CONFOUNDED ASYMMETRICAL FACTORIAL DESIGNS CONSTRUCTION AND ANALYSIS

By

SANTY GEORGE

THESIS

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Dedicated to

My beloved Professor

Late Dr. P. U. Surendran

DECLARATION

I hereby declare that this thesis entitled "CONFOUNDED ASYMMETRICAL FACTORIAL DESIGNS-CONSTRUCTION AND ANALYSIS" is a bonafide record of research work done by me during the course of research and that the thesis has not previously formed the basis for the award to me of any other degree, diploma, associateship, fellowship, or other similar title, of any other University or Society.

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SANTY GEORGE

CERTIFICATE

Certified that this thesis, entitled "CONFOUNDED ASYMMETRICAL FACTORIAL DESIGNS - CONSTRUCTION AND ANALYSIS" is a record of research work done independently by Kumari Santy George under my guidance and supervision and that it has not previously formed the basis for the award of any degree, fellowship, or associateship to her.

K. C. George

Dr. K.C. GEORGE,
(Chairman, Advisory Board),
Professor and Head,
Department of Statistics.

Place: Mannuthy,
Date : 31-1-84

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C O N T E N T S

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Introduction

INTRODUCTION

Factorial experiments is one of the important developments in the field of design of experiments initiated by many statistical research workers in the beginning of the twentieth century. As the symmetrical factorial experiment need a large number of treatment combinations for its complete layout, it faced a lot of difficulties in the initial stages of its introduction. People who worked in this aspect were mainly Fisher, Yates, Bose, Kishen, Nair, Rao, Das etc. In order to apply this design in a more efficient manner, a special technique known as confounding was introduced by the same authors. After the introduction of confounded factorial experiments, the layout of the experiment and the efficiency of its analysis increased considerably. Hence, this confounded factorial experiment became more prevalent technique in design of experiments especially in the field of agriculture. A lot of literature is available in this aspect by many authors. The prominent among them are Yates, Kempthorne, Cochran and Cox, Nair, Rao, Kishen, Das etc.

The factorial experiment confounded or not confounded require the application of each factor in equal levels.

Hence a large number of treatment combinations are needed while making the levels of each factor equal, eventhough, we may not require all these levels in most of the situations. This means for the sake of balanced arrangement an experimentor has to face lot of inconveniences by way of taking unwanted levels of different factors. This certainly is a main disadvantage of the symmetrical factorial experiment.

Many research workers started thinking in this line and arrived at a common decision of including only the needed levels of the various factors under consideration. This means the symmetry of the previous factorial experiment cannot be maintained. Only the needed levels will be taken into consideration while taking the factorial combinations of various factors. This concept led to the introduction of asymmetrical factorial experiments. Confounding also is practised in order to reduce the block size in asymmetrical factorial design. The workers in this field are mainly Yates, Nair, Rao, Kempthorne, Zelen, Good, Kishen, Srivastava, Li, Das, Tyagi, Sardana, Raghava Rao, Banerjee, Dean, John and many others. Most of these authors constructed asymmetrical factorial designs which are suited to the specific fields of their investigations and generalisation of their results within that field only. For example, Yates (1937) has given the construction and analysis of asymmetrical factorial designs involving factors at two or three levels or powers of these levels only. But, Chakravarti (1956) has given a general type of asymmetrical factorial design viz., $s_1^{m_1} \times s_2^{m_2} \times \ldots s_g^{m_g}$ through orthogonal arrays, but the construction imposes a lot of restrictions, hence cannot be practised in all situations. Hence an easy method of construction and analysis of asymmetrical factorial design for a general situation is not available.

In the present investigation an attempt is made with the objective to construct asymmetrical factorial layouts suiting many of the situations which the previous authors had not attempted. Another objective of the present investigation is to give an easy and efficient analysis to any type of asymmetrical factorial layout.

These objectives have been met by constructing asymmetrical factorial layouts by means of four approaches viz.,

1. using Galois field,
2. $p \times q \times t$ designs from $p \times p$ designs $(p > q \geq t)$,
3. using factors at two levels and
4. using balanced designs.

An easy and modified analysis by means of sum and difference method in the line of Yates modified by Good is also attempted. Many examples of asymmetrical factorial designs suiting to different levels of factors are also worked out. Finally two practical examples of asymmetrical factorial designs from the field of agriculture have been analysed by the new technique of analysis developed in the present investigation.

Review of Literature

REVIEW OF LITERATURE

In agricultural experiments the yield or response will be effected by a number of factors. Experiments to test the response of each factor at different levels are often of interest to the experimenter. These can be tested by conducting different experiments. A more precise test can be obtained by using a class of experiments known as factorial experiments.

Factorial experiments are experiments where the treatments consists of all possible combinations of two or more factors at two or more levels. If the number of levels of each factor is the same then, it is a symmetrical factorial; otherwise it is an asymmetrical factorial.

In practical situations the use of symmetrical factorial is limited as in most cases it will result in unimportant or unwanted level testings or exclusion of some important levels.

In factorial experiments all possible treatment combinations must be applied to experimental units. It is inconvenient to conduct the experiment with large blocks. When there are large number of factors or number of levels of factors is large, the total number of treatment combinations also will be large. One of the devices resorted in such circumstance is confounding. Confounding is inextricably mixing up main effects or interactions with block effects. In otherwords, information on some effects or interactions is sacrificed to obtain others more precisely.

Confounding in symmetrical factorial is well developed through the works of Fisher, Yates, Das, Bose, Kishen, Nair, Rao and others. Asymmetrical factorial has achieved attention only in recent past. The people mainly worked on this aspect are Yates, Nair, Rao, Kishen, Das, Srivastava, Dey and many others. Still a general technique to construct confounded asymmetrical factorials do not exist.

Confounded asymmetrical factorials is introduced in the literature by Yates (1937). Yates has given method of confounding with factors at two and three levels and all factorials reducible to them. The design is obtained by confounding as far as possible, the highest order interactions. These designs involve partial confounding of more important interactions also. The confounded degrees of freedom in any replication is divided between different sets of treatment degrees of freedom. The fraction of information sacrificed on the more important interactions is quite small. In order to balance the design the number of replications used is some multiple of number of replications required for a balanced arrangement. The analysis given here is using orthogonal contrasts.

Nair and Rao (1948) have given combinatorial set up of asymmetrical experiments. The arrangement is such that

(i) mutually orthogonal estimates are obtained for various main effects and interactions,

(ii) degrees of freedom confounding is the same for every component of particular main effect or interaction.

These arrangements are called balanced confounded arrangements. The analysis of a two factor confounded arrangement is given. Method of least squares is made use of here. Analysis given here consists of

(i) estimation of treatment differences,

(ii) efficiency and amount of information and

(iii) tests of significance.

Kempthorne (1952) had attempted to extent confounding in symmetrical factorial experiments with levels as a prime number to asymmetrical factorial with levels as different powers of same prime. He had obtained confounded $2^2 \times 4^2$ without confounding any main effect, in blocks of eight plots.

Orthogonal arrays are made use of by Chakravarti (1956) for constructing fractional replicates of asymmetrical factorial. For this, the asymmetric factorial is grouped into different groups with factors at same number of levels falling into a group. In this paper a factorial design of the type $s_1^{m_1} \times s_2^{m_2} \times \dots s_g^{m_g}$ is considered. Then orthogonal arrays $(N_i, m_i, s_i, k_i + d_i - 1, \lambda i)$ are constructed with N_i assemblies, m_i constraints, strength $k_i + d_i - 1$, index λi and with elements as members of $GF(s_i)$. Fractional replicates of the factorial is obtained by taking the product of these arrays. It has been shown that from such a derived array it is possible to estimate all main effects and interactions involving $r = \prod_{i=1}^{g} r_i$ factors $(0 < r \leqslant gk,\ 0 \leqslant r_i \leqslant k_i)$ becomes measurable, where, r_1 factors are chosen from the first set of m_1 factors and r_2 from second set of m_2 factors and so on. Orthogonal contrasts are obtained for various main effects and estimable interactions.

Zelen (1958) constructed confounded asymmetrical factorials using group divisible designs. The factors are first grouped into two groups with factors A_s at m_s levels falling into one group and factors B_r at n_r

levels forming the other group (s=1, ... g; r=1, ... l)

$$m = \prod_{s=1}^{g} m_s, \quad n = \prod_{r=1}^{l} n_r \text{ and } \quad v = mn.$$

The v treatments are grouped into m groups of n members each. Two treatments belonging to the same class are first associates and belonging to different classes are second associates. So, there are (n-1) first associates and n(m-1) second associates for each treatment. The treatment combinations are arranged in an mxn array, assigning treatment combinations of A factors to the columns and B factors to the rows. The resulting design will be a PBIBD with two association classes. The analysis cited by the author is based on method of least squares.

Good (1958) has given interaction algorithm for asymmetrical factorial experiments. Matrix M_i corresponding to factor A_i with t_i levels is taken from Yates Tables (1937). An asymmetrical factorial with levels $t_1, t_2, \ldots t_n$ is considered. Direct product of matrices M_i, (i = 1, ... n) will yield to matrix A. Interaction contrasts are obtained as

$$X = AY$$

where, X denote interaction contrast vector, Y the vector of yields arranged in standard order. Inverse algorithm is obtained by taking direct product of inverses of matrices M_i and is

$$Y = A^{-1}X$$

Kishen and Srivastava (1959) made use of Galois field for constructing confounded asymmetrical factorial experiments. Construction of $s_1 \times s_2 \times \dots s_n$ (where, $s_1 \geqslant s_2, \dots \geqslant s_n$ and s_1 a prime number) with s_1 blocks in each replication is obtained as follows. Suitable polynomials are chosen that will take only s_1 values in the Galois field. For confounding a k factor interaction involving F_1, the blocks are obtained by taking s_1 flats of the pencil

$$x_1 + (a_{12}\, x_{12} + \dots\ a_{1k-1}\, x_{1k-1}) = \alpha,\ \alpha \in GF(s_1),$$

$$a_{1r} \in GF(s_1),\ r = 2, 3, \dots\ k-1.$$

Li (1944) constructed 5 x 2 x 2 design in 10 plot blocks with five replications. The method used by Li is as follows:

Designate by α the treatment combinations (0,0) and (1,1) of last two factors and by β the level combinations (0,1) and (1,0). Two αs and three βs are distributed over five levels of first factor to get one blocks. In the next block the role of α and β are interchanged.

The blocks together will give one replication of the treatments. In the same way other blocks of the other replications are also obtained. Shah (1960) proved that the design given by Li is only partially balanced. He suggested an alternative method which differs only in allotment of αs and βs. He has obtained a balanced but not resoluable design.

Das (1960) has given a method of construction and analysis of asymmetrical factorials $s_1 \times s^m$ and $s_1 \times s_2 \times s^m$ through fractional replicates in symmetrical factorial. Construction of $s_1 \times s^m$ factorial is as follows:

Attach p pseudo factors (each at s levels and are denoted by X_1, X_2 etc) to the factor of asymmetry (say X). p is chosen such that $s^{p-1} < s_1 \leq s^p$. Regular factors are denoted by A, B etc. Construct a confounded symmetrical factorial s^{m+p} in s^L blocks of s^{p+k} plots each $k = M-L$. Care should be taken not to confound any main effect or interaction of pseudo factors alone. This design is called 'parent design' and the set of confounded interactions is called 'confounding set'.

Omit $s^p - s_1$ treatment combinations of pseudo factors and rename s_1 factors as s_1 levels of first factor X. These combinations are called 'y omitted combinations' and the factorial will be s_1/s^p fraction of original factorial.

If any interaction of the parent design contains $p'>0$ pseudo factors together with some real factors then it will correspond to that interaction in which the pseudo factor interaction is replaced by X.

A replicate of $s_1 \times s_2 \times s^m$ design can be obtained in the same way from the parent design s^M in blocks of $s^{p_1+p_2+k}$ plots where p_1 and p_2 are obtained from $s^{p_1-1} < s_1 \leq s^{p_1}$ and $s^{p_2-1} < s_2 \leq s^{p_2}$ and $M = p_1 + p_2 + m$. The p_1 factors corresponding to X are called 'x- pseudo factors' and corresponding to Y are called 'y- pseudo factors'.

The set of all main effects and interactions of pseudo factors confounded in y omitted combinations is called 'partitioning set'.

The set formed by (i) partitioning set (ii) confounding set (iii) interaction between the two, from which interaction with real factor only is omitted, is called 'total confounded set.'

Single replicates has a complex analysis. So by taking a suitable set of confounded interactions the design is balanced. In parent design it is possible to get more than one confounding sets such that (i) each set corresponds to the same set of interaction of the asymmetric design (ii) each set give rise to the same total confounded set. These sets are called 'similar sets'. If there are n similar sets then a balanced design can be obtained by taking them in n different replications. If $s_1 = s^p$, balanced design will be obtained with single replication.

