim}{A}_{M.S} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \qquad (2.28)$$

Thus by substituting the vectors $\underline{v}^{(1)}$, $\underline{v}^{(2)}$, $\underline{v}^{(3)}$, and $\underline{v}^{(4)}$ as given in (2.15), (2.21), (2.22) and (2.23), the column vectors $\underline{V}^{(1)}_{M.S}$, $\underline{V}^{(2)}_{M.S}$, $\underline{V}^{(3)}_{M.S}$ and $\underline{V}^{(4)}_{M.S}$ of frequencies of mother-son pairs in the first four generations of half-sib mating can be obtained as given below.

$$\underline{V}^{(1)}_{M.S} = \begin{bmatrix} p^2 \\ 0 \\ pq \\ pq \\ 0 \\ q^2 \end{bmatrix} \qquad (2.29)$$

$$\underline{V}^{(2)}_{M.S} = \begin{bmatrix} \frac{1}{8}p(1+7p) \\ 0 \\ \frac{7pq}{8} \\ \frac{7pq}{8} \\ 0 \\ \frac{1}{8}q(1+7q) \end{bmatrix} \qquad (2.30)$$

$$\underline{V}^{(3)}_{M.S} = \begin{bmatrix} \frac{1}{32}p(7+25p) \\ 0 \\ \frac{25}{32}pq \\ \frac{25}{32}pq \\ 0 \\ \frac{1}{32}q(7+25q) \end{bmatrix} \qquad (2.31)$$

$$\underline{V}^{(4)}_{M.S} = \begin{bmatrix} \frac{1}{128}p(39+89p) \\ 0 \\ \frac{89}{128}pq \\ \frac{89}{128}pq \\ 0 \\ \frac{1}{128}q(39+89q) \end{bmatrix} \qquad (2.32)$$

The correlation coefficient of mother-son pairs in the first, second, third and fourth generations of half-sib mating can be obtained from the correlation table formed from $\underline{V}^{(1)}_{M.S}$, $\underline{V}^{(2)}_{M.S}$, $\underline{V}^{(3)}_{M.S}$, and $\underline{V}^{(4)}_{M.S}$ respectively as

$$r^{(1)}_{H.S\ M.S} = 0.707 \quad --- \quad (2,33)$$

$$r^{(2)}_{H.S\ M.S} = 0.750 \quad --- \quad (2.34)$$

$$r^{(3)}_{H.S\ M.S} = 0.781 \quad --- \quad (2.35)$$

$$r^{(4)}_{H.S\ M.S} = 0.808 \quad --- \quad (2.36)$$

Proceeding similarly the correlation coefficients of mother-son pairs in the fifth, sixth, seventh etc. generations of half-sib mating can be calculated. The correlation coefficients of mother-son pairs in the first ten generations of half-sib mating is given in table (2.27). These correlations are represented graphically in Fig.5 by curve (5).

2.23 <u>FATHER-DAUGHTER CORRELATION</u>

The column vector, $\underline{V}^{(n)}_{F.D}$, of the joint distribution of father-daughter pairs in the n^{th} generation of half-sib mating can be obtained as

$$\underline{V}^{(n)}_{F.D} = \underset{\sim}{A}_{F.D}\ \underline{v}^{(n)} \quad --- \quad (2.37)$$

where

$\underset{\sim}{A}_{F.D}$ is the generation matrix of father-daughter pairs, as defined by George (1974) as

$$\underset{\sim}{A}_{F.D} = \begin{bmatrix} 1 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 1 \end{bmatrix} \qquad (2.38)$$

Substituting the values of $\underline{v}^{(n)}$, $(n = 1, 2, 3, 4)$, as given in (2.15), (2.21), (2.22), (2.23) and $\underset{\sim}{A}_{F.D}$, the the column vectors $\underline{V}^{(n)}_{F.D}$ $(n = 1, 2, 3, 4)$ of frequencies of mother-son pairs in the first four generation of half-sib mating can be obtained as given below.

