STATISTICAL INVESTIGATIONS ON THE ANALYSIS OF DATA OF LONG TERM MANURIAL TRIALS ON PADDY

By

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THESIS

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To my husband

DECLARATION

I hereby declare that this thesis entitled "STATISTICAL INVESTIGATIONS ON THE ANALYSIS OF DATA OF LONG TERM MANURIAL TRIALS ON PADDY" is a bonafide record of research work done by me during the course of research and that the thesis has not been previously formed the basis for the award to me of any degree, diploma, associateship, fellowship or other similar title of any other University or Society.

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CERTIFICATE

Certified that this thesis, entitled "STATISTICAL INVESTIGATIONS ON THE ANALYSIS OF DATA OF LONG TERM MANURIAL TRIALS ON PADDY" is a record of research work done independently by Mrs. Rani John V., under my guidance and supervision and that it has not previously formed the basis for the award of any degree, fellowship or associateship to her.

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CONTENTS

Introduction

INTRODUCTION

Long term experiments are those which are continued on the same set of plots for a long period with a pre planned sequence of treatments. The objective of conducting such experiments is to study the long term effects of the given treatments on soil fertility and on economic returns. Treatments may be applied every year or periodically in a regular schedule. The same crop may be repeated season after season like that in rice, or once planted remain for several years as in perennial trees.

The subject(s) of long term experiment is complex and utmost care is needed in the statistical analysis of data from such trials. Although a few methods have been suggested from time to time for the analysis of data of long term trials none of them appears to be full proof and self sufficient. Hence it would be worthwhile to make an empirical comparison of various available methods of data analysis relating to such experiments and to develop alternate procedures if any, indicating their superiority over the existing methods.

The prime objective of any agricultural experiment is to provide data for the comparison of the efficiency of treatments. The experiment is usually planned by adopting a suitable design and the data are then analysed using the well known procedure of analysis of variance. However, simple comparisons among treatments are not always sufficient as it may sometimes be important to ensure that the superiority of a particular treatment persists from year after year or from place to place or both. Moreover, while examining the data collected from experiments it is of interest to see how far the individual treatments are stable under varying environments. The results of a single experiment conducted in any particular year cannot be totally relied upon as they are subjected to the environmental conditions of the experimental field which fluctuate from year after year or season after season. Hence in order to draw valid conclusions one has to repeat the experiment on the same or different field for a number of years or seasons with the same set of treatments adopting the same cultural and other agronomic practices. The treatment effects are then averaged over the entire range of seasons so as to provide more stable information.

This study is restricted to the case of long term experimentation with a fixed set of treatments under a system of continuous cropping. The data from such experiments can be studied at the end of each year and the results have to be combined after a few years. But several statistical problems may creep up in the process of assessing the overall merit of treatments from such trials due to possible violations from the basic assumptions. Additivity, normality, independence and homosedasticity are the major assumptions implied in the analysis of data using a linear model. The departure from the assumptions affects both the level of significance and power of statistical test. The true type 1 error may be lesser than the specified one and as a result too many significant differences between treatments may be reported. The power is affected in that a more powerful test could be obtained if a correct statistical model were adopted.

The assumption of independence of error is generally critical. Proper randomisation of the experiment introduces independence in the assessment of treatments to experimental units and the resulting experimental errors may be regarded as independent. But in many long term experiments randomisation remains unchanged year after year and that the observations in successive years or seasons are highly correlated. This type of auto correlation among the error terms of successive years will definitely affect the precision of overall treatment comparisons. The Fisherian technique of fitting a fifth degree polynomial to such data and adjusting the effects of treatments on the basis of the expected response does not appear to be sound. The usual method of treating the data on long term trials as special cases of groups of experiments is faulty and unrealistic because this type of analysis also makes use of the assumption of independence of error terms. Analysis of data from groups of experiments introduces added difficulties in the sense that no general test for overall treatment comparisons appear to be available in cases where error variances are heterogeneous and interaction effect is absent. It has been pointed out by Rao (1975) that about 30 per cent of the field trials with heterogeneous error variances belong to this category. The conclusions drawn on individual treatment means from such experiments using chi-square and 't' tests do not appear to be adequate and wholly reliable. In such cases, a possible transformation of data into a suitable scale may be attempted. However, this does not offer complete and satisfactory solution to the problem as it is very difficult to find out the right type of transformation for a given set of data. It is therefore necessary to find an alternative to the method of groups of experiments so as to draw fairly accurate inferences regarding treatments.

Another possibility suggested by many workers in dealing with such experiments is to consider them as special cases of a split plot arrangement with years or seasons as subplots, within each treatment mainplot. But split plot design requires the random arrangement of set of subplot treatments within each main plot and that cannot be expected in the case of trials repeated over several seasons. In such experiments one has to confront with a systematic arrangement of seasons in chronological order under each main plot. Here also the assumption of independence of error terms does not seem to be wholly valid. Even if we assume that the set of experimental years constitute a random sample of years from a population of years, the systematic occurrence of seasons makes the estimates biased.

Another approach to the same problem is in the direction of activities to reduce the risk due to doubt about the correctness of the basic assumptions. This involves the use of methods which do not depend on the exact nature or form of the basic distributions. Only broad assumptions like the distributions are continuous are needed in some cases. These methods are known as non parametric methods, and they mainly depend on ranks and order of the observations rather than their exact values. Therefore, certain amount of lack of precision creeps in. But, if the assumptions are not correct or not known to be correct, one is compelled to seek such methods so as to draw valid inferences from the data even at the cost of sacrificing certain amount of precision. In this connection the method proposed by Rai and Rao (1980) requires special mention. This method has been developed as an alternative to the analysis of data on groups of experiments and has certain distinct advantages over the other method. Non parametric methods do not require any stringent assumptions on the nature of the underlying universe. The only assumption required for the method proposed by Rai and Rao is that the sampling distribution of the means of ranks of the data is approximately normal. But, the method is applicable only for cases where the number of replications per experiment is four or more. Further, the amount of information lost will be more when there are only a few treatments.

It is proposed to develop a new non-parametric method for the analysis of data from long term trials. The work by Friedman (1937) on the two way analysis of ranked data is an important milestone in this direction. An attempt has been made in this study to suitably extend the Friedman's analysis of variance technique to the case of a three way classification with years as the additional factor. The suggested method utilises none of the usual assumptions required for the analysis of variance.

A viable alternative to the same problem is through the use of stability analysis. Stability in performance is one of the desirable properties of any treatment repeated over several seasons or years. A number of statistical methods are now known for estimation of phenotypic stability. Among them the method suggested by Eberhart and Russell (1966) appears to be the most popular. This method involves the use of regression coefficient of yield on environmental index as a measure of overall phenotypic stability of the treatments and judging their performance on the basis of the stability parameters.

Stability of treatments in changing environment can also be measured through non-parametric methods. They are easy to use, distribution free and are not expected to be as sensitive to errors of measurements as that of their counterparts. Furthermore, addition or deletion of one or a few observations is not as likely to cause great variation in the estimates as could be the case for parametric stability measures. The stability of each treatment can be assessed on the basis of such measures and the long term effects of treatments can be assessed on the basis of such parameters.

From the point of view of the farmer, the treatment which gives him better satisfaction than others is preferred often. Farmers differ in their resource position, profit orientation, risk bearing ability and decision making ability. Hence in fertilizer trials the usual practice of making blanket recommendations for all types of farmers have been widely criticised. A conservative farmer may require a recommendation which will not incur him a loss in years of stress. At the other extreme, the business minded farmer may require a dose of nutrient that will assure him the maximum possible return in a given time interval. Thus specific recommendations have to be formulated for different types of farmers with varying decision environments. The principle of game theory is very useful in the choice of an optimum strategy under such risky environments.