A method when s_1 is non-prime is also given. Let $s = rt$. To construct $s_1 \times s^m$ construct $r \times s^m$ and attach t levels to each treatment combinations and rename rt treatment combinations of first two factors as s_1 levels of X.

Kishen and Tyagi (1964) constructed confounded asymmetrical factorial experiments through pairwise balanced designs. They constructed $q \times 2 \times 2$ design making use of pairwise balanced design with q treatments (0,0) and (1,1) of last two factors are denoted by X_0 and (0,1) and (1,0) combinations by X_1. They obtained the design by writing X_0, X_1 in the pattern of pairwise balanced design and filling the remaining places with X_1, X_0. For constructing $q \times 3^2$, J_0, J_1, J_2 are arranged in a pairwise balanced design and the remaining places are filled with J_1, J_2, J_0.

Another method of constructing $q \times 2^2$ is, in a pairwise balanced design when

$b = 2r$, and

$k_1 = k_2 = q/2$ when q is even

$k_1 = (q-1)/2$ and $k_2 = (q+1)/2$ when q is odd.

Then, arrange X_0 in the PB design and fill the remaining places with X_1. Here only half the replication is necessary for balance compared to the previous one. For constructing $q \times 3 \times 3$ design the use of resolvable pairwise balanced design will reduce the number of replications required for balancing considerably.

Resolvable PB designs are resorted to, for constructing balanced confounded $q \times p^2$ $(q > p \geqslant 4)$ and p prime or prime power.

Pseudo factors are made use for constructing $l \times s \times s$ $(l = s^m)$, s, being prime in blocks of ls plots each.

Balanced confounded asymmetrical factorial designs of the class $q \times t \times s$ $(t = s^m)$ from $q \times s \times s$ designs also is constructed.

Sardana and Das (1965) constructed $p \times 3 \times 2$ designs from confounded $p \times p$ designs. A balanced confounded $p \times p$ in p plot blocks and (p-1) replications is constructed. Collapsing (p-3) levels of the second factor (B) will result in px3 in three plot blocks. Two levels of the third factor C is attached to every treatment combinations in a block. The resulting design will be q x 3 x 2 in six plot blocks and with (p-1) replications and the design will be a balanced one.

The analysis of p x 3 x 2 design in six plot blocks and with (p-1) replications also have been attempted here.

Das and Rao (1967) introduced a new method of confounding $3^n \times 2^m$ factorials from 2^M factorial in 2^k plot blocks by confounding suitable interactions. Group the first 2n factors into pairs. The levels are denoted by -1 and 1. By adding the levels corresponding to each pair will yield to n factors at three levels and the remaining m factors at two levels. An advantage of this method is that some degrees of freedom will be left for error. Analysis of the design suggested by the authors is a modification of Yates addition subtraction method. The analysis using contrasts also have been attempted.

Banerjee and Das (1969) constructed confounded asymmetrical factorials through an association with 2^n factorial designs. Corresponding to p_i levels of a factor A_i a number n_i is obtained such that

$$2^{n_i-1} < p_i \leq 2^{n_i} .$$

The effects and interactions of first (n_i-1) factors are confounded in 2^n factorial after denoting the levels by -1 and 1. The blocks are arranged in such a way that first 2^{n_i-2} blocks has combinations with level -1 of the first factor. First $2p_i - 2^{n_i}$ levels of A_i, are assigned to $p_i - 2^{n_i-1}$ blocks and the remaining $2^{n_i} - p_i$ levels are assigned to each of the remaining blocks.
An asymmetrical $p_1 \times p_2 \times \dots p_k$ experiment is constructed by taking a 2^n confounded design where $\sum_i n_i = n$.
They have also obtained contrasts for estimating various effects andiinteractions in 5 x 7 and 6 x 7 factorials.

Construction of a confounded $q \times s$ factorial with main effect B partially confounded is given by Tyagi and Jha (1969), where, $s = lm$, q and m are any positive integers and l is a prime or prime power. For construction, a balanced $l \times m$ design in m plot blocks is constructed with l-1 replications. Then q levels of the first factor are associated with each treatment and rename the lm levels as $lm = s$ levels of the second factor.

q x 6 partially balanced designs are constructed using a balanced confounded asymmetrical factorial 3 x 2 and pairwise balanced designs. Least square estimates of effects and interactions also is given.

Confounded $q \times 2 \times 2$ designs are constructed by Tyagi (1971). The procedure adopted by him is as follows. In a pairwise balanced design with q treatments $(i,0,0)$ and $(i,1,1)$ treatment combinations are alloted in a block if, the i^{th} treatment occurs in that block. Otherwise $(i,0,1)$ and $(i,1,0)$ are alloted in that block. Then $b-2r$ blocks with $(i,0,0)$ and $(i,1,1)$ or $2r-b$ blocks with $(i,0,1)$ and $(i,1,0)$ are added to the design according as $2r \lessgtr b$. The design obtained will be balanced confounded asymmetrical factorial design.

Raghava Rao (1971) constructed $3^m \times 2^n$ in $3^{m-1}\ 2^n$ and $3^m 2^{n-1}$ plot blocks and $v \times s^m$ in vs^{m-1} plot blocks using pencils and $(m-1)$ flats of $EG(m,s)$. A problem of confounding in $t^m \times s^n$ where $t = p^{\alpha}$, $s = p^{\beta}$ and p is a prime also has been solved following a method similar to Das (1960). α pseudo factors are associated to factors at t levels and β factors to each factors at s levels. Confounding in $p^{m\alpha+n\beta}$ is done using some well known methods with sufficient care taken not to confound main effects of original factors.

Ray (1972) obtained $p^m \times q^n$ in blocks of size $p^t \times q^n$ where m, n, t are integers, p prime power, q a prime number $p = g^b$, g prime number, b an integer. A design with $(mb + n)$ factors mb factors at g levels and n factors at q levels is constructed in $g^{bt}q^{n}$ plot blocks mb factors at g levels are grouped into m groups of b factors each and the p levels of original factors are assigned to this.

Dean and John (1975) constructed single replicate design for asymmetrical factorial experiments using group divisible designs. Construction of $v = \prod_{i=1}^{n} m_i$, in b blocks of k plots each is given. Using a single initial treatment the initial block is constructed as follows.

μ is taken as the least common multiple of $m_1, m_2, \ldots m_n$ ua denote the combinations obtained by multiplying an n-tuple a by u and taking each ua_i as mod m_i where, $a = (a_1, a_2, \ldots a_n)$. t = highest common factor of $(\mu, a_1 \mu/m_1, \ldots a_n \mu/m_n)$ then, 0, a, 2a, ... $(\mu/t-1)a$ will form the initial block with $k = \mu/t$.

If there are p generators say $b_1, b_2, \ldots b_p$. Then

$$q = \prod_{i=1}^{p} q_i$$

Initial blocks will have

$u_1b_1 + u_2b_2 + \ldots u_pb_p$ as the general element

$(u_i = 0, 1, \ldots q_i-1; \ i = 1, \ldots p)$.

Construction of 3 x 6 designs using rectangular design has been given by Aggarwal and Virk (1976). A rectangular partially balanced incomplete block design with parameters $v = (3)(6)$, $b = (6)(5)$, $r = 5$, $k = 3$, $\lambda_1 = \lambda_2 = 0$ and $\lambda_3 = 1$ is constructed using a balanced array (30,3,6,2; 0,1). A detailed analysis of the same is given using method of least squares.

Banerjee (1977) tackled the problem of constructing 5 x 7 factorial in fewer replications and its analysis. A symmetrical 2^6 factorial is used for the construction of the same. A 2^3 experiment is constructed in two plot blocks. The first two levels are associated to the first block. The remaining levels each to the remaining blocks. In the case of the factor at 7 levels, first three blocks are used to denote the first six levels and last block the last level. Confounding is done in 2^6 factorial and the analysis is carried out by the association between symmetric and asymmetric factorials.

Lewis (1979) constructed asymmetrical resolution III fractions from generalised cyclic designs. Any block of the design will give orthogonal estimates.

Another method of construction of balanced asymmetrical factorial has been given by Das (1979). He considered $p_1 \times p_2 \dots p_t = p$ where, p_i is the number of levels of i^{th} factor F_i. For the method of construction given here $p = NR$, where R is the block size and N a prime power say s^k. The design is constructed using an association with symmetric factorial. Factors at s levels are called real factors and others are called factor of asymmetry. The p_i levels of the factor of asymmetry are represented by, p_i elements of $GF(s)$ if $p_i \leqslant s$, or, p_i levels combinations of n_i pseudo factors each at s levels if, $s^{n_i-1} < p_i \leqslant s^{n_i}$.

Estimates of various effects and interactions are obtained by making use of the association between symmetrical and asymmetrical factorials. Analysis is done after adjusting to block effects.

Hardamard matrices are made use of by Anie and Dey (1981) for constructing fractions of asymmetrical factorial. They have obtained orthogonal main effect plans for 8×2^m factorial in $4n$ runs.

Rahul Mukerjee (1982) constructed balanced main effect plans for asymmetrical factorials using difference arrays. These difference arrays are constructed by cyclic rotation.

Agrawal and Dey (1983) made use of Hardamard matrices for constructing orthogonal main effect plans for $4^n \times 3^s \times 2^{3n-3\ (r+s)}$ factorial in $4n$ runs. This is an extension of method used by Anie and Dey (1981).

Materials and Methods

MATERIALS AND METHODS

The asymmetrical factorial design depends mainly on method of construction. Several methods are used for constructing such designs by different workers. In this present study it is attempted to construct confounded asymmetrical factorial designs through four different approaches.

1. using Galois field,
2. p x q x t designs from p x p designs,
3. using factors at two levels and
4. using balanced designs.

1. Construction using Galois Field

A field with finite number of elements is a Galois field. A Galois field with s elements is denoted by $GF(s)$, s will be a prime number or power of a prime number. If s is a prime number the elements of $GF(s)$ will be $0, 1, \ldots s-1$. If s is not a prime but power of a prime number the elements are members of the residue class of minimum function of the field. Minimum function of $GF(4)$ used for constructing designs here is

$$x^2 + x + 1$$

Kishen and Srivastava (1959) introduced the method of using Galois field for constructing confounded asymmetrical factorial designs. These designs require polynomials that will take only specific number of values (which are the number of levels of different factors) in GF(s).

In the present investigation it is shown that x^d will take only $(s-1)/d + 1$ distinct values in GF(s), where, d is a divisor of (s-1) and designs are constructed using this. A general method of obtaining these polynomials by inverting the matrix with elements as elements of GF(s) arranged in the standard order also is given.

2. Construction of $p \times q \times t$ designs from $p \times p$ designs

Sardana and Das (1961) constructed $p \times 3 \times 2$ designs by constructing confounded $p \times p$ with p-1 replications. p-3 levels of the second factor are collapsed to get a $p \times 3$ design in three plot blocks. To this design, two levels of the third factor are associated.

In the present study an attempt is made to construct $p \times q \times t$, $(p > q \geq t)$ confounded factorial design making use of the Sardana and Das's approach, by constructing a $p \times p$ confounded design and collapsing the last p-q levels of the second factor. The resulting design will be p x q x t in q x t plot blocks and with p-1 replications.

3. Construction using factors at two levels

A. Das and Rao (1967) constructed 3 x 3 x 2 design in eight plot blocks using an association with 2^5 design. In the present study construction of confounded $4^p \times 3^q \times 2^r$ is attempted. The method of construction adopted here is as follows.

Associate two pseudo factors each at two levels to, factors at three and four levels. Construct a confounded $2^{2p+2q+r}$ design in 2^k plot blocks. Group the first 2(p+q) factors in pairs. Rename the four combinations of two factors as four levels of p factors and three levels of q factors as follows:

Levels of pseudo factors		Levels of factor at four levels	Levels of factor at three levels
0	0	0	0
0	1	1	1
1	0	2	1
1	1	3	2

B. Banerjee and Das (1967) constructed asymmetrical factorial from symmetrical 2^n factorial by suitably designating the levels of each of the factors of asymmetrical design by one or more combinations of a certain number of factors each at two levels. The same technique is used for constructing asymmetrical factorial with one factor at 13 levels. Contrasts of the asymmetrical factorial also is given. The technique adopted here is as given below. A 2^4 factorial confounding all main effects and interactions of first three factors in two plot blocks is constructed. The blocks are arranged in such a way that first four blocks has the lower level of the first factor. Designate first five blocks as first ten levels of the factor and remaining three blocks are used for representing the remaining three levels.

4. Construction using balanced designs

Tyagi (1971) constructed confounded asymmetrical factorials using balanced designs. This method is made use of for constructing a $4 \times 2 \times 2$ design in eight plot blocks and with three replications. A $7 \times 2 \times 2$ factorial design with four replications is also obtained by using the same method.

Analysis

Yates (1937) analysed 2^n design by addition subtraction method. This was modified by Good (1958). Good has given the algorithm for analysing asymmetrical factorials.