$$\underline{V}^{(1)}_{F.D} = \begin{bmatrix} p^2 \\ 0 \\ pq \\ pq \\ 0 \\ q^2 \end{bmatrix} \qquad (2.39)$$

$$\underline{V}^{(2)}_{F.D} = \begin{bmatrix} \frac{1}{8}p(1+7p) \\ 0 \\ \frac{7}{8}pq \\ \frac{7}{8}pq \\ 0 \\ \frac{1}{8}q(1+7q) \end{bmatrix} \qquad (2.40)$$

$$\underline{V}^{(3)}_{F.D} = \begin{bmatrix} \frac{1}{32}p(7+25p) \\ 0 \\ \frac{25}{32}pq \\ \frac{25}{32}pq \\ 0 \\ \frac{1}{32}q(7+25q) \end{bmatrix} \qquad (2.41)$$

$$\underline{V}^{(4)}_{F.D.} = \begin{bmatrix} \frac{1}{128}p(39+89p) \\ 0 \\ \frac{89}{128}pq \\ \frac{89}{128}pq \\ 0 \\ \frac{1}{128}q(39+89q) \end{bmatrix} \qquad (2.42)$$

The correlation coefficients of father-daughter pairs in the first four generations of half-sib mating can be calculated directly from the correlation tables formed from $\underline{V}^{(1)}_{F.D}$, $\underline{V}^{(2)}_{F.D}$, $\underline{V}^{(3)}_{F.D}$, $\underline{V}^{(4)}_{F.D}$, respectively as given below.

$$ {}_{H.S}r^{(1)}_{F.D} = 0.707 \qquad (2.43)$$

$$ {}_{H.S}r^{(2)}_{F.D} = 0.750 \qquad (2.44)$$

$$ {}_{H.S}r^{(3)}_{F.D} = 0.781 \qquad (2.45)$$

$$ {}_{H.S}r^{(4)}_{F.D} = 0.808 \qquad (2.46)$$

In a similar manner, the correlation coefficients of father-daughter pairs in the fifth, sixth, seventh, etc. generations of half-sib mating can be calculated. The correlation coefficients of father-daughter pairs in the first ten generations of brother-sister mating are given in table (2.27). These correlations are represented graphically in Fig.5 by curve (5).

Table 2.27

Correlations of the various relative pairs in the sex-linked gene case, under different generations of half-sib mating system.

Correlation	Generation (n) 0	1	2	3	4	5	6	7	8	9	10
Brother-sister correlation $_{H.S}r^{(n)}_{B.S}$	0	0.177	0.292	0.390	0.472	0.540	0.598	0.648	0.691	0.728	0.761
Brother-sister correlation $_{H.S}r^{(n)}_{B.B}$	0	0.250	0.313	0.391	0.457	0.517	0.570	0.617	0.659	0.696	0.729
Sister-sister correlation $_{H.S}r^{(n)}_{S.S}$	0.5	0.625	0.694	0.750	0.792	0.825	0.851	0.873	0.891	0.906	0.918
Mother-daughter correlation $_{H.S}r^{(n)}_{M.S}$	0.5	0.500	0.589	0.667	0.725	0.770	0.805	0.834	0.858	0.878	0.893
Mother-son correlation $_{H.S}r^{(n)}_{M.S}$	0.707	0.707	0.750	0.781	0.808	0.831	0.851	0.869	0.884	0.897	0.909
Father-daughter correlation $_{H.S}r^{(n)}_{F.D}$	0.707	0.707	0.750	0.781	0.808	0.831	0.851	0.869	0.884	0.897	0.909

Note: 0^{th} generation stands for random mating.