From the very nature of long term experiments, multivariate techniques afford themselves as efficient tools for the analysis and interpretation of data. Among the different multivariate techniques, the principal component analysis is considered to be the most versatile and popular. It consists of transforming a set of correlated variables into a few uncorrelated linear components. The advantages of principal component analysis are that it does not require an underlying statistical model to explain the error structure and no assumption is made about the probability distribution of the original variables. Principal component analysis is concerned with reducing the dimensionality of the data. The first principal component serves as a weighted index of yearly responses and is expected to explain maximum amount of variability in the data. The percentage variation explained by the treatment totals in the aggregate data will be definitely less than that accounted by the first principal component. Further, the problem of lack of independence of error terms in the linear model can also be solved through the use of principal components as the dependent structure of error terms in successive years is lost by replacing a single index value for the entire time series data from each of the different plots. Thus principal component analysis can definitely be recommended for the analysis of data from groups of experiments. But the use of principal component analysis to interpret the results of a long term trial has not been reported so far. So this study is also aimed at applying the technique of principal component analysis for the analysis of data on long term manurial trials.

As several methods have been suggested for the analysis of data of long term trials it is desirable to have an empirical comparison of all the proposed methods on the basis of actual field data to know their relative efficiencies and the degree of mutual concordance. The results of such analysis will definitely indicate the appropriate technique to be adopted for the analysis and interpretation of data on long term manurial trials.

Another problem with regard to the analysis and interpretation of data from long term fertilizer trials is to select an appropriate mathematical function to represent yield fertilizer relationships. The fitted response function is then studied to get the optimum and economic doses of nutrients and the expected returns.

Among the different mathematical functions which are used to describe the response pattern of fertilizer on crop yield, the quadratic polynomial is the most popular. Fitting a quadratic response surface is simple as only linear estimation is involved and the usual technique of analysis of variance and tests of significance can be immediately applied with this surface. This function also takes account of declining yields in part of the range of doses tried. The standard errors of estimated optimum doses based on quadratic surface are less than those of other surfaces. But a disadvantage of using the second order response function is that they are symmetrical in shape about its stationary value. There are many instances in fertilizer research where response curve is not symmetrical about the stationary value. The quadratic function is not efficient to represent the response pattern in such experiments. In certain other cases the curve ceases to decline beyond the optimum value and one is confronted with an assymptotic nature of response within the range of nutrients. There are also instances where the curve will have more than one stationary value. In such cases polynomial of higher degree than two can be used to represent response pattern. In the case of fertilizer responses of assymptotic nature Holliday function, Nelder's polynomial etc. may be recommended. When the ordinary polynomials fail to fit the data, some transformations such as square root transformation, logarithemic transformation etc. can be used to get more reliable results. Since a variety of functions could be used to describe the response pattern, choice of a suitable mathematical model for describing the dose-yield relationship is an important aspect of fertilizer use research. It is also necessary to develop alternate models to represent the response pattern in particular situations where the ordinary models fail to describe the proposed relationship.

As different functions may be fitted to the data of different seasons or years in repeated experiments, the function which gives a satisfactory fit in most of the seasons or years can be considered to be a better choice than others.

As nitrogen and phosphorous are the two nutrients which have been generally tried and which have shown response in rice, this investigation is confined to these two nutrients alone. All the experiments considered in the study pertain to rice crop since suitable experiments on other crops are extremely few.

In view of the facts described in the previous paragraphs, the present study is aimed at the following objectives.

1. To empirically evaluate the relative efficiencies of various statistical techniques involved in the analysis of data from long term fertilizer experiments and suggest suitable methods for specific situation based on various criteria.

2. To develop more reliable and subtle methods of data analysis in the case of long term experiments and to compare the utility of such methods with the existing methods.

3. To compare the relative efficiencies of different mathematical models in describing the yield-fertilizer response relationship in paddy and to determine the optimum levels of inputs in a realistic manner taking into account such factors as cost of input, cost of output etc.

4. To develop alternate models for describing the relationship between yield and fertilizer response in paddy and explore their superiority over the existing models.

Review of Literature

REVIEW OF LITERATURE

Although comprehensive studies on the various problems of data analysis in the field of long term manurial trials on paddy or other crops are very few in India or abroad, many reports on the fitting of response surface models to fertilizer trial data are available. A short review of the available literature on the subject is furnished below under two sub-headings viz. (A) Analysis of data of long term experiments and (B) fitting response models.

A. Analysis of data of long term experiments

In an attempt to obtain information on the various treatment effects under a system of continuous cropping Fisher (1924) applied the method of orthogonal polynomials to an experiment on wheat involving various fertilizer treatments. He fitted a fifth degree polynomial to represent the relation between yield and annual rainfall. However, since the equation is of limited utility in predicting the limiting response and since the biological explanation of a mechanism which will generate a polynomial does not appear to be reasonable, some other functions of time should be used to explain the yield from the plot in the i^{th} year. Further, his method did not provide a test of significance to make an objective comparison among treatment means.

Yates and Cochran (1938) made an attempt to analyse data from sets of experiments involving the same or similar treatments carried out at a number of places, or in a number of years. They pointed out that the ordinary analysis of variance procedure suitable for dealing with the results of a single experiment may require modification, owing to lack of equality in the errors of different experiments and non-homogeneity of the components of the interaction of the treatments with places and times.

Patterson (1939) considered the problem of field experimentation with perennial crops and suggested that certain modifications have to be effected in the statistical analysis of long term data on perennial crops. He recommended the use of split plot design for the analysis of long term experiments with years assigned to sub-plots and treatments assigned to main plots.

The problems arising in the analysis of data from long term experiments containing different crop rotations were investigated by Yates (1954). The method was illustrated by application to rice pasture experiment containing rotations of different lengths and with different proportions of rice to pasture. When the design of the experiment was such that each block contained plots which sometimes carried a given crop but did not all carry the crop in the same set of years the year-block totals were not found to be orthogonal to the plot-totals. He recommended the method of fitting constants to obtain seperate estimates of plot error and plot x year error which were free of year x block interactions.

Danford et al.(1960) made the analysis of repeated measurement experiments and found that assymptotically the univariate and multivariate tests were identical.

Finlay and Wilkinson (1963) studied the adaptability of crop varieties to different seasons or places. The linear regression of y_{ij} on x_j could be written as

$$y_{ij} - \bar{y}_i = b\ (\bar{x}_j - \bar{\bar{x}})$$

where y_{ij} is the mean yield of the i^{th} variety in the j^{th} place $\bar{x}_j$ is the mean of the yields of all varieties at the j^{th} place, $\bar{y}_i$ is the mean yield of the i^{th} variety in the experiments, $\bar{\bar{x}}$ is the grand mean of the yields of all varieties in all the experiments and b is the regression coefficient. A variety had average, better than average or less than average adaptability according as b is one, less than one or greater than one.

Eberhart and Russell (1966) developed stability parameters for comparing varieties. The model $y_{ij} = \mu_i + \beta_i I_j + \delta_{ij}$ defined, stability parameters that might be used to describe the performance of a variety over a series of environments. y_{ij} is the mean of the i^{th} variety at the j^{th} environment. μ_i is the mean of the i^{th} variety over all environments, β_i is the regression coefficient that measures the response of the i^{th} variety to varying environments, δ_{ij} is the deviation from regression of the i^{th} variety at the j^{th} environment, and I_j is the environmental index.