In this study the analysis is done by a simplified and modified form of method of sums and difference introduced by Yates with Good's modification.

Results

RESULTS

The main objective of the present investigation was to construct confounded asymmetrical factorial designs. Four different techniques were used here.

1. Construction of Confounded Asymmetrical Factorial Designs using Galois Field

To construct asymmetrical factorial by confounding certain effects, it was sufficient if we replace some of the factors by suitable polynomials, such that these polynomials would take only desired number of values in the Galois field $GF(s)$. Two methods of constructing these polynomials were explained here. Constructions are based on two lemmas.

<u>Lemma (1).</u>

If $GF(s)$ is a Galois field with s elements and d is a divisor of $s-1$, then x^d can assume only $(s-1)/d +1$ distinct values in $GF(s)$ as x assumes all the s values in $GF(s)$ where $s = p^n$ and p is a prime and n any integer.

<u>Proof.</u>

Let s elements of $GF(s)$ be denoted by $0, \alpha, \alpha^2, \ldots \alpha^{s-1}$, where, α is a primitive element of $GF(s)$. Since d is a divisor of $(s-1)$,

$s-1 = md$, where m is an integer.

Let $x = \alpha^k$, $k = 1, \ldots, s-1$.

The different values x^d can assume are

$$\alpha^d, \quad \alpha^{2d}, \ldots, \quad \alpha^{(s-1)d}.$$

These can be rewritten as

$$\alpha^d, \quad \alpha^{2d}, \ldots \quad \alpha^{(m-1)d}, \quad \alpha^{md}, \ldots \quad \alpha^{(md)d}.$$

But for $GF(s)$

$$\alpha^{s-1} = \alpha^{md} = 1,$$

so that the values x^d can assume are only

$$\alpha^d, \quad \alpha^{2d}, \ldots \quad \alpha^{(m-1)d}, \quad 1.$$

x^d will take the value zero, when x takes the value zero. In otherwords, x^d will take only $m+1$, which is $(s-1)/d + 1$ values in $GF(s)$ while x takes all the s values in $GF(s)$.

<u>Lemma (2)</u>.

If S and T are two square matrices of order $s-1$ such that

$$S = \begin{bmatrix} \alpha_1 & \alpha_1^2 & \cdots & \alpha_1^{s-1} \\ \alpha_2 & \alpha_2^2 & \cdots & \alpha_2^{s-1} \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_{s-1} & \alpha_{s-1}^2 & \cdots & \alpha_{s-1}^{s-1} \end{bmatrix} \quad \text{and}$$

$$T = \frac{1}{p-1}\begin{pmatrix} \alpha_1^{s-2} & \alpha_2^{s-2} & \cdots & \alpha_{s-1}^{s-2} \\ \alpha_1^{s-3} & \alpha_2^{s-3} & \cdots & \alpha_{s-1}^{s-3} \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_1 & \alpha_2 & \cdots & \alpha_{s-1} \\ 1 & 1 & \cdots & 1 \end{pmatrix}$$

then S and T are inverses of each other.

Proof.

In order to show that T is the inverse of S, it is enough to show that ST is an identity matrix.

The t^{th} row k^{th} element of ST be r_{tk}.

Then two cases arise

Case 1. When $t = k$ is,

$$r_{tk} = (\alpha_t \alpha_k^{s-2} + \alpha_t^2 \alpha_k^{s-3} + \cdots + \alpha_t^{s-1}\alpha_k + \alpha_t^{s-1})/(p-1)$$

$$= (\alpha_t/\alpha_k)\alpha_k^{s-1} + (\alpha_t^2/\alpha_k^2)\alpha_k^{s-1} + \cdots + (\alpha_t^{s-1}/\alpha_k^{s-1})\alpha_k^{s-1} + 1)/(p-1)$$

But in GF(s), $\alpha_j^{s-1} = 1$ for $j = 1, 2, \ldots s-1$ and α_t/α_k will be an element of GF(s) say x. Hence, it is possible to write r_{tk} in the following form

$$r_{tk} = (x + x^2 + \cdots \; x^{s-2} + 1)/(p-1)$$

ie, $r_{tk} = \frac{x^{s-1} - 1}{(x-1)(p-1)} = 0$, since $x^{s-1} = 1$.

<u>Case 2</u>. When $t = k$.

$$r_{tt} = (\alpha_t \alpha_t^{s-2} + \dots \alpha_t^{s-2} \alpha_t + \alpha_t^{s-1})/(p-1)$$

$$= (1 + 1 + \dots 1)$$

$$= (s-1)/(p-1) = 1,$$

making use of properties of $GF(s)$.

This shows that $ST = I$ an identity matrix or T and S are inverses of each other.

<u>Construction</u>.

Let us take a polynomial as,

$$a_1 x + a_2 x^2 + \dots \; a_{s-1} x^{s-1}.$$

when x takes the values $\alpha_1, \alpha_2 \dots \alpha_{s-1}$ in $GF(s)$, this polynomial will take the values

$$a_1 \alpha_1 + a_2 \alpha_1^2 + \quad \dots \quad a_{s-1} \alpha_1^{s-1}$$

$$a_1 \alpha_2 + a_2 \alpha_2^2 + \quad \dots \quad a_{s-1} \alpha_2^{s-1}$$

. . . .

$$a_1 \alpha_{s-1} + a_2 \alpha_{s-1}^2 + \quad \dots \quad a_{s-1} \alpha_{s-1}^{s-1}$$

In order to restrict these values to a desired number which is the number of levels of the factor it is enough to solve for a_is by multiplying the matrix T with the vector which consists of the levels of the factor as elements of GF(s).

Example 1.

Lemma (1) was made use of for constructing a 3 x 3 x 2 design in blocks of size six confounding ABC the three factor interaction. Here, the polynomial used for constructing the blocks was

$x_1 + x_2 + x_3^2$, here the factor x_3 is replaced by x_3^2 and it takes only two values zero and one in GF(3). x_1 and x_2 takes all the three values of GF(3). Then the different blocks satisfying the polynomial were as follows

(0,0,0)	(1,0,0)	(2,0,0)
(0,2,1)	(1,2,1)	(2,2,1)
(1,2,0)	(2,2,0)	(0,2,0)
(1,1,1)	(2,1,1)	(0,1,1)
(2,0,1)	(0,0,1)	(1,0,1)
(2,1,0)	(0,1,0)	(1,1,0)

Suppose we considered arrangement confounding ABC and AB^2C then we would get a balanced arrangement. The polynomials were

$x_1 + x_2 + x_3^2$ and

$x_1 + 2x_2 + x_3^2$

Corresponding blocks of the two replications were

Replication I			Replication II		
(0,0,0)	(1,0,0)	(2,0,0)	(0,0,0)	(1,0,0)	(2,0,0)
(0,2,1)	(1,2,1)	(2,2,1)	(0,1,1)	(1,1,1)	(2,1,1)
(1,1,1)	(2,1,1)	(0,1,1)	(1,1,0)	(2,1,0)	(0,1,0)
(1,2,0)	(2,2,0)	(0,2,0)	(1,2,1)	(2,2,1)	(0,2,1)
(2,0,1)	(0,0,1)	(1,0,1)	(2,0,1)	(0,0,1)	(1,0,1)
(2,1,0)	(0,1,0)	(1,1,0)	(2,2,0)	(0,2,0)	(1,2,0)

This design would be the same as one obtained by confounding ABC and AB^2C in $3 \times 3 \times 3$ designs and collapsing the last level of the third factor.

Example 2.

$3 \times 2 \times 2$ in four plot blocks confounding ABC was obtained as

(0,0,0)	(1,0,0)	(2,0,0)
(1,1,1)	(2,1,1)	(0,1,1)
(2,0,1)	(0,0,1)	(1,0,1)
(2,1,0)	(0,1,0)	(1,1,0)

This was constructed by taking the polynomial $x_1 + x_2^2 + x_3^2$, where x_2^2 and x_3^2 were polynomials that takes only two values zero and one in GF(3).

Example 3.

$4 \times 2 \times 2$ in four plot blocks confounding ABC with three degrees of freedom. The polynomial for constructing such a confounding design was

$$x_1 + x_2^3 + x_3^3 .$$

Here the factors x_2 and x_3 were replaced by x_2^2 and x_3^3 respectively so that these would take only two values zero and one in GF(4).

Different blocks of the design were

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)
(0,1,1)	(1,1,1)	(2,1,1)	(3,1,1)
(1,0,1)	(0,0,1)	(3,0,1)	(2,0,1)
(1,1,0)	(0,1,0)	(3,1,0)	(2,1,0)

It could be seen that the design obtained above was same as one obtained by dropping higher levels of B and C in a $4 \times 4 \times 4$ design confounding ABC.

Example 4.

$4 \times 4 \times 2$ design confounding ABC in eight plot blocks. The polynomial for constructing such a design was

$$x_1 + x_2 + x_3^3 .$$

Here, the factor x_3 is replaced by x_3^3 which takes only two values in GF(4) while x_1 and x_2 take all the four values and the resulting blocks were

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)
(1,1,0)	(0,1,0)	(3,1,0)	(2,1,0)
(1,0,1)	(0,0,1)	(3,0,1)	(2,0,1)
(0,1,1)	(1,1,1)	(2,1,1)	(3,1,1)
(2,2,0)	(3,2,0)	(0,2,0)	(1,2,0)
(3,3,0)	(2,3,0)	(1,3,0)	(0,3,0)
(2,3,1)	(3,3,1)	(0,3,1)	(1,3,1)
(3,2,1)	(2,2,1)	(1,2,1)	(0,2,1)

Example 5.

5 x 3 x 2 design in six plot blocks is constructed. The polynomial for constructing the blocks was

$$x_1 + x_2^2 + x_3^4 .$$

Here, x_2 and x_3 were replaced by x_2^2 and x_3^4 respectively so that x_2^2 would take only three values and x_3^4 would take only two values in the Galois field GF(5) where x_1 took all the five values of it. The blocks were obtained as

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)	(4,0,0)
(2,2,1)	(3,2,1)	(4,2,1)	(0,2,1)	(1,2,1)
(3,1,1)	(4,1,1)	(0,1,1)	(1,1,1)	(2,1,1)
(3,2,0)	(4,2,0)	(0,2,0)	(1,2,0)	(2,2,0)
(4,0,1)	(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)
(4,1,0)	(0,1,0)	(1,1,0)	(2,1,0)	(3,1,0)

Example 6.

$4 \times 4 \times 3$ in 12 plot blocks. Here two was not a divisor of three so that second lemma was made use of for getting a polynomial in x_3 that took only three values zero, one and α of GF(4).

In the polynomial

$$a_1x_3 + a_2x_3^2 + a_3x_3^3$$

solved for a_1, a_2 and a_3 from the following equation

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 1 & \alpha^2 & \alpha^4 \\ 1 & \alpha & \alpha^2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ \alpha \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha \\ \alpha+1 \end{pmatrix}$$

Hence x_3 was replaced by

$\alpha x_3^2 + (\alpha+1)x_3^3$, so that it would take only three values in GF (4).

The polynomial for constructing the blocks was

$$x_1 + x_2 + \alpha x_3^2 + (\alpha+1)\, x_3^3.$$

The blocks obtained from these polynomials after converting the levels into natural numbers were

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)
(0,2,2)	(1,2,2)	(2,2,2)	(3,2,2)
(0,1,1)	(1,1,1)	(2,1,1)	(3,1,1)
(1,1,0)	(0,1,0)	(3,1,0)	(2,1,0)
(1,0,1)	(0,0,1)	(3,0,1)	(2,0,1)
(1,3,2)	(0,3,2)	(3,3,2)	(2,3,2)
(2,0,2)	(3,0,2)	(0,0,2)	(1,0,2)
(2,2,0)	(3,2,0)	(0,2,0)	(1,2,0)
(2,3,1)	(3,3,1)	(0,3,1)	(1,3,1)
(3,1,2)	(2,1,2)	(1,1,2)	(0,1,2)
(3,2,1)	(2,2,1)	(1,2,1)	(0,2,1)
(3,3,0)	(2,3,0)	(1,3,0)	(0,3,0)

Example 7.

$7 \times 4 \times 3$ in twelve plot blocks was constructed using the polynomial $x_1 + x_2^2 + x_3^3$ where the factors x_2 and x_3 were replaced by x_2^2 and x_3^3 respectively so that x_2^2 would take only four values and x_3^3 took only three values in GF(7).