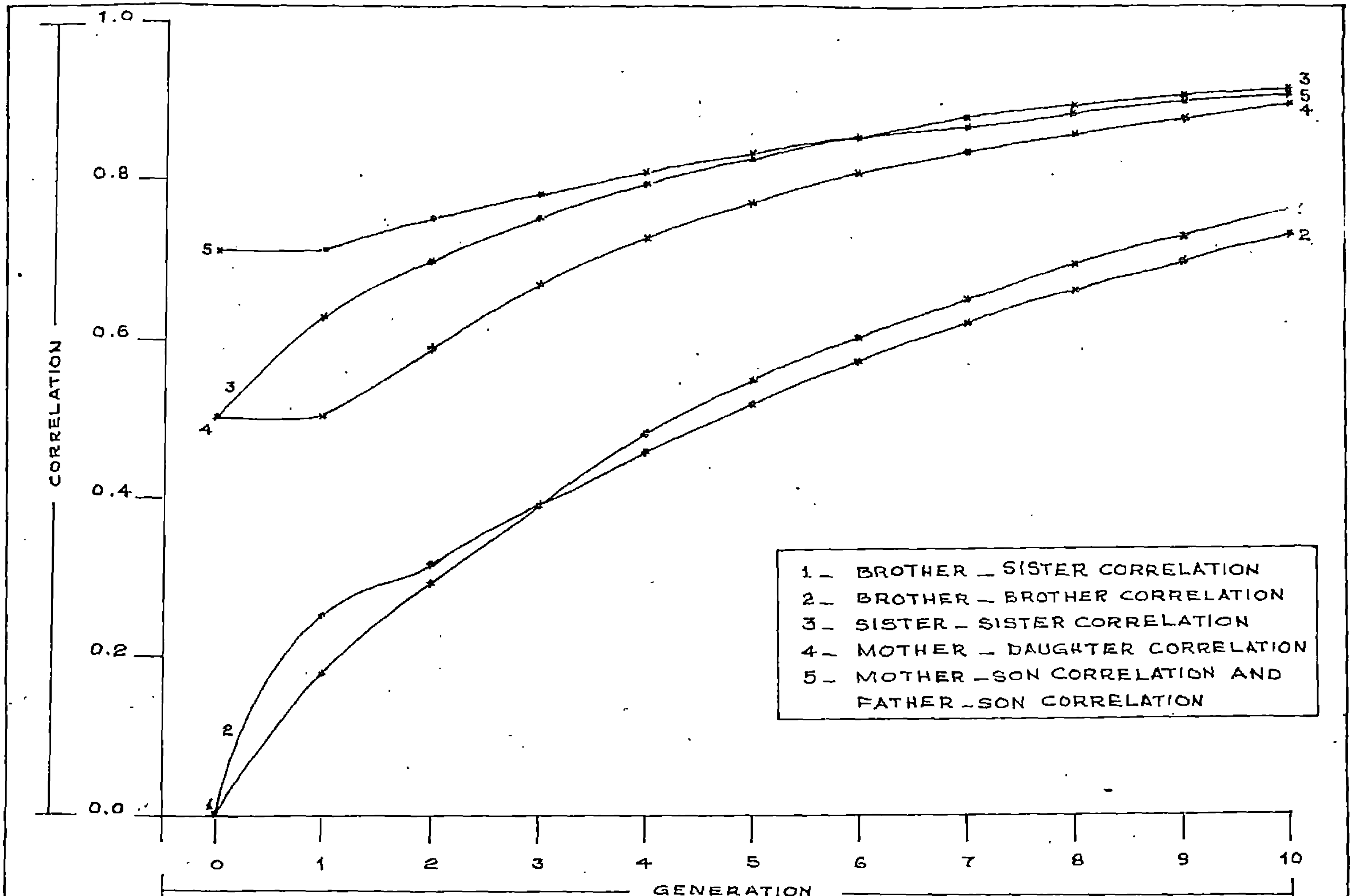

FIG: 5. HALF - SIB CORRELATIONS AND PARENT - OFFSPRING CORRELATIONS IN THE SEX - LINKED GENE CASE, UNDER HALF-SIB MATING SYSTEM

SUMMARY AND CONCLUSIONS

SUMMARY AND CONCLUSION

The objectives of the present investigation entitled 'a study of half-sib correlations and parent-offspring correlations under half-sib mating system', was mainly three-fold, viz., (i) to derive the joint distribution - correlation table as well as the correlation of half-sib pairs under different generations of half-sib mating, in the case of autosomal genes, assuming single locus with two alleles; (ii) to derive the joint distribution - correlation table as well as the correlation of parent-offspring pairs under different generations of half-sib mating, in the case of autosomal genes, assuming single locus with two alleles and (iii) to derive the joint distribution correlation table and the correlation between both the parents and several offspring as well as one parent and several offspring under different generations of half-sib mating, in the case of autosomal genes, assuming single locus with two alleles.

Over and above this, an attempt was also made to study the joint distribution - correlation table as well as the correlation between the different half-sib pairs, viz., brother - sister, brother-brother and sister - sister and also the correlations of different parent-offspring pairs, viz., mother-daughter, mother-son and father-daughter, in the sex-linked gene case.