Methods of multivariate analyses were used to analyse data from experiments with repeated measurements by Cole and Grizzle (1966). They found that multivariate techniques had the same power, scope and flexibility as their univariate counterparts.

Agarwal and Heady (1969) developed a theory for statistical decision making under uncertainty. They made a comparative study of the four existing theories of decision making viz. Wald's maximin criterion, Laplace's principle of insufficient reason, Hurwicz 'optimism-pessimism' criterion, Savage's regret criterion and suggested the new theory of choice - the criterion of benefit which blended the properties of these four models.

An alternate approach for interpretation of data collected from groups of experiments was developed by Hawlo and Das (1978). The method consisted in obtaining a treatment index as an average of the treatment yield and an environmental index as an average of the environment. An inverse of the regression coefficient of treatment index on environment index was taken as a measure of stability of the treatment with changing environment.

Attempts were made by Khosla et al. (1979) to study the behavior of experimental errors and presence of treatment x year interaction in the case of groups of experiments.

Friedman (1937) developed a non parametric two way analysis of variance technique based on ranked data. Later, a non parametric method for the analysis of data of long term trials was devised by Rai and Rao (1980). They developed a test criterion for which the sampling distribution approached a chisquare distribution. The method was applicable to a wide class of problems to which the analysis of variance could not be validly applied.

Krishnan et al. (1982) made a comparison of two methods of analysis of data relating to permanent manurial trials on paddy. The data on Jaya variety of rice were analysed both by the method of stability coefficients and by the method of analysis of groups of experiments. The results obtained by the two methods revealed that they were equivalent.

Application of principle of game theory to a fertilizer experiment on coorge mandarin was discussed by Ramachander et al. (1982). The pay-off in the form of yield and net-returns was considered. It was assumed that resources were not a limiting factor in choosing the strategies. The treatments applied were considered as strategies and the yield of different years were considered as the nature's strategies.

Nassar and Huhn (1987) proposed tests of significance for non parametric measures of phenotypic stability. The statistical properties and tests of significance for two non parametric measures of phenotypic stability ie, mean of the absolute rank differences of a genotype over the environments and variance among the ranks over the environments, were also investigated.

Prabhakaran et al. (1988) applied the principle of game theory for interpretation of data of long term fertilizer trial on WCT coconut in red loam soils of Kerala. They made specific recommendations for farmers with varying decision environments using different criterion such as Wald's maximin criterion, Laplace's principle of insufficient reason, Hurvicz 'optimism-pessimism' criterion, Savage's regret criterion and Agarwal's excess benefit criterion.

B. Fitting of response models

Justus Von Liebig's law of the minimum was the first attempt to define a fundamental relationship between fertilizer or nutrient inputs and crop yields. Liebig (1855) stated that crop yields were proportional to the amount of nutrients supplied to them and when all nutrients were present in sufficient quantity the addition of one or more would not increase crop yield. Von Liebig did not suggest an algebraical model to represent the relationship.

Mitscherlich (1909) defined an algebraic form of fertilizer yield relationship. He proposed a non linear function to represent the relationship between nutrient intake and crop yield. With the aid of Baule, a mathematician, he proposed the equation

$$\log A = \log (A-Y) = Ca$$

to explain fertilizer response allowing marginal productivity. In this A is the total yield when the nutrient Y is not deficient and C is the proportionality constant which indicates the rate at which marginal yields decline.

Spillman (1924) proposed an exponential yield equation similar to that of Mitscherlich which is given by $Y = M-AR^x$ where M is the maximum yield attainable by increasing the nutrient input x. A is a constant defining the maximum response attainable from use of x and R is the coefficient defining the ratio by which marginal productivity of x increases.

Briggs (1925) suggested the use of hyperbola of the form $Y = \frac{(x+b)E}{x+b+E}$ where E is the maximum yield. b is the quantity of x in the soil and h is the optimal supply of input.

A modified statement of the equation proposed by Balmukund (1928) based on Maskels resistance formula is expressed as

$$Y^{-1} = a(b+x)^{-1} + c$$

where a, b, c are constants and in the case of fertilizers b is the nutrients in the soil and x is the amount of nutrients added.

Boresch (1928) modified Liebig's law and developed an algebraical model $Y = a + bx$ where Y is the total yield, a is the yield in absence of application of x, the nutrients supplied.

Crowther and Yates (1941) emphasised that final conclusions on fertilizer response must be based on a series of experiments conducted in different years on different crops under varying soil and farm situations. They used the modified Mitscherlich's formula which is given by

$$Y = Y_0 + d\ (1-10^{-kx})$$

where Y_0 is the yield without fertilizer, d is the limiting response, x is the quantity of nutrient added and k( ) is a constant.

Sukhatme (1941) used a quadratic equation to fit respose data for rice and Panse *et al.* (1951) used it for cotton.

Johnson (1953) emphasised that in the case of single input, quadratic and square root polynomials were better than other forms with some preference to the square root quadratic attributed to its non symmetrical and flatter shape by xy plane.

Gomes (1953) used the Mitscherlich's regression equation in the analysis of experiments with fertilizers which is given by $y = A\left[1-10^{-c(x+b)}\right]$ where A measures a maximum yield which could not be exceeded by the use of the fertilizer in consideration. C measures the efficiency of the fertilizer and b measures the soil content of the fertilizer in the control plots in a form assimilable by the plant.

Halter *et al.*(1957) proposed the function $y = cx^a e^{bx}$ which was a hybrid combination of power function and exponential function. a, b, c are constants and x is the nutrient added.

Bleasdale and Nelder (1960) proposed an equation $Y = \frac{\rho}{(a+b\,\rho^c)^d}$ with constants a, b, c and d. It was usually satisfactory to take $c=1$. By taking $d=1$, $d>1$ the assymptotic and parabolic responses were obtained.

Holliday (1960) found that dry matter yield had an assymptotic relationship with plant populations. He attempted to describe the assymptotic type of relationship by a function of the form

$$Ya = \frac{Ax}{1+Abx} \quad \text{where}$$

Ya is the yield per unit area, x is the number of plants per unit area and A is the apparent maximum yield attainable by an individual plant in the particular environment.

Abraham and Rao (1966) studied the functional relationship between doses of fertilizers and the yield of paddy crop. They compared the efficiencies of different mathematical models in describing the response surface for paddy crop based on empirical data. The five response functions Mitscherlich, Resistance formula, Cobb-Douglas, quadratic and square root formula were considered. It was found that in the general absence of interaction for most of the cases the quadratic surface, could be fitted. Resistance formula gave uniformly better fit when interaction was present. Estimates of the nutrients available in the soil were made using Mitscherlich, Resistance and Cobb-Douglas functions.

Nelder (1966) discussed about inverse polynomial response functions. If $x_1, x_2, \ldots\ldots\ldots x_k$ represent the levels of k experimental factors and y is the mean response, then the inverse polynomial response function is defined by

$$\frac{x_1 \cdot x_2 \ldots\ldots x_k}{y} = \text{polynomial in } (x_1, x_2, \ldots\ldots x_k).$$

The goodness of fit of ordinary and inverse polynomials was compared and the inverse kind shown to have some advantages.

Church (1966) presented a method of reducing a curvilinear response to a set of numbers which described a curve. He made an analysis of such numbers including reconstruction of the curves.

Inverse polynomial response surfaces applied to data from plant nutrition experiments was proposed by Clarke (1968). Inverse polynomial surfaces of linear and quadratic type were compared, the latter often succeeding even in cases where a maximum was not reached.