The blocks obtained were

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)	(4,0,0)	(5,0,0)	(6,0,0)
(2,3,2)	(3,3,2)	(4,3,2)	(5,3,2)	(6,3,2)	(0,3,2)	(1,3,2)
(3,2,2)	(4,2,2)	(5,2,2)	(6,2,2)	(0,2,2)	(1,2,2)	(2,2,2)
(3,3,1)	(4,3,1)	(5,3,1)	(6,3,1)	(0,3,1)	(1,3,1)	(2,3,1)
(4,1,2)	(5,1,2)	(6,1,2)	(0,1,2)	(1,1,2)	(2,1,2)	(3,1,2)
(4,3,0)	(5,3,0)	(6,3,0)	(0,3,0)	(1,3,0)	(2,3,0)	(3,3,0)
(5,1,1)	(6,1,1)	(0,1,1)	(1,1,1)	(2,1,1)	(3,1,1)	(4,1,1)
(5,2,0)	(6,2,0)	(0,2,0)	(1,2,0)	(2,2,0)	(3,2,0)	(4,2,0)
(5,0,2)	(6,0,2)	(0,0,2)	(1,0,2)	(2,0,2)	(3,0,2)	(4,0,2)
(6,0,1)	(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)	(4,0,1)	(5,0,1)
(6,1,0)	(0,1,0)	(1,1,0)	(2,1,0)	(3,1,0)	(4,1,0)	(5,1,0)

Example 8.

$4 \times 3 \times 2 \times 2$ in 12 plot blocks. The polynomial for constructing the blocks was

$$x_1 + \alpha x_2^2 + (\alpha+1)\, x_2^3 + x_3^3 + x_4^3$$

For constructing this polynomial the factor x_2 was replaced by $\alpha x_2^2 + (\alpha+1)x_2^3$ and x_3 by x_3^3 and x_4 by x_4^3 so that they would take three, two and two values respectively in GF(4). The blocks obtained by making use of this polynomial were

(0,0,0,0)	(1,0,0,0)	(2,0,0,0)	(3,0,0,0)
(0,0,1,1)	(1,0,1,1)	(2,0,1,1)	(3,0,1,1)
(0,1,1,0)	(1,1,1,0)	(2,1,1,0)	(3,1,1,0)
(0,1,0,1)	(1,1,0,1)	(2,1,0,1)	(3,1,0,1)
(1,0,1,0)	(0,0,1,0)	(3,0,1,0)	(2,0,1,0)
(1,0,0,1)	(0,0,0,1)	(3,0,0,1)	(2,0,0,1)
(1,1,0,0)	(0,1,0,0)	(3,1,0,0)	(2,1,0,0)
(1,1,1,1)	(0,1,1,1)	(3,1,1,1)	(2,1,1,1)
(2,2,0,0)	(3,2,0,0)	(0,2,0,0)	(1,2,0,0)
(2,2,1,1)	(3,2,1,1)	(0,2,1,1)	(1,2,1,1)
(3,2,0,1)	(2,2,0,1)	(1,2,0,1)	(0,2,0,1)
(3,2,1,0)	(2,2,1,0)	(1,2,1,0)	(0,2,1,0)

2. Construction of p x q x t designs from p x p designs

As explained in the materials and methods, first a p x p confounded factorial design in (p-1) replications was constructed. Collapsing the last (p-q) levels of the second factor and attaching the t levels of the third factor to each of the treatment combinations an asymmetrical confounded factorial design of size p x q x t in qt plot blocks with (p-1) replications was obtained.

Example I.

4 x 3 x 2 design in six plot blocks. A confounded 4 x 4 symmetrical factorial layout in four plot blocks with three replications was constructed first. Let the two factors be A and B. Last level of B was collapsed to obtain a 4 x 3 design in three plot blocks. Two levels of the third factor C were associated to each treatment of the 4 x 3 layout in every block. The design thus obtained was a 4 x 3 x 2 asymmetrical factorial layout in six plot blocks with three replications partially confounding A and AB. The three replications of the 4 x 3 x 2 layout were as given below.

Replication I

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)
(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)
(1,1,0)	(0,1,0)	(3,1,0)	(2,1,0)
(1,1,1)	(0,1,1)	(3,1,1)	(2,1,1)
(2,2,0)	(3,2,0)	(0,2,0)	(1,2,0)
(2,2,1)	(3,2,1)	(0,2,1)	(1,2,1)

Replication II.

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)
(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)
(2,1,0)	(3,1,0)	(0,1,0)	(1,1,0)
(2,1,1)	(3,1,1)	(0,1,1)	(1,1,1)
(3,2,0)	(2,2,0)	(1,2,0)	(0,2,0)
(3,2,1)	(2,2,1)	(1,2,1)	(0,2,1)

Replication III.

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)
(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)
(1,2,0)	(0,2,0)	(3,2,0)	(2,2,0)
(1,2,1)	(0,2,1)	(3,2,1)	(2,2,1)
(3,1,0)	(2,1,0)	(1,1,0)	(0,1,0)
(3,1,1)	(2,1,1)	(1,1,1)	(0,1,1)

Example 2.

Construction of confounded $5 \times 4 \times 3$ asymmetrical factorial design. A confounded 5×5 symmetrical factorial design in five plot blocks with four replications was constructed first. Let the factors be A and B. The last level of B was collapsed to get a 5×4 asymmetrical layout in four plot blocks. The three levels of the third factor were associated to each treatment combination in every block. The resulting design was a $5 \times 4 \times 3$ asymmetrical layout in twelve plot blocks and with four replications partially confounding A and AB and were as given below.

Replication I.

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)	(4,0,0)
(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)	(4,0,1)
(0,0,2)	(1,0,2)	(2,0,2)	(3,0,2)	(4,0,2)
(2,3,0)	(3,3,0)	(4,3,0)	(0,3,0)	(1,3,0)
(2,3,1)	(3,3,1)	(4,3,1)	(0,3,1)	(1,3,1)
(2,3,2)	(3,3,2)	(4,3,2)	(0,3,2)	(1,3,2)
(3,2,0)	(4,2,0)	(0,2,0)	(1,2,0)	(2,2,0)
(3,2,1)	(4,2,1)	(0,2,1)	(1,2,1)	(2,2,1)
(3,2,2)	(4,2,2)	(0,2,2)	(1,2,2)	(2,2,2)
(4,1,0)	(0,1,0)	(1,1,0)	(2,1,0)	(3,1,0)
(4,1,1)	(0,1,1)	(1,1,1)	(2,1,1)	(3,1,1)
(4,1,2)	(0,1,2)	(1,1,2)	(2,1,2)	(3,1,2)

Replication II.

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)	(4,0,0)
(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)	(4,0,1)
(0,0,2)	(1,0,2)	(2,0,2)	(3,0,2)	(4,0,2)
(1,2,0)	(2,2,0)	(3,2,0)	(4,2,0)	(0,2,0)
(1,2,1)	(2,2,1)	(3,2,1)	(4,2,1)	(0,2,1)
(1,2,2)	(2,2,2)	(3,2,2)	(4,2,2)	(0,2,2)
(3,1,0)	(4,1,0)	(0,1,0)	(1,1,0)	(2,1,0)
(3,1,1)	(4,1,1)	(0,1,1)	(1,1,1)	(2,1,1)
(3,1,2)	(4,1,2)	(0,1,2)	(1,1,2)	(2,1,2)
(4,3,0)	(0,3,0)	(1,3,0)	(2,3,0)	(3,3,0)
(4,3,1)	(0,3,1)	(1,3,1)	(2,3,1)	(3,3,1)
(4,3,2)	(0,3,2)	(1,3,2)	(2,3,2)	(3,3,2)

Replication III.

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)	(4,0,0)
(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)	(4,0,1)
(0,0,2)	(1,0,2)	(2,0,2)	(3,0,2)	(4,0,2)
(1,3,0)	(2,3,0)	(3,3,0)	(4,3,0)	(0,3,0)
(1,3,1)	(2,3,1)	(3,3,1)	(4,3,1)	(0,3,1)
(1,3,2)	(2,3,2)	(3,3,2)	(4,3,2)	(0,3,2)
(2,1,0)	(3,1,0)	(4,1,0)	(0,1,0)	(1,1,0)
(2,1,1)	(3,1,1)	(4,1,1)	(0,1,1)	(1,1,1)
(2,1,2)	(3,1,2)	(4,1,2)	(0,1,2)	(1,1,2)
(4,2,0)	(0,2,0)	(1,2,0)	(2,2,0)	(3,2,0)
(4,2,1)	(0,2,1)	(1,2,1)	(2,2,1)	(3,2,1)
(4,2,2)	(0,2,2)	(1,2,2)	(2,2,2)	(3,2,2)

Replication IV.

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)	(4,0,0)
(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)	(4,0,1)
(0,0,2)	(1,0,2)	(2,0,2)	(3,0,2)	(4,0,2)
(2,2,0)	(3,2,0)	(4,2,0)	(0,2,0)	(1,2,0)
(2,2,1)	(3,2,1)	(4,2,1)	(0,2,1)	(1,2,1)
(2,2,2)	(3,2,2)	(4,2,2)	(0,2,2)	(1,2,2)
(3,3,0)	(4,3,0)	(0,3,0)	(1,3,0)	(2,3,0)
(3,3,1)	(4,3,1)	(0,3,1)	(1,3,1)	(2,3,1)
(3,3,2)	(4,3,2)	(0,3,2)	(1,3,2)	(2,3,2)
(4,1,0)	(0,1,0)	(1,1,0)	(2,1,0)	(3,1,0)
(4,1,1)	(0,1,1)	(1,1,1)	(2,1,1)	(3,1,1)
(4,1,2)	(0,1,2)	(1,1,2)	(2,1,2)	(3,1,2)

Example 3.

Construction of 7 x 6 x 3 confounded asymmetrical factorial design. A confounded 7 x 7 symmetric factorial design in seven plot blocks with six replications was constructed. The last level of the second factor was collapsed so as to result in a 7 x6 asymmetrical layout in six plot blocks. The three levels of the third factor were associated to each treatment combinations in a block. Thus an asymmetrical 7 x 6 x 3 factorial were obtained. The blocks of the design were

Replication I.

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)	(4,0,0)	(5,0,0)	(6,0,0)
(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)	(4,0,1)	(5,0,1)	(6,0,1)
(0,0,2)	(1,0,2)	(2,0,2)	(3,0,2)	(4,0,2)	(5,0,2)	(6,0,2)
(2,5,0)	(3,5,0)	(4,5,0)	(5,5,0)	(6,5,0)	(0,5,0)	(1,5,0)
(2,5,1)	(3,5,1)	(4,5,1)	(5,5,1)	(6,5,1)	(0,5,1)	(1,5,1)
(2,5,2)	(3,5,2)	(4,5,2)	(5,5,2)	(6,5,2)	(0,5,2)	(1,5,2)
(3,4,1)	(4,4,1)	(5,4,1)	(6,4,1)	(0,4,1)	(1,4,1)	(2,4,1)
(3,4,0)	(4,4,0)	(5,4,0)	(6,4,0)	(0,4,0)	(1,4,0)	(2,4,0)
(3,4,2)	(4,4,2)	(5,4,2)	(6,4,2)	(0,4,2)	(1,4,2)	(2,4,2)
(4,3,0)	(5,3,0)	(6,3,0)	(0,3,0)	(1,3,0)	(2,3,0)	(3,3,0)
(4,3,1)	(5,3,1)	(6,3,1)	(0,3,1)	(1,3,1)	(2,3,1)	(3,3,1)
(4,3,2)	(5,3,2)	(6,3,2)	(0,3,2)	(1,3,2)	(2,3,2)	(3,3,2)
(5,2,0)	(6,2,0)	(0,2,0)	(1,2,0)	(2,2,0)	(3,2,0)	(4,2,0)
(5,2,1)	(6,2,1)	(0,2,1)	(1,2,1)	(2,2,1)	(3,2,1)	(4,2,1)
(5,2,2)	(6,2,2)	(0,2,2)	(1,2,2)	(2,2,2)	(3,2,2)	(4,2,2)
(6,1,0)	(0,1,0)	(1,1,0)	(2,1,0)	(3,1,0)	(4,1,0)	(5,1,0)
(6,1,1)	(0,1,1)	(1,1,1)	(2,1,1)	(3,1,1)	(4,1,1)	(5,1,1)
(6,1,2)	(0,1,2)	(1,1,2)	(2,1,2)	(3,1,2)	(4,1,2)	(5,1,2)

Replication II.