Very little work had been done in the case of half-sib mating system especially to derive the joint distribution and hence to obtain the correlation coefficient of different relative pairs. As the generation matrix in this system cannot be easily obtained, George (1979) made an attempt to derive the joint distribution of half-sib pairs from the first principles, in the autosomal gene case. He confined his study to the first three generations of half-sib mating.

In the first chapter, the joint distribution of half-sib pairs under the fourth generation of half-sib mating was derived from first principles and the corresponding correlation was worked out therefrom. Further, a series of half-sib correlation coefficients for the first ten generations of half-sib mating was obtained as given in table (1.16). It could be observed from this table that the correlation coefficient steadily increases from 0.250 under random mating to 0.858 under the tenth generation of half-sib mating. This is fully in agreement with the principles of inbreeding. The chapter further concerned with the derivation of the joint distribution of parent-offspring pairs and the calculation of parent-offspring correlation, adopting the generation matrix approach. The correlation coefficients of parent-offspring pairs in the first ten generations of half-sib mating were worked out and given in table (1.16). The table indicated that the correlation increases from 0.5 under random mating to 0.908, in the tenth generation of half-sib mating, which is also in full agreement with the phenomena of inbreeding. These two correlations, half-sib correlation and parent-offspring correlation, were exhibited graphically in Fig.2 by curve (2) and curve (1) respectively. A comparative study of these two curves revealed that the parent-offspring correlations are comparatively of higher order than half-sib correlations. The former was double the latter, under random mating; but the gap was found to reduce as the number of generation increased. In the tenth generation, the difference between the two correlations was only by 0.05. This shows that half-sib correlation increases at a faster rate than parent-offspring correlation, as the inbreeding goes on.

Under section 1.31 in chapter 1, a general term for the correlation between both the parents and k offspring under each of the first ten generations was derived (Table 2.18) in the lines of George (1979). By giving different values to k from 1 to 10 the correlation coefficients between both the parents and k offspring for the first ten generations of half-sib mating were worked out as given in table (1.19). These correlations were also exhibited graphically in Fig.3. The correlations were found to be of very high order ranging from 0.707 to 0.994 and beyond the 5^{th} generation of half-sib mating, the correlations were of the order 0.9, even when k = 1. This indicated that the correlations attain a saturation point in twelve generations or so. From Fig.3

it could be noted that the correlation increases very rapidly for smaller values of k ie., when k = 1 to 5, and for further values of k, the increase in correlation is very negligible. However, in all generations, there is a tendency shown by the correlation to attain unity as the number of offspring increases.

Under section 1.32, the correlation between one parent and k offspring was dealt with. The expressions for the correlation coefficient for the first ten generations of half-sib mating, in the general case of k offspring, was derived as given in table (2.21). By giving values to k from 1 to 10 the correlations between parent and k offspring for the first ten generations of half-sib mating were worked out as given in table (2.22). Here also, as in the previous case, the correlation was found to increase as n, the number of generations and k, the number of offspring, increase. But the range of correlations was rather wide - from 0.50 to 0.953. These correlations were exhibited graphically in Fig.4. Which revealed that the correlations increase rather steadily from k = 1 to 5 and beyond k = 5 the increase was almost in a linear fashion.

A comparative study of these two correlations, as given in table (2.19) and (2.21), indicated that the correlations in both parent case are comparatively of higher order than that of one parent case. By comparing the graphs of these two correlations(Fig.3 and Fig.4)it was observed that the trend in Fig.4 is the same as that in Fig.3, but in the latter case the curves were slightly depressed all along, since the rate of increase was rather less in this case.

Chapter 3 concerned with the correlations of different relative pairs, viz., brother-sister, brother-brother, sister-sister, mother-daughter, mother-son and father-daughter, in the case of sex-linked genes. In all these six cases, the joint distributions were derived and the correlation coefficients worked out for the first ten generations. The correlations of the different relative pairs in the first ten generations were given in table (3.27). From this table it was noted that the mother-son correlations and the father-daughter correlations are identical and are of very high order ranging from 0.707 to 0.909. But, as the rate of increase was relatively high in the case of sister-sister correlation, it exceeded even the mother-son correlation after the sixth generation. However, the sister-sister correlation was found to be of much higher order than brother-brother and brother-sister correlations, at every generation of half-sib mating. It was also noted that the correlations of all these six types of relative pairs will be tending to unity as the number of generations increases indefinitely. Further, the mother-daughter correlations in the sex-linked gene case were the same as the parent-offspring correlations in the autosomal gene case except for a shift of one generation.