Inverse polynomial response surfaces applied to data from plant nutrition experiments were further discussed by Clarke and Esan (1971). Curves of the form $y^{-1} = ax^{-1} + b$ and $y^{-1} = ax^{-1} + b + cx$ in which y is the crop yield and x is the level of fertilizer applied, gave two useful shapes of relation between y and x. When several fertilizers, $x_1, x_2 \ldots\ldots$

were used in an experiment, these curves might be generalised into surfaces, where various combinations of the two types of relations could be included.

Snee (1972) made a study on the analysis of response curve data, and developed a better model which combined the univariate analysis of variance and principal component analysis.

A family of linear plateau models involving intersecting straight lines and concomitant experimental designs useful in evaluating response to fertilizer nutrients was proposed by Anderson and Nelson (1975). They found that for multi-nutrient experiments a complete factorial experiment with a number of levels of each nutrient was the best design for evaluating the model and then estimating the optimal nutrient levels.

A mixture model with inverse terms was proposed by Draper and John (1975). They suggested a type of model which combined Scheffe polynomials and inverse terms.

Perrin (1976) established that the linear response plateau models proposed by Anderson and Nelson were inferior to quadratic models.

Barnes <u>et al</u>. (1976) obtained a dynamic model for the effects of potassium and nitrogen fertilizers on the growth and nutrient uptake of crops. The model had the ability to forecast the effect of different weather conditions on crop response and the interaction between the effects of nitrogen and potassium fertilizers on the growth and chemical composition of plants.

The response function approach to the effect of fertilizers on crop yield was discussed by Thornley (1978). The problem was first considered in general terms and expressions were derived for the maximum yield and the economic yield. The theory was then applied to the inverse polynomial function, which was used to describe the response to various levels of nitrogen, phosphorous and potassium fertilizers.

Tonk and Singh (1982) obtained a method for analysis of response curve data. The procedure combined the analysis of variance model and the modified principal component analysis. The method consisted in developing certain statistics which described the level and shape of the curves. These statistics were then used to determine the effects of the treatments on the curve.

Gupta and Nigam (1982) discussed about models useful for approximating fertilizer response relationships. They found that if the observations had a long tail to the right, then the performance of second degree inverse polynomial was better than the ordinary second degree polynomial. For symmetric situations the two polynomials behaved equally well. For negatively skewed observations, the performance of the ordinary polynomial was better than the performance of the inverse polynomial.

Materials and Methods

MATERIALS AND METHODS

This chapter has been written as two sections under the subheadings (A) Analysis of data of long term experiments and (B) Fitting of response models.

A. Analysis of data of long term experiments

The data relating to the permanent manurial trials with Jaya variety of rice during the period 1973 to 1987 for the kharif and rabi seasons were collected from the Regional Agricultural Research Station, Pattambi. The informations on rabi crop in the years 1982, 1984 and 1986 were not available due to the incidence of drought. So these years were not considered in the study. Data on rabi and kharif crop were pooled for each year to get a time series of yearly production of paddy for a period of 12 years. The experiment was laid out in a 4 replicate randomised block design with 8 treatments. A uniform spacing of 15 cm x 15 cm was adopted. The gross plot size was 7.8 x 5.25 sq.m. and the net plot size was 7.6 x 4.95 sq.m. The treatments are given below.

1. Cattle manure at 18000 kg/ha to supply 90 kg N/ha
2. Green leaf at 18000 kg/ha to supply 90 kg N/ha
3. Cattle manure at 9000 kg/ha + Green leaf at 9000 kg/ha to supply 90 kg N/ha

4. Ammonium sulphate to supply 90 kg N/ha
5. Cattle manure at 9000 kg/ha + Ammonium sulphate to supply 45 kg N/ha + 45 kg P_2O_5/ha + 45 kg K_2O as M.O.P. (Muriate of Potash).
6. Green leaf at 9000 kg/ha + Ammonium sulphate to supply 45 kg N/ha + Super phosphate to supply 45 kg P_2O_5/ha + 45 kg K_2O as M.O.P.
7. Cattle manure 4500 kg/ha + Green leaf 4500 kg/ha + 45 kg N as Ammonium sulphate + 45 kg P_2O_5/ha + 45 kg K_2O/ha
8. Ammonium sulphate to supply 45 kg N/ha + super phosphate to supply 45 kg P_2O_5/ha + M.O.P. to supply 45 kg K_2O/ha

(Ammonium sulphate to be applied half as basal and the rest as top dressing at panicle initiation)

The same experiment was repeated over season to season in different years. The responses to these treatments may remain steady or may depend upon the season. In this study several methods have been attempted for the analysis of data from long term experiments. They are discussed below.

1. Analysis of data as in groups of experiments

The data for each of the 12 years are analysed separately in the usual way as in a randomised block design. The method of analysis has been derived from the following model.

$$y_{ij} = \mu + \alpha_i + \beta_j + e_{ij}$$

where y_{ij} is the observation of the i^{th} treatment ($i = 1, 2, ...t$) in the j^{th} block ($j = 1, 2 .....r$), α_i is the effect due to i^{th} treatment, β_j is the effect due to j^{th} block and e_{ij} is the random error component which is assumed to be independently and normally distributed with zero mean and constant variance σ^2. The structure of the analysis of variance of randomised block design with t treatments and r replications is given below.

ANOVA

Source	d.f.	M.S.
Replications	r-1	S_r^2
Treatments	t-1	S_t^2
Error	(r-1)(t-1)	S^2
Total	rt-1	

Homogeneity of error mean squares is then tested using Bartlet's test. If the error mean squares are homogeneous, pooled analysis is done. In this method pooled error mean square is to be used to find 'F' ratio for treatments and blocks in case the year x treatment interaction effect is non significant. If on the other hand the interaction effect is significant the treatments are tested against interaction mean square. When errors are heterogeneous the method of weighted analysis is applied to test the significance of the effect of interaction. This is done by assigning a weight w_i to each experiment and the weights are calculated as $w_i = \frac{r}{s_i^2}$ where s_i^2 is the corresponding error mean square. Using those weights for each year the quantities $w_i p_i$ where p_i's are the year totals and for each treatment and quantities $\Sigma w_i t_i$ where t_i's are means for each treatment at each year are calculated. If G be the sum of $\Sigma w_i t_i$ over all the treatments, s_i be the crude sum of squares obtained for each year then the various items in the analysis of variance are calculated as below.

ANOVA

Source	d.f.	S.S.
Treatments	t-1	$\sum_i \frac{(\sum_i w_i t_i)^2}{\sum_i w_i} - C$
Years	p-1	$\frac{1}{t} \sum_i (w_i p_i^2) - C$
Interaction	(p-1)(t-1)	I
Total	pt-1	$\sum_i w_i s_i - C$

where p is the number of years and $C = \frac{G^2}{t \Sigma w_i}$, the correction factor.

The sum of squares for interaction, I is calculated by subtracting from the total sum of squares, the sum of squares for the years and treatments. In order to test the significance of interaction, the sum of squares of interaction is transformed into a chi-square variate using the formula, $\chi^2 = \frac{(n-4)\,(n-2)I}{n(n+t-3)}$. This follows a chi-square distribution with $\frac{(p-1)(t-1)(n-4)}{n+t-3}$ degrees of freedom where n is the uniform error degree of freedom.

In case the interaction is significant the means of the treatments for the different years may be set out in a two-way table and the simple analysis of variance is carried out.