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)	(4,0,0)	(5,0,0)	(6,0,0)
(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)	(4,0,1)	(5,0,1)	(6,0,1)
(0,0,2)	(1,0,2)	(2,0,2)	(3,0,2)	(4,0,2)	(5,0,2)	(6,0,2)
(1,3,0)	(2,3,0)	(3,3,0)	(4,3,0)	(5,3,0)	(6,3,0)	(0,3,0)
(1,3,1)	(2,3,1)	(3,3,1)	(4,3,1)	(5,3,1)	(6,3,1)	(0,3,1)
(1,3,2)	(2,3,2)	(3,3,2)	(4,3,2)	(5,3,2)	(6,3,2)	(0,3,2)
(3,2,0)	(4,2,0)	(5,2,0)	(6,2,0)	(0,2,0)	(1,2,0)	(2,2,0)
(3,2,1)	(4,2,1)	(5,2,1)	(6,2,1)	(0,2,1)	(1,2,1)	(2,2,1)
(3,2,2)	(4,2,2)	(5,2,2)	(6,2,2)	(0,2,2)	(1,2,2)	(2,2,2)
(4,5,0)	(5,5,0)	(6,5,0)	(0,5,0)	(1,5,0)	(2,5,0)	(3,5,0)
(4,5,1)	(5,5,1)	(6,5,1)	(0,5,1)	(1,5,1)	(2,5,1)	(3,5,1)
(4,5,2)	(5,5,2)	(6,5,2)	(0,5,2)	(1,5,2)	(2,5,2)	(3,5,2)
(5,1,0)	(6,1,0)	(0,1,0)	(1,1,0)	(2,1,0)	(3,1,0)	(4,1,0)
(5,1,1)	(6,1,1)	(0,1,1)	(1,1,1)	(2,1,1)	(3,1,1)	(4,1,1)
(5,1,2)	(6,1,2)	(0,1,2)	(1,1,2)	(2,1,2)	(3,1,2)	(4,1,2)
(6,4,0)	(0,4,0)	(1,4,0)	(2,4,0)	(3,4,0)	(4,4,0)	(5,4,0)
(6,4,1)	(0,4,1)	(1,4,1)	(2,4,1)	(3,4,1)	(4,4,1)	(5,4,1)
(6,4,2)	(0,4,2)	(1,4,2)	(2,4,2)	(3,4,2)	(4,4,2)	(5,4,2)

Replication III.

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)	(4,0,0)	(5,0,0)	(6,0,0)
(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)	(4,0,1)	(5,0,1)	(6,0,1)
(0,0,2)	(1,0,2)	(2,0,2)	(3,0,2)	(4,0,2)	(5,0,2)	(6,0,2)
(1,2,0)	(2,2,0)	(3,2,0)	(4,2,0)	(5,2,0)	(6,2,0)	(0,2,0)
(1,2,1)	(2,2,1)	(3,2,1)	(4,2,1)	(5,2,1)	(6,2,1)	(0,2,1)
(1,2,2)	(2,2,2)	(3,2,2)	(4,2,2)	(5,2,2)	(6,2,2)	(0,2,2)
(2,4,0)	(3,4,0)	(4,4,0)	(5,4,0)	(6,4,0)	(0,4,0)	(1,4,0)
(2,4,1)	(3,4,1)	(4,4,1)	(5,4,1)	(6,4,1)	(0,4,1)	(1,4,1)
(2,4,2)	(3,4,2)	(4,4,2)	(5,4,2)	(6,4,2)	(0,4,2)	(1,4,2)
(4,1,0)	(5,1,0)	(6,1,0)	(0,1,0)	(1,1,0)	(2,1,0)	(3,1,0)
(4,1,1)	(5,1,1)	(6,1,1)	(0,1,1)	(1,1,1)	(2,1,1)	(3,1,1)
(4,1,2)	(5,1,2)	(6,1,2)	(0,1,2)	(1,1,2)	(2,1,2)	(3,1,2)
(5,3,0)	(6,3,0)	(0,3,0)	(1,3,0)	(2,3,0)	(3,3,0)	(4,3,0)
(5,3,1)	(6,3,1)	(0,3,1)	(1,3,1)	(2,3,1)	(3,3,1)	(4,3,1)
(5,3,2)	(6,3,2)	(0,3,2)	(1,3,2)	(2,3,2)	(3,3,2)	(4,3,2)
(6,5,0)	(0,5,0)	(1,5,0)	(2,5,0)	(3,5,0)	(4,5,0)	(5,5,0)
(6,5,1)	(0,5,1)	(1,5,1)	(2,5,1)	(3,5,1)	(4,5,1)	(5,5,1)
(6,5,2)	(0,5,2)	(1,5,2)	(2,5,2)	(3,5,2)	(4,5,2)	(5,5,2)

Replication IV.

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)	(4,0,0)	(5,0,0)	(6,0,0)
(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)	(4,0,1)	(5,0,1)	(6,0,1)
(0,0,2)	(1,0,2)	(2,0,2)	(3,0,2)	(4,0,2)	(5,0,2)	(6,0,2)
(1,5,0)	(2,5,0)	(3,5,0)	(4,5,0)	(5,5,0)	(6,5,0)	(0,5,0)
(1,5,1)	(2,5,1)	(3,5,1)	(4,5,1)	(5,5,1)	(6,5,1)	(0,5,1)
(1,5,2)	(2,5,2)	(3,5,2)	(4,5,2)	(5,5,2)	(6,5,2)	(0,5,2)
(2,3,0)	(3,3,0)	(4,3,0)	(5,3,0)	(6,3,0)	(0,3,0)	(1,3,0)
(2,3,1)	(3,3,1)	(4,3,1)	(5,3,1)	(6,3,1)	(0,3,1)	(1,3,1)
(2,3,2)	(3,3,2)	(4,3,2)	(5,3,2)	(6,3,2)	(0,3,2)	(1,3,2)
(3,1,0)	(4,1,0)	(5,1,0)	(6,1,0)	(0,1,0)	(1,1,0)	(2,1,0)
(3,1,1)	(4,1,1)	(5,1,1)	(6,1,1)	(0,1,1)	(1,1,1)	(2,1,1)
(3,1,2)	(4,1,2)	(5,1,2)	(6,1,2)	(0,1,2)	(1,1,2)	(2,1,2)
(5,4,0)	(6,4,0)	(0,4,0)	(1,4,0)	(2,4,0)	(3,4,0)	(4,4,0)
(5,4,1)	(6,4,1)	(0,4,1)	(1,4,1)	(2,4,1)	(3,4,1)	(4,4,1)
(5,4,2)	(6,4,2)	(0,4,2)	(1,4,2)	(2,4,2)	(3,4,2)	(4,4,2)
(6,2,0)	(0,2,0)	(1,2,0)	(2,2,0)	(3,2,0)	(4,2,0)	(5,2,0)
(6,2,1)	(0,2,1)	(1,2,1)	(2,2,1)	(3,2,1)	(4,2,1)	(5,2,1)
(6,2,2)	(0,2,2)	(1,2,2)	(2,2,2)	(3,2,2)	(4,2,2)	(5,4,2)

Replication V.

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)	(4,0,0)	(5,0,0)	(6,0,0)
(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)	(4,0,1)	(5,0,1)	(6,0,1)
(0,0,2)	(1,0,2)	(2,0,2)	(3,0,2)	(4,0,2)	(5,0,2)	(6,0,2)
(1,4,0)	(2,4,0)	(3,4,0)	(4,4,0)	(5,4,0)	(6,4,0)	(0,4,0)
(1,4,1)	(2,4,1)	(3,4,1)	(4,4,1)	(5,4,1)	(6,4,1)	(0,4,1)
(1,4,2)	(2,4,2)	(3,4,2)	(4,4,2)	(5,4,2)	(6,4,2)	(0,4,2)
(2,1,0)	(3,1,0)	(4,1,0)	(5,1,0)	(6,1,0)	(0,1,0)	(1,1,0)
(2,1,1)	(3,1,1)	(4,1,1)	(5,1,1)	(6,1,1)	(0,1,1)	(1,1,1)
(2,1,2)	(3,1,2)	(4,1,2)	(5,1,2)	(6,1,2)	(0,1,2)	(1,1,2)
(3,5,0)	(4,5,0)	(5,5,0)	(6,5,0)	(0,5,0)	(1,5,0)	(2,5,0)
(3,5,1)	(4,5,1)	(5,5,1)	(6,5,1)	(0,5,1)	(1,5,1)	(2,5,1)
(3,5,2)	(4,5,2)	(5,5,2)	(6,5,2)	(0,5,2)	(1,5,2)	(2,5,2)
(4,9,0)	(5,5,0)	(6,2,0)	(0,2,0)	(1,2,0)	(2,2,0)	(3,2,0)
(4,2,1)	(5,2,1)	(6,2,1)	(0,2,1)	(1,2,1)	(2,2,1)	(3,2,1)
(4,2,2)	(5,2,2)	(6,2,2)	(0,2,2)	(1,2,2)	(2,2,2)	(3,2,2)
(6,3,0)	(0,3,0)	(1,3,0)	(2,3,0)	(3,3,0)	(4,3,0)	(5,3,0)
(6,3,1)	(0,3,1)	(1,3,1)	(2,3,1)	(3,3,1)	(4,3,1)	(5,3,1)
(6,3,2)	(0,3,2)	(1,3,2)	(2,3,2)	(3,3,2)	(4,3,2)	(5,3,2)

Replication VI.

(0,0,0)	(1,0,0)	(2,0,0)	(3,0,0)	(4,0,0)	(5,0,0)	(6,0,0)
(0,0,1)	(1,0,1)	(2,0,1)	(3,0,1)	(4,0,1)	(5,0,1)	(6,0,1)
(0,0,2)	(1,0,2)	(2,0,2)	(3,0,2)	(4,0,2)	(5,0,2)	(6,0,2)
(1,1,0)	(2,1,0)	(3,1,0)	(4,1,0)	(5,1,0)	(6,1,0)	(0,1,0)
(1,1,1)	(2,1,1)	(3,1,1)	(4,1,1)	(5,1,1)	(6,1,1)	(0,1,1)
(1,1,2)	(2,1,2)	(3,1,2)	(4,1,2)	(5,1,2)	(6,1,2)	(0,1,2)
(2,2,0)	(3,2,0)	(4,2,0)	(5,2,0)	(6,2,0)	(0,2,0)	(1,2,0)
(2,2,1)	(3,2,1)	(4,2,1)	(5,2,1)	(6,2,1)	(0,2,1)	(1,2,1)
(2,2,2)	(3,2,2)	(4,2,2)	(5,2,2)	(6,2,2)	(0,2,2)	(1,2,2)
(3,3,0)	(4,3,0)	(5,3,0)	(6,3,0)	(0,3,0)	(1,3,0)	(2,3,0)
(3,3,1)	(4,3,1)	(5,3,1)	(6,3,1)	(0,3,1)	(1,3,1)	(2,3,1)
(3,3,2)	(4,3,2)	(5,3,2)	(6,3,2)	(0,3,2)	(1,3,2)	(2,3,2)
(4,4,0)	(5,4,0)	(6,4,0)	(0,4,0)	(1,4,0)	(2,4,0)	(3,4,0)
(4,4,1)	(5,4,1)	(6,4,1)	(0,4,1)	(1,4,1)	(2,4,1)	(3,4,1)
(4,4,2)	(5,4,2)	(6,4,2)	(0,4,2)	(1,4,2)	(2,4,2)	(3,4,2)
(5,5,0)	(6,5,0)	(0,5,0)	(1,5,0)	(2,5,0)	(3,5,0)	(4,5,0)
(5,5,1)	(6,5,1)	(0,5,1)	(1,5,1)	(2,5,1)	(3,5,1)	(4,5,1)
(5,5,2)	(6,5,2)	(0,5,2)	(1,5,2)	(2,5,2)	(3,5,2)	(4,5,2)

3. Construction of asymmetrical factorial design using factors at two levels

A. Construction in the line of Das and Rao.

As already explained in the materials and methods a general confounded asymmetrical factorial design of order $4^p \times 3^q \times 2^r$ was constructed following the line of Das and Rao.

First a $2^{2p+2q+r}$ confounded symmetric factorial design in 2^k plot blocks were constructed. The first $2p + 2q$ factors were paired. The four combinations of two factors were renamed as four levels of p factors and three levels of q factors as shown below

Levels of pseudo factors		Levels of factor at four levels	Levels of factor at three levels
0	0	0	0
0	1	1	1
1	0	2	1
1	1	3	2

Substituting these new levels a $4^p \times 3^q \times 2^r$ factorial design was constructed.

Example 1.

4 x 4 x 2 design in eight plot blocks. A confounded 2^5 factorial design confounding ABCD and BCE was constructed in eight plot blocks. Four combinations of first two pairs of factors are renamed as four levels of factors of asymmetry. Thus a 4 x 4 x 2 design in eight plot blocks was obtained as shown below

(2,1,0)	(2,0,0)	(2,1,1)	(2,0,1)
(1,1,1)	(1,0,1)	(1,1,0)	(1,0,0)
(0,3,1)	(0,2,1)	(0,3,0)	(0,2,0)
(3,0,1)	(3,1,1)	(3,0,0)	(3,1,0)
(2,2,1)	(2,3,1)	(2,2,0)	(2,3,0)
(1,2,0)	(1,3,0)	(1,2,1)	(1,3,1)
(3,3,0)	(3,2,0)	(3,3,1)	(3,2,1)
(0,0,0)	(0,1,0)	(0,0,1)	(0,1,1)

Example 2.