Another interesting point noted was that the joint distribution and correlation coefficient of brother-brother pairs can be easily worked out in the same lines explained by Li and Sacks (1954).

A comparative study of all these six types of correlations was also made graphically as shown in Fig.5. From this figure it can be observed that the correlation under random mating and under the first generation of half-sib mating remains the same in the case of mother-daughter pairs as well as mother-son pairs. The brother-brother correlation was noticed to be the lowest order correlation among all and there was a constriction between generations (1) and (2) as the rate of increase in correlation reduced considerably after the second generation of half-sib mating.

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A STUDY OF HALF-SIB CORRELATIONS AND PARENT-OFFSPRING CORRELATIONS UNDER HALF-SIB MATING SYSTEM

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ABSTRACT OF THE THESIS

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ABSTRACT

A purely theoretical investigation entitled 'a study of half-sib correlations and parent-offspring correlations under half-sib mating system' was carried out with the following objectives.

i) to derive the joint distribution - correlation table as well as the correlation of half-sib pairs under different generations of half-sib mating, in the case of autosomal genes, assuming single locus with two alleles.

ii) to derive the joint distribution - correlation table as well as the correlation of parent-offspring pairs under different generations of half-sib mating, in the case of autosomal genes, assuming single locus with two alleles.

iii) to derive the joint distribution - correlation table and the correlation between both the parents and several offspring as well as the correlation between one parent and several offspring under different generations of half-sib mating, in the case of autosomal genes, assuming single locus with two alleles.

iv) to derive the joint distribution - correlation table as well as the correlation of the different 'half-sib pairs, viz., brother-sister, brother-brother and sister-sister and also the correlations of different parent-offspring pairs, viz., mother-daughter, mother-son and father-daughter, in the sex-linked gene case.

As the generation matrix of half-sib pairs is too tedious to derive, the joint distribution of half-sib pairs, both in the autosomal gene case and sex-linked gene case, was derived from first principles and the correlation was calculated from the correlation table, assuming additive genic effects and by using the product-moment correlation coefficient formula. The correlations were worked out for the first ten generations of half-sib mating.

The joint distribution of parent-offspring pairs under half-sib mating system, both in the autosomal gene case and sex-linked gene case was derived by using the generation matrix approach. Here also, the correlations were worked out for the first ten generations of half-sib mating.

A comparative study of the half-sib correlations and parent-offspring correlations, conducted both numerically and graphically, revealed that even though the half-sib correlations are of lower order than parent-offspring correlations, the rate of increase in the former is much higher than the rate of increase in the latter.

Among the correlations of various half-sib pairs in the sex-linked gene case, the correlation of sister-sister pairs was found to be of the highest order. It was also observed that the correlation of brother-brother pairs can be easily obtained by the I.T.O method developed by Li and Sacks (1954). Regarding the various parent-offspring pairs, mother-son correlations and father-daughter correlations were found to be the same and there was no correlation between father and son which is quite logical in the case of half-sib mating as the son receives his father's y-chromosome.

A comparative study of the half-sib correlations and parent-offspring correlations in the sex-linked gene case was carried out both numerically and graphically. It was noted that even though the mother-son correlations and the father-daughter correlations are of very high order ranging from 0.707 to 0.909, the sister-sister correlation exceeds them after the sixth generation. From this it is obvious that the correlation between sister-sister pairs increases much rapidly than the correlation between mother-son or father-daughter pairs. It was also noted that the correlations of all these six types of relative pairs tend to unity as the number of generations increases indefinitely.

A study was also made on the correlation between both the parents and k offspring as well as that between one parent and k offspring, in the case of autosomal genes. A general term (ie. when number of offspring is k) for the correlation in each of the above cases was derived for each of the ten generation of half-sib mating. The correlation were found to increase with the number of generations and k, the number of offspring. It was also noted that the correlations in both parent case are comparatively of higher order than that of one parent case and the rate of increase in the former was much greater than the rate of increase in the latter. The numerical values of these correlations were worked out for k = 1 to 10 and also for the first ten generations of half-sib mating as given in table (1.19) and (1.22) respectively.