ANOVA

Source	d.f.
Treatments	t-1
Years	p-1
Interaction	(t-1) (p-1)
Pooled error	p(r-1) (t-1)

The treatment mean square is then compared with the interaction mean square to test the significance of treatment differences.

2. Analysis of split plot design

The analysis is performed by considering the treatments as main plots and years as sub plots. The method of of analysis has been derived from the following model.

$$y_{ijk} = \mu + \alpha_i + \tau_j + (\tau\alpha)_{ij} + \beta_k + (\tau\beta)_{jk} + (\alpha\beta)_{ik} + (\tau\alpha\beta)_{ijk}$$

where y_{ijk} is the observation in the i^{th} block receiving j^{th} main plot treatment and k^{th} subplot treatment, μ is the general mean, α_i, τ_j and β_k are the fixed effects of i^{th} block, j^{th} mainplot treatment and k^{th} subplot treatment respectively. $(\tau\alpha)_{ij}$ is the interaction effect of the i^{th} block and j^{th} main plot treatment which is termed as the main plot error. $(\tau\beta)_{jk}$ is the interaction effect of j^{th} main plot treatment and k^{th} subplot treatment. $(\alpha\beta)_{ik}$ and $(\tau\alpha\beta)_{ijk}$ are two error components associated with subplot, together known as subplot error. The error components in the model are assumed to be independently and normally distributed with zero mean and constant variance σ^2.

Let there be r replications, p main plots and q sub plots under each main plot. Then $i = 1, 2, \ldots\ldots r$, $j = 1, 2, \ldots\ldots p$, $k = 1, 2, \ldots\ldots q$. If R_i, M_j and S_k are the total of all observations in the i^{th} replication, j^{th} treatment and k^{th} year respectively then analysis of variance of the design is given below.

ANOVA

Source	d.f.	S.S.	M.S.
Replications	r-1	$\sum_i \frac{R_i^2}{pq} - CF$	s_r^2
Treatments	p-1	$\sum_j \frac{M_j^2}{rq} - CF$	s_m^2
Error (a)	(r-1)(p-1)	T.T.S.S. (1)	s_a^2
Years	q-1	$\sum_k \frac{S_k^2}{rp} - CF$	s_s^2
Treatment x year interaction	(p-1)(q-1)	T.T.S.S. (2)	s_{mxs}^2
Error (b)	p(r-1)(q-1)		s_b^2
Total	rpq-1	$\sum_i \sum_j \sum_k Y_{ijk}^2 - CF$	

T.T.S.S.(1) is obtained by considering the replications and treatments as a two way table. If a_{ij} is the observation in the ij^{th} cell, T.T.S.S.(1) $= \sum_i \sum_j \frac{a_{ij}}{q} - \sum_i \frac{R_i^2}{pq} - \sum_j \frac{M_j^2}{rq} + CF.$

T.T.S.S.(2) is obtained by considering the treatments and years as a two way table.

If b_{jk} is the observation in the jk^{th} cell,

$$T.T.S.S.(2) = \sum_j \sum_k \frac{b_{jk}^2}{r} - \sum_j \frac{M_j^2}{rq} - \sum_k \frac{S_k^2}{rp} + CF$$

$C.F. = \frac{(\sum_i \sum_j \sum_k y_{ijk})^2}{rpq}$, the correction factor. Sum of squares for error (b) is obtained by subtracting all the other sum of squares from the total sum of squares.

Standard error for the difference between two treatment means $= \sqrt{\frac{2Sa^2}{rq}} = S_e$

Critical difference, CD is given by $t_\alpha \sqrt{\frac{S_e^2}{r}}$ where t_α is the tabled value of students 't' with whole plot error degrees of freedom for α per cent level of significance.

Then the treatment means are compared with CD to test the significance of treatment differences.

3. Principal component analysis

Consider the random variables $x_1, x_2, \ldots\ldots x_p$ which have a multivariate distribution with mean vector μ and correlation matrix Σ. Assume that the elements of μ and Σ are finite. The rank of Σ is p and there will be p characteristics roots. Let the characteristics roots be $\lambda_1, \lambda_2, \ldots\ldots\ldots \lambda_p$ such that $\lambda_1 > \lambda_2 > \ldots\ldots\ldots > \lambda_p$ and they are all distinct.

Let there be N treatments repeated over p years. The observations can be written as the N x p data matrix.

$$X = \begin{bmatrix} x_{11} & x_{12} & \ldots\ldots & x_{1p} \\ \cdot & \cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot & \cdot \\ x_{N1} & x_{N2} & \cdot\ \cdot\ \cdot & x_{Np} \end{bmatrix}$$

Each x_{ij} could be transformed into a standard score z_{ij} as

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j} \qquad - (1)$$

where $\bar{x}_j$ is the mean and s_j is the standard deviation of x_{ij}, $i = 1, 2, \ldots\ldots N$.

The covariance matrix calculated from $Z = (z_{ij})$ will be the correlation matrix of the original data matrix and will be of order $p \times p$

The first principal component of the observations z is that linear compound

$$y_1 = a_{11} z_1 + a_{21} z_2 + \cdots \cdots a_{p1} z_p = a_1' z$$

so that $a_1' a_1 = 1$ and variance of y_1 must be maximum. The coefficients of this linear compound must satisfy the p simultaneous linear equations $(\Sigma - \lambda_1 I)\, a_1 = 0$. The value of λ_1 must be so chosen as to make $|\Sigma - \lambda_1 I| = 0$. λ_1 is thus a characteristic root of the correlation matrix and a_1 is its associated characteristic vector.

The second principal component is that linear compound

$$y_2 = a_{12} z_1 + a_{22} z_2 + \cdots \cdots a_{p2} z_p = a_2' z$$

whose coefficients has been chosen subject to the constraints $a_2' a_2 = 1$, $a_1' a_2 = 0$ so that the variance of y_2 is a maximum.

The second vector must satisfy $(\Sigma - \lambda_2 I)\, a_2 = 0$. λ_2 is thus the second characteristic root and a_2 is its associated characteristic vector. Similarly all other characteristic roots and characteristic vectors can be found out so that $\lambda_1 + \lambda_2 + \ldots\ldots + \lambda_p = \text{trace}\,\Sigma = p$.

The first principal component serves as that linear combination of years which explains maximum variation among the treatments. This is simply a weighted index of seasonal components, the weights being the coefficients in the associated eigen vector. The process provides a unique value for each treatment in the set of treatments and this is obtained by multiplying the transformed matrix z with the eigin vector a_1. This value of the derived composite variable known as the index value acts as an index of performance of the specific treatment in relation to the other treatments and thus helps in the descrimination between treatments. The treatments are then ranked on the basis of the derived indices and the best treatment is recommended for adoption.

But the method described above fails to provide a statistical test of significance among treatment effects. A more general approach is to derive the first principal component from the original Nr x p matrix of observations where r is the number of replications for each treatment. Standardised values are then obtained by applying the relevant transformation described in (1). P eigen values and corresponding eigen vectors are generated and the eigin vector corresponding to the largest eigen value is designated as the first principal component. It is given by

$$y_1 = \sum_{j=1}^{p} a_{j1} \, z_j$$

Then by multiplying the Nr x p matrix of standardised values with the largest eigen vector (principal component) of order p an index value matrix of order Nr x 1 is obtained which can be arranged into a two way table of N treatments and r replications. Data of the two way layout can be analysed as in a randomised block design. The analysis of variance of the resulting data is given below.