3 x 3 x 2 x 2 in eight plot blocks. A confounded 2^6 factorial design confounding ABD, CDE and ACF was constructed. The four combinations of first two pairs of factors were designated as three levels of factors of asymmetry. This resulted in a 3 x 3 x 2 x 2 design in eight plot blocks, as given below

(1,1,1,1)	(1,1,1,0)	(1,1,0,1)	(1,1,0,0)
(1,1,1,0)	(1,1,1,1)	(1,1,0,0)	(1,1,0,1)
(0,1,1,1)	(0,1,1,0)	(0,1,0,1)	(0,1,0,0)
(2,0,0,1)	(2,0,0,2)	(2,0,1,1)	(2,0,1,0)
(1,2,0,0)	(1,2,0,1)	(1,2,1,0)	(1,2,1,1)
(1,2,0,1)	(1,2,0,0)	(1,2,1,1)	(1,2,1,0)
(2,1,1,0)	(2,1,1,1)	(2,1,0,0)	(2,1,0,1)
(0,0,0,0)	(0,0,0,1)	(0,0,1,0)	(0,0,1,1)

B. Construction in the line of Banerjee and Das.

A confounded asymmetrical factorial design with one factor at p levels following the line of Banerjee and Das (1969) were also constructed from symmetrical 2^n factorial, where, $2^{n-1} < p \leqslant 2^n$. The method of construction were as follows

First a 2^n factorial layout confounding main effects and interactions in two plot blocks were constructed in such a way that the first half blocks and the lower level of the first factor. The first $p-2^{n-1}$ blocks were used for designating $2p-2^n$ levels and the remaining blocks were used for designating one level each. The contrasts of the asymmetrical factorial were obtained by taking the contrasts of the 2^n symmetrical design.

A confounded 2^4 design in two plot blocks were constructed. The levels are denoted by -1 and 1. Arrange the blocks in such a way that the first four blocks had combinations with first factor at lower level. The first five blocks were used for designating the first ten levels and remaining blocks for designating each level. The levels of the asymmetrical factorial and combination of the 2^4 factorial with corresponding contrasts were as shown in table 1.

Example1.

Construction of 13 x 3 x 2 asymmetrical confounded factorial layout confounding the three factor interaction. A confounded 2^7 factorial confounding x_1y_1z and $x_2x_3y_2$ was constructed first, where x_1, x_2, x_3 and x_4 corresponds to the factor at 13 levels, y_1 and y_2 correspond to the second factor at three levels and z was taken as the factor at two levels. The treatment combinations thus obtained were renamed as 13 levels of first factor as shown in the table 1 and as the three levels of the second factor. The resulting layout was as shown in table 2.

Table 1. Table showing the levels and contrasts of the asymmetrical factorial layout with the first factor at 13 levels

Combinations of symmetric design	Levels of asymmetrical factor	Contrast of the asymmetrical design											
-1 -1 -1 -1	0	+	+	+	+	+	+	-	-	+	0	0	0
-1 -1 -1 1	1	+	+	+	+	+	+	-	+	-	0	0	0
-1 1 -1 -1	2	+	-	+	-	+	-	+	-	+	0	0	0
-1 1 -1 1	3	+	-	+	-	+	-	+	+	-	0	0	0
-1 -1 1 -1	4	+	+	-	+	-	-	-	0	0	+	-	0
-1 -1 1 1	5	+	+	-	+	-	-	-	0	0	-	+	0
-1 1 1 -1	6	+	-	-	-	-	+	+	0	0	+	-	0
-1 1 1 1	7	+	-	-	-	-	+	+	0	0	-	+	0
1 -1 -1 -1	8	-	+	+	-	-	+	-	0	0	0	0	+
1 -1 -1 1	9	-	+	+	-	-	+	-	0	0	0	0	-
1 1 -1 -1	10	-	-	+	+	-	-	+	0	0	0	0	0
1 1 -1 1	10	-	-	+	+	-	-	+	0	0	0	0	0
1 -1 1 -1	11	-	+	-	-	+	-	+	0	0	0	0	0
1 -1 1 1	11	-	+	-	-	+	-	+	0	0	0	0	0
1 1 1 -1	12	-	-	-	+	+	+	-	0	0	0	0	0
1 1 1 1	12	-	-	-	+	+	+	-	0	0	0	0	0

Table 2. Layout of a 13 x 3 x 2 asymmetrical factorial design

(0,0,0)	(0,0,1)	(0,1,0)	(0,1,1)
(0,2,1)	(0,2,0)	(0,2,1)	(0,2,0)
(1,0,0)	(1,0,1)	(1,1,0)	(1,1,1)
(1,2,1)	(1,2,0)	(1,2,1)	(1,2,0)
(2,1,0)	(2,1,1)	(2,0,0)	(2,0,1)
(2,2,1)	(2,2,0)	(2,2,1)	(2,2,0)
(3,1,0)	(3,1,1)	(3,0,0)	(3,0,1)
(3,2,1)	(3,2,0)	(3,2,1)	(3,2,0)
(4,1,0)	(4,1,1)	(4,0,0)	(4,0,1)
(4,2,1)	(4,2,0)	(4,2,1)	(4,2,0)
(5,1,0)	(5,1,1)	(5,0,0)	(5,0,1)
(5,2,0)	(5,2,1)	(5,2,0)	(5,2,1)
(6,0,0)	(6,0,1)	(6,1,0)	(6,1,1)
(6,2,1)	(6,2,0)	(6,2,1)	(6,2,0)
(7,0,0)	(7,0,1)	(7,1,0)	(7,1,1)
(7,2,1)	(7,2,0)	(7,2,1)	(7,2,0)
(8,0,1)	(8,0,0)	(8,1,1)	(8,1,0)
(8,2,0)	(8,2,1)	(8,2,0)	(8,2,1)
(9,0,1)	(9,0,0)	(9,1,1)	(9,1,0)
(9,2,0)	(9,2,1)	(9,2,0)	(9,2,1)
(10,1,1)	(10,1,0)	(10,0,1)	(10,0,0)
(10,1,1)	(10,1,0)	(10,0,1)	(10,0,0)
(10,2,0)	(10,2,1)	(10,2,0)	(10,2,1)
(10,2,0)	(10,2,1)	(10,2,0)	(10,2,1)
(11,1,1)	(11,1,0)	(11,0,1)	(11,0,0)
(11,1,1)	(11,1,0)	(11,0,1)	(11,0,0)
(11,2,0)	(11,2,1)	(11,2,0)	(11,2,1)
(11,2,0)	(11,2,1)	(11,2,0)	(11,2,1)
(12,0,1)	(12,0,0)	(12,1,1)	(12,1,0)
(12,0,1)	(12,0,0)	(12,1,1)	(12,1,0)
(12,2,0)	(12,2,1)	(12,2,0)	(12,2,1)
(12,2,0)	(12,2,1)	(12,2,0)	(12,2,1)

4. Construction of confounded asymmetrical factorial designs using balanced designs

Following the line of Tyagi (1971) $p \times 2 \times 2$ designs were constructed using a balanced design with p treatments. In the incident matrix of the design one was replaced by α and zero by β. The p treatments were then identified as p levels of the first factor. The combinations (0,0) and (1,1) of the last two treatments were then attached to a level where α occur and (0,1) and (1,0) were attached to a level where β occurs. Now, (2r-b) rows with β were added to this incidence matrix if $b < 2r$, 2r-b rows with α were added otherwise.

Example.

A $4 \times 2 \times 2$ balanced design was constructed using a balanced design with four treatment, six blocks, three replications, with block size two and with $\lambda = 1$. The incidence matrix of the above design was

$$N = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \qquad M = \begin{bmatrix} \alpha & \alpha & \beta & \beta \\ \beta & \beta & \alpha & \alpha \\ \alpha & \beta & \alpha & \beta \\ \beta & \alpha & \beta & \alpha \\ \alpha & \beta & \beta & \alpha \\ \beta & \alpha & \alpha & \beta \end{bmatrix}$$

where the matrix M was obtained by replacing one by α and zero by β. Corresponding blocks were obtained as shown below

(0,0,0)	(0,0,1)	(0,0,0)	(0,0,1)	(0,0,0)	(0,0,1)
(0,1,1)	(0,1,0)	(0,1,1)	(0,1,0)	(0,1,1)	(0,1,0)
(1,0,0)	(1,0,1)	(1,0,1)	(1,0,0)	(1,0,1)	(1,0,0)
(1,1,1)	(1,1,0)	(1,1,0)	(1,1,1)	(1,1,0)	(1,1,1)
(2,0,1)	(2,0,0)	(2,0,0)	(2,0,1)	(2,0,1)	(2,0,0)
(2,1,0)	(2,1,1)	(2,1,1)	(2,1,0)	(2,1,0)	(2,1,1)
(3,0,1)	(3,0,0)	(3,0,1)	(3,0,0)	(3,0,0)	(3,0,1)
(3,1,0)	(3,1,1)	(3,1,0)	(3,1,1)	(3,1,1)	(3,1,0)

Example 2.

7 x 2 x 2 confounded asymmetrical balanced design was obtained by taking the incidence matrix N of a BIBD with seven treatment as shown below

$$N = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

The transformed matrix M obtained by replacing zero by β

and one by α and augmenting a row vector with α as elements was as follows

$$M = \begin{vmatrix} \alpha & \beta & \beta & \beta & \alpha & \alpha & \beta \\ \beta & \alpha & \beta & \beta & \beta & \alpha & \alpha \\ \beta & \beta & \alpha & \alpha & \beta & \alpha & \beta \\ \beta & \beta & \beta & \alpha & \alpha & \beta & \alpha \\ \beta & \alpha & \alpha & \beta & \alpha & \beta & \beta \\ \alpha & \beta & \alpha & \beta & \beta & \beta & \alpha \\ \alpha & \alpha & \beta & \alpha & \beta & \beta & \beta \\ \alpha & \alpha & \alpha & \alpha & \alpha & \alpha & \alpha \end{vmatrix}$$

Corresponding blocks obtained were

(0,0,0)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,1)	(0,0,0)	(0,0,0)	(0,0,0)
(0,1,1)	(0,1,0)	(0,1,0)	(0,1,0)	(0,1,0)	(0,1,1)	(0,1,1)	(0,1,1)
(1,0,1)	(1,0,0)	(1,0,1)	(1,0,1)	(1,0,0)	(1,0,1)	(1,0,0)	(1,0,0)
(1,1,0)	(1,1,1)	(1,1,0)	(1,1,0)	(1,1,1)	(1,1,0)	(1,1,1)	(1,1,1)
(2,0,1)	(2,0,1)	(2,0,1)	(2,0,1)	(2,0,0)	(2,0,0)	(2,0,1)	(2,0,0)
(2,1,0)	(2,1,0)	(2,1,0)	(2,1,0)	(2,1,1)	(2,1,1)	(2,1,0)	(2,1,1)
(3,0,1)	(3,0,1)	(3,0,0)	(3,0,0)	(3,0,1)	(3,0,1)	(3,0,0)	(3,0,0)
(3,1,0)	(3,1,0)	(3,1,1)	(3,1,1)	(3,1,0)	(3,1,0)	(3,1,1)	(3,1,1)
(4,0,0)	(4,0,1)	(4,0,1)	(4,0,0)	(4,0,0)	(4,0,1)	(4,0,1)	(4,0,0)
(4,1,1)	(4,1,0)	(4,1,0)	(4,1,1)	(4,1,1)	(4,1,0)	(4,1,0)	(4,1,1)
(5,0,0)	(5,0,0)	(5,0,0)	(5,0,1)	(5,0,1)	(5,0,1)	(5,0,1)	(5,0,0)
(5,1,1)	(5,1,1)	(5,1,1)	(5,1,0)	(5,1,0)	(5,1,0)	(5,1,0)	(5,1,1)
(6,0,1)	(6,0,0)	(6,0,1)	(6,0,0)	(6,0,1)	(6,0,0)	(6,0,1)	(6,0,0)
(6,1,0)	(6,1,1)	(6,1,0)	(6,1,1)	(6,1,0)	(6,1,1)	(6,1,0)	(6,1,1)

Analysis

In the present study an attempt is made to generalise the method of sum and difference in the line of Yates (1937) modified by Good (1958). This method is much simpler than that of Good.

A general analysis of a three factor asymmetrical design is explained here. This method can be easily extended to designs with any number of factors.

Let F_1, F_2 and F_3 be three factors at levels p, q and t respectively ($p \geqslant q \geqslant t$). Arrange pqt treatment combinations in the dictionary sequence with F_1 preceeding F_2 and is succeeded by F_3. Write the sum of responses for each treatment combinations in all replications against them. Group these numbers into qt groups of p items each in the same order as they are written. These group sums will form 1/p fraction of the next column ie, the third column. In the next 1/p fraction linear contrasts corresponding to number p, of these groups are written. Next 1/p fraction will be formed by quandratic contrasts and next fraction cubic contrast and so on. Orthogonal contrasts can be obtained from Fisher and Yates Tables (1938). Fourth column is obtained from the third column in a similar fashion as third column is obtained from the second column, but the grouping here is done into pt groups of q items and contrasts also correspond to the number q. Following the same line with the number t, from the fourth column fifth column is obtained. The fifth column will consists of contrasts of the final design.

Divisors of contrast squares are obtained by taking Kronecker product A of matrices M_1, M_2 and M_3 in the reverse order, where, M_1 is a p x p matrix with all elements in the first row as unity and coefficients of contrasts in the remaining rows. Similarly M_2 is a q x q matrix and M_3 a t x t matrix whose elements are taken similar to that of M_1. A will be a pqt x pqt matrix with all elements of first row as unity. These matrices are same as M_1 matrices given by Good (1958). Diagonal elements of AA' when multiplied with number of replications will provide divisors of different contrasts.

While doing the entire procedure, care should be taken not to violate the order.

<u>A particular case.</u>

Consider the case p = 4, q = 3 and t = 2, which results in a 4 x 3 x 2 design. Write the different treatment combinations in the standard order. In the second column corresponding treatment totals in different replications are to be written. This column is grouped into groups of four treatments. These group totals form first one fourth of third column. Remaining portions are filled with linear quadratic and cubic contrasts respectively. For constructing the fourth column third column is grouped into groups of three items and their sums, linear and quadratic contracts are taken. This column is again paired and sums and differences of these pairs will given the contrasts of the final design. Matrix A is obtained by taking the Kronecker product of matrices M_1, M_2 and M_3. Here

$$M_1 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -3 & -1 & 1 & 3 \\ 1 & -1 & -1 & 1 \\ -1 & 3 & -3 & 1 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{pmatrix} \quad \text{and} \quad M_3 = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$A = M_3 \otimes M_2 \otimes M_1.$$

The procedure for finding the contrasts by the sum and difference method is as shown in the table 3.

Table 3. *Method of sum and difference for the calculation of contrasts*

1	2	3	4	5	6
x1	x1+x2+x3+x4 (y1)	y1+y2+y3 (z1)	z1+z2 (c1)	24r (d1)	$c1^2/d1$
x2	x5+x6+x7+x8 (y2)	y4+y5+y6 (z2)	z3+z4 (c2)	120r (d2)	$c2^2/d2$
x3	x9+x10+x11+x12 (y3)	y7+y8+y9 (z3)	z5+z6 (c3)	24r (d3)	$c3^2/d3$
x4	x13+x14+x15+x16 (y4)	y10+y11+y12 (z4)	z7+z8 (c4)	120r (d4)	$c4^2/d4$
x5	x17+x18+x19+x20 (y5)	y13+y14+y15 (z5)	z9+z10 (c5)	16r (d5)	$c5^2/d5$
x6	x21+x22+x23+x24 (y6)	y16+y17+y18 (z6)	z11+z12 (c6)	80r (d6)	$c6^2/d6$
x7	-3x1-x2+x3+3x4 (y7)	y19+y20+y21 (z7)	z13+z14 (c7)	16r (d7)	$c7^2/d7$
x8	-3x5-x6+x7+3x8 (y8)	y22+y23+y24 (z8)	z15+z16 (c8)	80r (d8)	$c8^2/d8$
x9	-3x9-x10+x11+3x12 (y9)	-y1+y3 (z9)	z17+z18 (c9)	48r (d9)	$c9^2/d9$
x10	-3x13-x14+x15+3x16 (y10)	-y4+y6 (z10)	z19+z20 (c10)	240r (d10)	$c10^2/d10$
x11	-3x17-x18+x19+3x20 (y11)	-y7+y9 (z11)	z21+z22 (c11)	48r (d11)	$c11^2/d11$
x12	-3x21-x22+x23+3x24 (y12)	-y10+y12 (z12)	z23+z24 (c12)	240r (d12)	$c12^2/d12$

(contd...)

Table 3. Contd......

1	2	3	4	5	6
x13	x1-x2-x3+x4 (y13)	-y13+y15 (z13)	-z1+z2 (c13)	24r (d13)	$c13^2/d13$
x14	x5-x6-x7+x8 (y14)	-y16+y18 (z14)	-z3+z4 (c14)	120r (d14)	$c14^2/d14$
x15	x9-x10-x11+x12 (y15)	-y19+y21 (z15)	-z5+z6 (c15)	24r (d15)	$c15^2/d15$
x16	x13-x14-x15+x16 (y16)	-y22+y24 (z16)	-z7+z8 (c16)	120r (d16)	$c16^2/d16$
x17	x17-x18-x19+x20 (y17)	-y1-2y2+y3 (z17)	-z9+z10 (c17)	16r (d17)	$c17^2/d17$
x18	x21-x22-x23+x24 (y18)	y4-2y5+y6 (z18)	-z11+z12 (c18)	80r (d18)	$c18^2/d18$
x19	-x1+3x2-3x3+x4 (y19)	y7-2y8+y9 (z19)	-z13+z14 (c19)	16r (d19)	$c19^2/d19$
x20	-x5+3x6-3x7+x8 (y20)	y10-2y11+y12 (z20)	-z15+z16 (c20)	80r (d20)	$c20^2/d20$
x21	-x9+3x10-3x11+x12 (y21)	y13-2y14+y15 (z21)	-z17+z18 (c21)	48r (d21)	$c21^2/d21$
x22	-x13+3x14-3x15+x16 (y22)	y16-2y17+y18 (z22)	-z19+z20 (c22)	240r (d22)	$c22^2/d22$
x23	-x17+3x18-3x19+x20 (y23)	y19-2y20+y21 (z23)	-z21+z22 (c23)	48r (d23)	$c23^2/d23$
x24	-x21+3x22-3x23+x24 (y24)	y22-2y23+y24 (z24)	-z23+z24 (c24)	240r (d24)	$c24^2/d24$

Note: r is the number of replications and d is the product of the diagonal element of the matrix AA' with the replication and matrix A is as given in table 4.

Table 4. A matrix for a 4 x 3 x 2 design

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-3	-1	1	3	-3	-1	1	3	-3	-1	1	3	-3	-1	1	3	-3	-1	1	3	-3	-1	1	3
1	-1	-1	1	-1	-1	-1	-1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
-1	3	-3	1	-1	3	-3	1	-1	3	-3	1	-1	3	-3	1	-1	3	-3	1	-1	3	-3	1
-1	-1	-1	-1	0	0	0	0	1	1	1	1	-1	-1	-1	-1	0	0	0	0	1	1	1	1
3	1	-1	-3	0	0	0	0	-3	-1	1	3	3	1	-1	-3	0	0	0	0	-3	-1	1	3
-1	1	1	-1	0	0	0	0	1	-1	-1	1	-1	1	1	-1	0	0	0	0	-1	-1	-1	1
1	-3	3	-1	0	0	0	0	-1	3	-3	1	1	-3	3	-1	0	0	0	0	-1	3	-3	1
1	1	1	1	-2	-2	-2	-2	1	1	1	1	1	1	1	1	-2	-2	-2	-2	1	1	1	1
-3	-1	1	3	6	2	-2	-6	-3	-1	1	3	-3	-1	1	3	6	2	-2	-6	-3	-1	1	3
1	-1	-1	1	-2	2	2	-2	1	-1	-1	1	1	-1	-1	1	-2	2	2	-2	1	-1	-1	1
-1	3	-3	1	2	-6	6	-2	-1	3	-3	1	-1	3	-3	1	2	-6	6	-2	-1	3	-3	1
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	-1	-3	3	1	-1	-3	3	1	-1	-3	-3	-1	1	3	-3	-1	1	3	-3	-1	1	3
-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
1	-3	3	-1	1	-3	3	-1	1	-3	3	-1	-1	3	-3	1	-1	3	-3	1	-1	3	-3	1
1	1	1	1	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	1	1	1	1
-3	-1	1	3	0	0	0	0	3	1	-1	-3	3	1	-1	-3	0	0	0	0	-3	-1	1	3
1	-1	-1	1	0	0	0	0	-1	1	1	-1	-1	1	1	-1	0	0	0	0	1	-1	-1	1
-1	3	-3	1	0	0	0	0	1	-3	3	-1	1	-3	3	-1	0	0	0	0	1	-3	3	-1
-1	-1	-1	-1	2	2	2	2	-1	-1	-1	-1	1	1	1	1	-2	-2	-2	-2	1	1	1	1
3	1	-1	-3	-6	-2	2	6	3	1	-1	-3	-3	-1	1	3	6	2	-2	-6	-3	-1	1	3
-1	1	1	-1	2	-2	-2	2	-1	-1	1	-1	1	-1	-1	1	-2	2	2	-2	1	-1	-1	1
1	-3	3	-1	-2	6	-6	2	1	-3	3	-1	-1	3	-3	1	2	-6	6	-2	1	3	-3	1

In the case of more than three factors say n factors the contrasts are obtained by similar to addition and subtraction method applied n times. The treatment totals are grouped into different sets, first in the order of M_1 matrix, second in the order of M_2 matrix etc. The contrasts will be obtained in the $(n+2)^{th}$ column and the divisors can be obtained from the A matrix.

$$A = M_n \otimes M_{n-1} \otimes \cdots M_1,$$

where, M_1, M_2 etc. are the matrices obtained by writing the first rows as unit elements and remaining rows as the coefficients of contrasts as given in Fisher and Yates Tables (1938) for the n different factors under consideration. The divisors are the diagonal elements of AA' multiplied by the number of replications.

Illustrative Example I.

Data on dry weight of shoots (in kg/ha) at panicle initiation stage of rice (*Oriza sativa*) from an experiment conducted by Abdul Salam (1983) at Tamil Nadu Agricultural University during North East Monsoon season was taken. The design was $4 \times 3 \times 2$ factorials with treatments four levels of N, three levels of P and two levels of Z. The data were as shown in table 5.

Table 5. Dry weight of shoots at panicle initiation stage of rice

Treatment	Replication I (kg/ha)	Replication II (kg/ha)	Total
n0p0z0	2700	2100	4800
n0p0z1	3150	2250	5400
n0p1z0	3250	2450	5700
n0p1z1	3300	2750	6050
n0p2z0	2700	3350	6050
n0p2z1	2800	2700	5500
n1p0z1	2850	2400	5250
n1p0z1	2800	2700	5500
n1p1z0	2950	3900	6650
n1p1z1	3600	5100	8700
n1p2z0	3300	4200	7500
n1p2z1	3350	4300	7650
n2p0z0	3400	3600	7000
n2p2z1	3550	3750	7300
n2p1z0	3700	3900	7600
n2p1z1	4000	4200	8200
n2p2z0	3900	3900	7800
n2p2z1	4050	4000	8050
n2p0z0	4200	4500	8700
n3p0z1	4500	4800	9300
n3p1z0	4800	5000	9800
n3p1z1	5105	5200	10305
n3p2z0	5200	5100	10300
n3p2z1	5200	5100	10300

Table 6. Table of contrasts

1	2	3	4	5	6
n0p0z0	4800	25750	87350	179605	48
n1p0z0	5250	29950	92255	80115	240
n2p0z0	7000	31650	39550	4805	48
n3p0z0	8700	27500	40565	11705	240
n0p1z0	5700	33255	8350	9900	32
n1p1z0	6850	31500	1455	900	160
n2p1z0	7600	13450	3850	-2000	32
n2p1z0	9800	13050	7855	9300	160
n0p2z0	6050	13050	5900	-10010	96
n1p2z0	7500	13500	4000	4170	480
n2p2z0	7300	12265	-400	3290	96
n3p2z0	10300	14800	1300	11110	480
n0p0z1	5400	1250	-200	4905	48
n1p0z1	5500	1050	-1800	1015	240
n2p0z1	7300	1050	4700	-1895	48
n3p0z1	9300	1900	5100	4005	240
n0p1z1	6050	-545	-2500	-1900	32
n1p1z1	8700	100	-7510	1700	160
n2p1z1	8200	-1350	400	-1600	32
n3p1z1	10305	1850	3770	400	160
n0p2z1	5500	3350	200	-5010	96
n1p2z1	7650	-1500	3090	3370	480
n2p2z1	8050	5755	-1700	2890	96
n3p2z1	10300	3600	-9410	-7710	480

A N O V A

Source	df	SS	MS	F
Block	1	174604.69	174604.69	0.999
Treatment				
N	3	27795249.00	9265083.00	53.05**
P	2	4106563.30	2053281.60	11.76**
Z	1	501229.68	501229.68	2.87
NP	6	1136440.60	789406.76	4.52
NZ	3	145939.06	48646.35	0.28
PZ	2	374271.87	187135.93	1.07
NPZ	6	333565.61	55594.27	0.31
Error	23	4017015.36	174652.84	
Total	47	38410274.48		

<u>Inference:</u> Change in the levels of N and P have got significant effect in shoot dry weight whereas change in the levels of Z has no effect and interactions are also not significant.