ANOVA

Source	d.f.	S.S.	M.S.
Replications	r-1	$\sum_i \frac{R_i^2}{N}$ - CF	s_r^2
Treatments	N-1	$\sum_j \frac{T_j^2}{r}$ - CF	s_t^2
Error	(r-1)(N-1)		s_e^2
Total	rN-1	$\sum_i \sum_j y_{ij}^2$ - CF	

R_i and T_j are the total of all observations in the i^{th} replication and j^{th} treatment respectively.

$CF = \left(\frac{\sum_i \sum_j y_{ij}}{rN} \right)^2$ the correction factor.

If the treatments are found to be significant, critical difference $CD_\alpha = t_\alpha \sqrt{\frac{s_e^2}{r}}$ can be calculated. Then the treatment means are compared with CD_α to test the significance of the treatment differences.

4. The non-parametric method proposed by Rai & Rao (1980)

The method is applicable to problems in which analysis of variance cannot validly be applied. It can also be used for the trials when the error variances are heterogeneous. The procedure involves first ranking of the observations in each replication of the individual experiment. If t treatments are compared in a replication the individual observations are ranked by giving rank 1 to the highest value, 2 to the next lower and so on. The smallest value of the observations will be given rank t. Ranking is done afresh for each replication and it will have variate value 1, 2, ...t. On the hypothesis that there is no significant difference between the treatments, the difference in the values in each replication for different treatments will arise solely from sampling fluctuations.

The set of ranks for each treatment represents a random sample from the discontinuous rectangular distribution. Suppose each treatment is replicated r times in a particular trial and the trial is repeated over p years. If the character under study is independent of the replication the set of ranks r_{ijk} being the rank of j^{th} treatment in the i^{th} replication of k^{th} experiment will represent a random sample of rp items from a discontinuous rectangular universe. Mean and variance of this universe are obtained as follows.

$$\text{Mean} = t\,\frac{(t+1)}{2\,t} = \frac{t+1}{2}$$

$$\text{Variance} = \frac{1}{t}\left[\frac{t(t+1)\,(2t+1)}{6} - \frac{t^2(t+1)^2}{4t}\right] = \frac{t^2-1}{12}$$

The mean rank of j^{th} treatment is given by

$$\bar{R}_j = \frac{1}{rp}\sum_{i=1}^{r}\sum_{k=1}^{p} r_{ijk}$$

It is known that the sampling distribution of the means of the ranks will be approximately normal. The sampling distribution of the mean ranks $\bar{R}_j$ will have the mean value $\bar{R}$ which is equal to $\frac{t+1}{2}$ and the variance σ^2 which is equal to $\frac{t^2-1}{12\,rp}$. The hypothesis that the means of the ranks of various treatments came from a single homogeneous normal population can be tested by the statistic

$$k = \sum_{j=1}^{t} \frac{(\bar{R}_j - \bar{R})^2}{\sigma^2}$$

By putting the values of $\bar{R}$ and σ^2 and taking $R_j = rp\,\bar{R}_j$ where R_j is the sum of ranks of the j^{th} treatment, we get the value of k in another form

$$K = \frac{1}{\sigma^2} \sum_{j=1}^{t} (\bar{R}_j - \bar{R})^2$$

$$= \frac{1}{\sigma^2} \sum_{j=1}^{t} (\bar{R}_j - \frac{t+1}{2})^2$$

$$= \frac{1}{\sigma^2} \left[\Sigma \bar{R}_j^{\,2} - \frac{t(t+1)^2}{4} \right]$$

$$= \frac{12\,rp}{t^2-1} \left[\Sigma \frac{R_j^{\,2}}{r^2p^2} - \frac{t(t+1)^2}{4} \right]$$

$$= \frac{12}{rp(t^2-1)} \Sigma R_j^{\,2} - \frac{3rpt(t+1)}{(t-1)}$$

This statistic is distributed as chi-square with (t-1) d.f. If k is significantly greater than the expected value, the mean ranks averaged over years differ significantly and there is significant difference in the treatment effects. The chi-square value representing the treatment x year interaction may be obtained as

$$k^1 = \frac{12}{r(t^2-1)} \left[\sum_{k=1}^{p} \sum_{j=1}^{t} (\sum_{i=1}^{r} r_{ijk})^2 - \frac{1}{p} \sum_{j=1}^{t} R_j^{\,2} \right]$$

which is distributed as a chi-square variate with (t-1)(p-1) d.f. The significance of this statistic indicates the presence of interaction of treatments with years.

The rank means of treatments can then be compared in the usual manner with the help of critical difference calculated by

$$CD_{(.05)} = \sqrt{\frac{(t^2-1)}{6\ rp}} \times 1.96$$

5. Extended Friedman's Analysis of variance by ranks

Consider a set of t treatments assigned randomly to the units in each of the r blocks of a randomised block layout. Let x_{ij} denote the observation in block i of treatment j (i=1, 2, ...r), j=1, 2, ...t). Since the observations in different blocks are independent the collection of entries in the various rows of the two way classification are independent. In order to determine whether the treatment (column) effects are all the same or not the analysis of variance technique is appropriate if the assumptions of normality, additivity and homogeneity of error variances are satisfied. If on any ground one is in doubt on the validity of these assumptions he may proceed to apply a non parametric test of equal treatment effects proposed by Friedman (1937). In this approach the observations in each row (block) are replaced by their rank order within that row. If R_{ij} denote the rank order of the j^{th} treatment in the i^{th} block and R_j, the rank total of the j^{th} column (treatment)

$$E\,(R_{ij}) = \frac{t+1}{2}$$

$$\text{Var}\,(R_{ij}) = \frac{t^2-1}{12}$$

$$\text{Cov}\,(R_{ij}, R_{ik}) = \frac{-(t+1)}{12}$$

Further by design assumptions, observations in different rows are independent.

The sum of squares of deviation of the observed column totals around its expected value $\frac{r(t+1)}{2}$ will be a measure of the difference in treatment effects. Therefore we shall consider the sampling distribution of the random variable S where

$$s = \sum_{j=1}^{t} \left(R_j - r\left(\frac{t+1}{2}\right)\right)^2$$

under the null hypothesis of no difference between treatments. The probability distribution of S is given by

$$f(s) = \frac{U_s}{(t!)^r}$$

where Us is the number of arrangements of ranks in a block which yields S as the sum of squares of column total deviations. Tables of the distribution of S for small values of t have been prepared by Kendall (1962). Outside the range of the existing tables an approximation is generally used for tests of significance. The expectation and variance of s are given by

$$E(s) = \frac{rt(t^2-1)}{12}$$

$$\text{Var }(s) = t^2 r \frac{(r-1)(t-1)(t+1)^2}{72}$$

Friedman (1937) has shown that a linear function of s which is denoted as χ^2_r is distributed approximately as a chi-square variate with (t-1) degrees of freedom.

$$\chi^2_r = \frac{12\, s}{rt(t+1)}$$

$$= \frac{12 \sum_{j=1}^{t} R_j^2}{rt(t+1)} - 3r\,(t+1)$$

The first two moments of χ^2_r are (t-1) and 2(t-1) which are the first two moments of a chi-square distribution with (t-1) degrees of freedom. The higher moments of χ^2_r also closely approximated by corresponding higher moments of the chi-square. Thus for all practical purposes χ^2_r can be considered to be a chi-square variable with (t-1) degrees of freedom. Numerical comparison have shown this to be a good approximation as long as $t > 7$.

The region for a test of equal treatment effects with level of significance, α is

$$F \in R \quad \text{for } f \geq \chi^2_{n-1,\alpha}$$

where R is the critical region and f the calculated value of χ^2_r.

The above approach is related to the classical analysis of variance using ranked data.

If S_T denote the total sum of squares of deviations of all the rt ranks around its average value then

$$S_T = \sum_{i=1}^{r} \sum_{j=1}^{t} (R_{ij} - \frac{t+1}{2})^2$$

$$= rt \frac{(t^2-1)}{12}$$

or $$\chi^2_r = \frac{(t-1)s}{S_T}$$

The total sum of squares of the ranked data can be partitioned into two components as follows.

$$S_T = \sum_{j=1}^{r} \sum_{j=1}^{t} (R_{ij} - \bar{R}_j + \bar{R}_j - \bar{R})^2$$

$$= \sum_{i} \sum_{j} (R_{ij} - \bar{R}_j)^2 + \frac{s}{r}$$

$$= S_R + \frac{s}{r}$$

where S_R is the residual sum of squares.

All these can be presented in the analysis of variance table as follows.

ANOVA

Source	d.f.	S.S.	M.S.
Between columns (Treatments)	$t-1$	s/r	M.S.T.
Between rows (blocks)	$r-1$	0	0
Residual	$(t-1)(r-1)$	$S_T - \frac{s}{r}$	MSE
Total	$tr-1$	S_T	

The additive property of chi-square enables us to extend this result to the case of three way tables with years as the additional factor. Let us assume that the set of experimental years represent a random sample from an infinite population of years. Then it is possible to calculate the Friedman's χ^2_r statistic to the data of each of the p years separately. On the assumption of independence these chi-square values can be pooled to get a total chi-square with P(t-1) degrees of freedom. This chi-square can be split into two components.

$$\chi^2_r T = \chi^2_r D + \chi^2_r H$$

where $\chi^2_r D$ is the deviation chi-square calculated from the column totals of the pooled data. It can be used to provide a general test of equality of treatment effects over all the p years. $\chi^2_r H$ - the heterogeneity chi-square is a component of interaction between seasons and treatments. A significant $\chi^2 H$ indicates the presence of treatment x year interaction. The relevant procedure is outlined below.

Years	S.S.	Chi-square	d.f.
1	S_1	χ^2_{r1}	t-1
2	S_2	χ^2_{r2}	t-1
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
P	S_p	χ^2_{rp}	t-1
Total	$S_T = \sum_{i=1}^{p} S_i$	$\chi^2_r T = \sum_{i=1}^{p} \chi^2_{ri}$	p(t-1)
Deviation	S_D	$\chi^2_r D = S_D \frac{12}{t(t+1)}$	t-1
Heterogeneity	$S_H = S_T - S_D$	$\chi^2_r H = \chi^2_r T - \chi^2_r D$	(p-1)(t-1)

- -

The results can also be presented in the form of an analysis of variance table as follows.

ANOVA

Source	d.f.	S.S.	χ^2
Treatments	$t-1$	S_D	χ^2_D
Replications	$r-1$	0	
Years	$p-1$	0	
Treatment x year interaction	$(t-1)(p-1)$	S_H	χ^2_H
Residual	$(r-1)(tp-1)$	S_R	
Total	$rtp-1$	S_G	

$$\text{where } S_D = \sum_j \frac{R_{.j.}^2}{rp} - rpt\,\frac{(t+1)^2}{4}$$

$$S_G = \sum_i \sum_j \sum_k R_{ijk}^2 - rpt\,\frac{(t+1)^2}{4}$$

$$= rtp\,\frac{t^2-1}{12}$$

$$S_H = \sum_j \sum_k \frac{R_{.jk}^2}{r} - \sum_j \frac{R_{.j.}^2}{rp}$$

$$\text{and } S_R = S_G - S_D - S_H$$

Zar (1982) gave a non parametric multiple comparison procedure to be adopted in two way analysis with ranks when the usual assumption of normality and homosedasticity are not satisfied. According to him rank sums are to be arranged in descending order. Critical ranges of different lengths have to be calculated by multiplying the standard error of treatment totals by the tabulated value of studentised range with number of means k and error degrees of freedom n. Then the Newman and Keul's procedure may be used for making multiple comparison. The S.E. is calculated by the expression,

$$SE(R_j) = \sqrt{\frac{rt(t+1)}{12}}$$

Among the different multiple comparison procedures Duncan's multiple range test is considered to be most precise and powerful and has been widely used. Thus it would be better to incorporate a non parametric multiple comparison procedure involving Duncan's multiple range test. For the overall comparison of treatment totals based on pooled data for P years, $SE(R_j) = \sqrt{\frac{rtp(t+1)}{12}}$. The critical ranges can be calculated from the expression, $W_j = D_{(n,f)}\ SE(R_j)$.

If treatment means are to be compared the expression becomes,

$$W_j = D_{(n,f)}\ SE(\bar{R}_j) \text{ where } SE(\bar{R}_j) = \sqrt{\frac{t(t+1)}{12\ rp}}$$

Here $D_{(n,f)}$ is the table value obtained from the Duncan's table with number of means n and error degrees of freedom f. A range of j treatment means can be compared by W_j.

6. Stability analysis proposed by Eberhart and Russell (1966)

Let there be 't' treatments whose performance has been tested in 's' years. Considering y_{ij} as the mean of the i^{th} treatment in the j^{th} year, Eberhart and Russell (1966) used the following model to study the stability of treatments under different environments.

$$y_{ij} = \mu_i + b_i I_j + \delta_{ij} \qquad i = 1, 2, \ldots\ldots t$$
$$j = 1, 2, \ldots\ldots s$$

where μ_i is the mean of i^{th} treatment over all the years, b_i is the regression coefficient that measures the response of i^{th} treatment to varying environments. I_j is the environmental index, obtained as deviation of the mean of all treatments at the j^{th} year from the grand mean and δ_{ij} is the deviation from regression of the i^{th} treatment in the j^{th} year.

$I_{j's}$ which are the independent variables on which $y_{ij's}$ are regressed were obtained as

$$I_j = \sum_{i=1}^{t} \frac{y_{ij}}{t} - \sum_{i=1}^{t} \sum_{j=1}^{s} \frac{y_{ij}}{st}$$

so that $\sum_{j=1}^{s} I_j = 0$

The two parameters of stability under this model are

$$b_i = \frac{\sum_{j=1}^{s} y_{ij} \; I_j}{\sum_{j=1}^{s} I_j^2}$$

$$s_{di}^2 = \sum_{j=1}^{s} \frac{\delta_{ij}^2}{s-2} - \frac{s_e^2}{r}$$

where $$\sum_{j=1}^{s} \delta_{ij}^2 = \sigma_{vi}^2 - b_i \sum_{j=1}^{s} y_{ij} \; I_j$$

$$\sigma_{vi}^2 = \sum_{j=1}^{s} y_{ij}^2 - \frac{y_{i.}^2}{s}$$

$$b_i \sum_{j=1}^{s} y_{ij} \; I_j = \frac{\left(\sum_{j=1}^{s} y_{ij} \; I_j\right)^2}{\sum_{j=1}^{s} I_j^2}$$

and r is the number of replications.

The analysis of variance under Eberhart and Russell model is given below.