<u>Illustrative Example II.</u>

The data on winged bean (<u>Psophocarpus tetrangonolobus</u>) obtained from a fertiliser trial conducted by Brillin (1983) at College of Agriculture, Vellayani had been taken. The design adopted was a 4 x 3 x 3 asymmetrical factorial confounding NK^2 in two replications. The data were as given in table 9.

Table 7. Weekly weight of winged bean

Treatment	Replication I	Replication II	Total
n0k0p2	690	910	1600
n0k0p1	630	1030	1660
n0k0p0	540	1225	1765
n2k2p3	565	1815	2380
n0k0p3	635	1475	2110
n2k2p2	945	2550	3495
n2k2p1	820	2855	3675
n1k1p1	660	1385	2045
n2k2p1	1185	2440	3625
n1k1p0	1100	3185	4285
n1k1p3	1690	3175	4865
n1k1p2	1040	3330	4370
n2k2p3	1320	1800	3120
n0k2p0	775	1590	2365
n1k0p1	1305	585	1890
n2k1p2	3880	3105	6985
n0k2p2	3060	2690	5750
n2k1p1	1200	2030	3230
n1k0p2	2570	2685	5255
n1k0p3	1335	3500	4835
n2k1p0	1450	2535	3985
n1k0p0	2470	1300	3770
n0k2p1	2785	2765	5550
n2k1p3	1240	3625	4865
n0k1p1	1175	2545	3720
n1k2p2	1745	5160	6905
n1k2p0	1870	2585	4455
n2k0p2	595	3935	4530
n0k1p0	700	2695	3395
n2k0p3	715	2040	2755
n1k2p1	735	4835	5570
n2k0p1	675	2550	3225
n0k1p2	1705	2930	4635
n1k2p3	345	2470	2815
n0k1p3	930	2010	2940
n2k0p0	1230	2702	3932

Table 8. Table of contrasts

1	2	3	4	5	6
n0k0p0	1765	7135	37327	137072	72
n0k0p1	1660	15750	49320	11284	360
n0k0p2	1600	14482	50425	-12688	72
n0k0p3	2110	14690	5309	-42072	360
n1k0p0	3770	15565	10010	8932	48
n1k0p1	1890	19065	-4035	-2156	240
n1k0p2	5255	16715	1007	2902	48
n1k0p3	4835	19745	-650	- 16102	240
n2k0p0	3932	13965	-13045	-16108	144
n2k0p1	3225	975	-13597	-9836	720
n2k0p2	4530	6560	-19980	-9658	144
n2k0p3	2755	-2226	-8495	20958	720
n0k1p0	3395	-450	7307	13098	48
n0k1p1	3720	4065	4375	-9344	240
n0k1p2	4635	6395	-2750	14052	48
n0k1p3	2940	2675	-3201	5102	240
n1k1p0	4285	-3585	6845	10057	32
n1k1p1	2045	-3125	-5900	2599	160
n1k1p2	4370	615	-1683	5613	32
n1k1p3	4865	1460	655	2317	160
n2k1p0	3985	-1068	3930	1113	96
n2k1p1	3230	-2020	-5617	21091	480
n2k1p2	6985	2735	-7185	5943	96
n2k1p3	4865	-1365	-3330	-5053	480
n0k2p0	2295	-5885	-9923	-10888	144
n0k2p1	5550	-5205	2625	-18746	720
n0k2p2	5750	-1955	-8810	-10738	144
n0k2p3	3120	525	-14371	17868	720
n1k2p0	4455	-9030	-2185	-4193	96
n1k2p1	5570	-5092	6720	-22691	480
n1k2p2	6905	-3200	-3373	937	96
n1k2p3	2815	-6395	-8855	5453	480
n2k2p0	3625	-10385	2570	-23983	288
n2k2p1	3675	225	13493	-3281	1440
n2k2p2	4285	-5645	-975	16907	288
n2k2p3	2380	-3075	8440	23383	1440

A N O V A

Source	df	SS	MS	F
Blocks	5	39946917.83	7989383.57	26.0**
Treatments				
N	2	3463955.10	1731977.50	5.8**
P	3	7506412.60	2502137.50	8.4**
K	2	4397370.50	2198685.20	7.4**
NP	6	2667307.90	444551.30	1.5
NK	2	5157894.20	2578947.10	8.6
PK	6	6318199.90	1053033.30	3.5**
NPK	12	4948029.66	412335.80	1.4
Error	33	9836386.31	298072.31	
Total	71	84242474.00		

Conclusions:

All main effects and NK and PK interactions were highly significant.

Discussion

DISCUSSION

The present investigation was aimed at the construction of asymmetrical factorial designs for different situations and also to give a general and easy analysis of this asymmetrical factorial layouts.

In this investigation four different methods of construction were attempted

1. construction of confounded asymmetrical factorial designs using Galois field,
2. construction of a $p \times q \times t$ design from a $p \times p$ design,
3. construction of asymmetrical factorial designs using factors at two levels and
4. construction of confounded asymmetrical designs using balanced designs.

In the first method of construction of asymmetric factorial designs using Galois field, two lemmas were derived.

The first lemma,

"If $GF(s)$ is a Galois field with s elements and d is a divisor of $s-1$, then x^d can assume only $(s-1)/d$ +1 distinct values in $GF(s)$ as x assumes all the s values of $GF(s)$".

was used to construct asymmetrical factorial designs in which the first factor would be having s levels where s is a prime or prime power and the levels of the second factor was taken as $(s-1)/d + 1$ where d is a suitable divisor of $s-1$.

Using the second lemma,

"If S and T are square matrices of order $s-1$ such that

$$S = \begin{pmatrix} \alpha_1 & \alpha_1^2 & \cdots & \alpha_1^{s-1} \\ \alpha_2 & \alpha_2^2 & \cdots & \alpha_2^{s-1} \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_{s-1} & \alpha_{s-1}^2 & \cdots & \alpha_{s-1}^{s-1} \end{pmatrix} \quad \text{and}$$

$$T = \frac{1}{p-1} \begin{pmatrix} \alpha_1^{s-2} & \alpha_s^{s-2} & \cdots & \alpha_{s-1}^{s-2} \\ \alpha_1^{s-3} & \alpha_2^{s-3} & \cdots & \alpha_{s-1}^{s-3} \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_1 & \alpha_2 & \cdots & \alpha_{s-1} \\ 1 & 1 & \cdots & 1 \end{pmatrix}$$

then, S and T are inverses of each other"

asymmetrical factorial designs were also constructed by taking two square matrices S and T of order s-1 such that T is the inverse of S. This method gave suitable polynomials which took restricted values while constructing the layouts of the asymmetrical factorials. Eight examples of construction of asymmetrical factorial designs using these two lemmas were given.

In the construction of $3 \times 3 \times 2$, $3 \times 2 \times 2$, $4 \times 2 \times 2$, $4 \times 4 \times 2$, $5 \times 3 \times 2$ and $7 \times 4 \times 3$ layouts the first lemma was made use of. Whereas, in the construction of $4 \times 4 \times 3$ and $4 \times 3 \times 2 \times 2$ the second lemma was made use of.

A general method of construction of $p \times q \times t$ designs from $p \times p$ designs $(p > q \geqslant t)$ was explained in the second method. This method was illustrated with the construction of three examples viz. construction of $4 \times 3 \times 2$, $5 \times 4 \times 3$ and $7 \times 6 \times 3$ asymmetrical layouts.

While constructing asymmetrical factorial designs using factors at two levels two types of construction were attempted.

A. following the line of Das and Rao,

B. following the line of Banerjee and Das.

In the method (A), a general procedure for construction of confounded asymmetrical factorial design $4^p \times 3^q \times 2^r$ was given. This method was illustrated by two examples viz.

constructions of $4 \times 4 \times 2$ and $3 \times 3 \times 2 \times 2$ asymmetrical layouts. In method (B) a general method of construction of asymmetrical factorial design through symmetrical 2^n factorial at p levels, where $2^{n-1} < p \leq 2^n$ was described. This was illustrated through an example of construction of $13 \times 3 \times 2$ asymmetrical confounded layout.

In the fourth method a general method of construction of confounded asymmetrical factorial design in the line of Tyagi (1977) had been explained. This method was illustrated with two examples viz. $4 \times 2 \times 2$ and $7 \times 2 \times 2$ balanced confounded asymmetrical layouts.

The present investigation included a general and simplified analysis in the line of sum and difference approach given by Yates (1937) modified by Good (1958). This method was more simple than that of Good. In this method different matrices say M_1, M_2, M_3 etc. according to the number of levels of various factors were considered. Each of this matrix was constituted by elements unity in the first row, coefficients of contrasts in the other rows according to the levels of that particular factor. The divisors of various contrasts obtained in the final column of operation were obtained from the elements of Kronecker product of these matrices taken in the reverse order. The method of sum and difference for calculation of contrasts and sum of squares were illustrated in table 2 for a 4 x 3 x 2 design.

Finally this analysis was illustrated by taking data from two agricultural experiments - one, dry weight of shoots at panicle initiation stage of rice, an experiment conducted by Abdul Salam (1983) at Tamil Nadu Agricultural University and another the data on yield of winged bean, a fertilizer trial conducted by Brillin (1983) at the College of Agriculture, Vellayani.

All these four methods of construction were modifications and generalisation of the previous existing methods. In this investigation proper care was also taken to give the easiest possible construction technique and analysis of asymmetrical factorial experiments.

Summary

SUMMARY

The objective of the present study were to construct confounded asymmetrical factorial designs suitable for practical experimental situations and to obtain their analysis.

In the present investigation confounded asymmetrical factorials were constructed using four different techniques.

1. using Galois field,

2. p x q x t designs from p x p designs,

3. using factors at two levels and

4. using balanced designs.

Using the first method eight layouts were constructed based on two lemmas. The layouts 3 x 3 x 2, 3 x 2 x 2, 4 x 2 x 2, 4 x 4 x 2, 5 x 3 x 2 and 7 x 4 x 3 were constructed based on lemma (1) whereas layouts 4 x 4 x 3 and 4 x 3 x 2 x 2 were constructed based on lemma (2).

A general method of construction of p x q x t ($p > q \geq t$) designs from p x p designs were explained with illustrative examples of construction of 4 x 3 x 2, 5 x 4 x 3 and 7 x 6 x 3 layouts.

Two different approaches were made while constructing asymmetrical factorials from symmetrical factorials with factors at two levels.

A. Following the line of Das and Rao and

B. Following the line of Banerjee and Das.

Das and Rao's technique was extended to construct a general $4^p \times 3^q \times 2^r$ confounded factorial. Two examples of construction of asymmetrical designs viz. $4 \times 4 \times 2$ and $3 \times 3 \times 2 \times 2$ were given to illustrate this.

In the line of Banerjee and Das a general method of construction of confounded asymmetrical factorial was described with special emphasis given to a factorial with one factor at 13 levels. The same was illustrated with $13 \times 3 \times 2$ confounded layout.

In the last method of construction a balanced confounded asymmetrical factorial design was constructed with a balanced incomplete block design (BIBD). $4 \times 2 \times 2$ confounded layout was obtained from a BIBD with four treatments and $7 \times 2 \times 2$ layout from a BIBD with seven treatments.

In the second part of the study a general and simplified analysis of factorial experiments applicable to both symmetrical and asymmetrical factorial was described in the line of Yates (1937) modified by Good (1958). This method is more simple than that of Good's method. The general method of analysis of $4 \times 3 \times 2$ design was described in detail. The method of analysis was illustrated with numerical example of a 4 x 3 x 2 design by taking data on dry weight of shoots at panicle initiation stage of rice, an experiment conducted by Abdul Salam (1983) at Tamil Nadu Agricultural University and a 4 x 3 x 3 design using data on yield on winged bean from an experiment conducted by Brillin (1983) at College of Agriculture, Vellayani.

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CONFOUNDED ASYMMETRICAL FACTORIAL DESIGNS CONSTRUCTION AND ANALYSIS

By

SANTY GEORGE

ABSTRACT OF A THESIS

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ABSTRACT

Experiments in agriculture require several types of designs. Situations in which treatments are combinations of factors with asymmetrical factors are many. When the total number of treatment combinations is large, confounding is practised in order to get more precise estimates. Confounding is inextricable mixing up interaction effects with block effects.

In the present study four different methods of construction of asymmetrical designs are attempted. In the first method polynomials in Galois field are used for construction. These polynomials are devised on the basis of two lemmas and following the line of Kishen and Srivastava (1959).

Second method of construction is obtained by following Sardana and Das (1965). A general three factor design is constructed.

In the third method factors at two levels are used for constructing asymmetrical designs following the line of Das and Rao (1967) and Banerjee and Das (1969).

Fourth method of construction of asymmetrical factorial designs are from balanced designs. This method of construction is in the line of Tyagi (1971).

A general method of analysis applicable to both symmetrical and asymmetrical designs also is established following the line of Yates (1937). This method of analysis have been illustrated by two examples from the field of agriculture.