ANOVA

Source	d.f.	SS	M.S
Total	$st-1$	$\sum_{i=1}^{t} \sum_{j=1}^{s} Y_{ij}^2 - CF$	
Treatments	$t-1$	$\frac{1}{s} \sum_{i=1}^{t} y_{i.}^2 - CF$	MS_1
Year + (Treatments x years)	$t(s-1)$	$\sum_{i=1}^{t} \sum_{j=1}^{s} y_{ij}^2 - \sum_{i=1}^{t} \frac{y_{i.}^2}{s}$	
Year (linear)	1	$\frac{1}{t} \frac{(\sum_{j=1}^{s} y_{.j} I_j)^2}{\sum_{j=1}^{s} I_j^2}$	
Treatment x year (linear)	$t-1$	$\sum_{i=1}^{t} \frac{(\sum_{j=1}^{s} y_{ij} I_j)^2}{\sum_{j=1}^{s} I_j^2}$ - S.S. due to year (linear)	MS_2
Pooled deviation	$t(s-2)$	$\sum_{i=1}^{t} \sum_{j=1}^{s} \delta_{ij}^2$	MS_3
Treatment 1	$s-2$	$\sum_{j=1}^{s} \delta_{1j}^2$	
.	.	.	
.	.	.	
.	.	.	
.	.	.	
Treatment t	$s-2$	$\sum_{j=1}^{s} \delta_{tj}^2$	
Pooled error	$s(t-1)(r-1)$		$\frac{s_e^2}{r}$

Here, the sum of squares due to year and treatment x year interaction is partitioned into sum of squares due to years (linear) treatments x years (linear) and deviation from the regression model with degrees of freedom one, (t-1) and t(s-2) respectively.

The following F tests are made use of

(1) $F = \frac{MS_2}{MS_3}$, to test the equality of regression coefficients

(2) $F = \frac{\sum_{j=1}^{s} \frac{\delta_{ij}^2}{(s-2)}}{s_e^2}$ to test the individual deviation from regression

A treatment with unit regression coefficient (bi=1) and S_{di}^2 not significantly different from zero ($s_{di}^2 = 0$) could be considered as stable.

To test whether the regression coefficient of individual treatments differed significantly from unity, the following 't' test can be applied.

$$t = \frac{b_i - 1}{S.E.(b_i)}$$

where $S.E.(b_i) = \left[\frac{\text{M.S. due to pooled deviation}}{\sum_{j=1}^{s} I_j^2}\right]^{1/2}$

7. Stability analysis using non parametric measures

Non parametric measures of stability are based on the ranks of treatments in each year. Consider a two-way table with k treatments and N years. Within each year j (j = 1, 2, ...N) the k observations x_{ij} (i= 1, 2, ...k) are ranked by giving the lowest value a rank of 1 and the highest value a rank of k. Let r_{ij} be the rank of treatment i in the j^{th} year. A treatment is said to be stable over years if its ranks are similar over years.

According to Nassar and Huhn (1987) two non parametric measures of stability are

$$S_i^{(1)} = 2 \sum_{j=1}^{N-1} \sum_{j'=j+1}^{N} \frac{|r_{ij} - r_{ij'}|}{N(N-1)}$$

which is the mean of the absolute rank differences of a treatment over the N years and

$$S_i^{(2)} = \sum_{j=1}^{N} \frac{(r_{ij} - \bar{r}_{i.})^2}{N-1} \text{ where } \bar{r}_{i.} = \sum_{j=1}^{N} \frac{r_{ij}}{N}$$

which gives the variance among the ranks over the N years. For a treatment with maximum stability, $S_i^{(1)}$ and $S_i^{(2)}$ must be equal to zero.

The analysis is done with the null hypothesis that all treatments are equally stable. This would arise under the assumption of no differences among treatments and no treatment-year interaction.

The observation x_{ij} of i^{th} treatment and j^{th} year can be expressed as

$$x_{ij} = \mu + \beta_j + e_{ij}$$

where μ is the overall population mean, β_j is the effect of year j and e_{ij} is the random error with mean zero and variance σ^2. Since treatments are ranked separately within each year, environmental effects have no influence on stability and therefore the model may also be expressed as

$$x_{ij} = \mu + e_{ij}$$

Differences among treatments would have an effect on the $S_i^{(1)}$ and $S_i^{(2)}$ stability measures and may lead to differences in stability among treatments even if there is no treatment-year interaction. To avoid this x_{ij} values are corrected

$$x_{ij}^{*} = x_{ij} - (\bar{x}_{i.} - \bar{x}_{..})$$

where $\bar{x}_{i.}$ is the marginal mean of i^{th} treatment and $\bar{x}_{..}$ is the overall mean in the K x N table. The stability measures $S_i^{(1)}$ and $S_i^{(2)}$ may be computed using the ranks based on the corrected values.

For a given treatment i, the ranks r_{ij} (j=1, 2, ...N) represent a random sample from a descrete uniform distribution over the range 1 to k. Under the null hypothesis, the means and variances for each of the statistics $S_i^{(1)}$ and $S_i^{(2)}$ may be computed as follows.

$$E\left(S_i^{(1)}\right) = \frac{K^2-1}{3K}$$

$$E\left(S_i^{(2)}\right) = \frac{K^2-1}{12}$$

$$\mathrm{Var}\left(S_i^{(2)}\right) = \frac{m4}{N} - \frac{N-3}{N(N-1)}\left[E\left(S_i^{(2)}\right)\right]^2$$

where $m4 = E(y-\mu)^4 = E(y^4) - 4\mu E(y^3) + 6\mu^2 E(y^2) - 3\mu^4$

with $\mu = E(y)$ and $y = S_i^{(2)}$

$$E(y^4) = \frac{(K+1)(2K+1)(3K^2+3K-1)}{30}$$

$$E(y^3) = \frac{K(K+1)^2}{4}$$

$$E(y^2) = \frac{(K+1)(2K+1)}{6}$$

$$\mu = \frac{K+1}{2}$$

The variance of the statistic $S_i^{(1)}$ for different combinations of N and K have been generated numerically by Nassar and Huhn (1987) and are given in tables.

If the distribution of the statistics $S_i^{(1)}$ and $S_i^{(2)}$ may be approximated by a normal distribution, the statistics

$$Z_i^{(m)} = \frac{\left[S_i^{(m)} - E\left(S_i^{(m)}\right)\right]^2}{\mathrm{Var}\left(S_i^{(m)}\right)}, \quad m = 1, 2$$

would have an approximate chi-square distribution with 1 degree of freedom.

Similarly, the statistic

$$S^{(m)} = \sum_{i=1}^{K} Z_i^{(m)} \quad m = 1, 2$$

may be approximated by a chi-square distribution with K degrees of freedom. If this chi-square test is significant the null hypothesis of equal stability among genotypes is rejected and one may proceed to make multiple comparisons among the $S_i^{(m)}$ values.

8. Analysis based on principle of game theory

Any long term experiment can be regarded as a game between the experimenter and nature. The treatments are the strategies at the command of the experimenter where as varying weather conditions are the strategies of nature. Problems for the experimenter is to choose the optimum strategies so as to win over nature.

A decision making problem under uncertainty has the following four basic components relating to the decision maker.

(a) an objective function (b) a set of strategies (c) pay offs associated with given strategies of the decision maker for each state of nature and (d) uncertainty about the state of nature likely to prevail in the period for which the decision is made.

Let $S = (s_1, s_2, \ldots s_i \ldots s_m)$ be the strategy set of the decision maker, $T = (t_1, t_2 \ldots t_j \ldots t_n)$ be the states of nature and $P = [p_{ij}]$ be the pay off matrix of the decision maker.

There are several approaches for the choice of the optimal strategies. Among them Wald's maximin criterion, Laplace's principle of insufficient reason, Savage's regret criterion, Hurwicz optimism-pessimism criterion and Agarwal's excess benefit criterion are the major criteria which are usually employed in arriving at optimal decisions under risk. All these criteria are expected to suggest the strategy set S that would maximise the expected utility of the decision maker under varying environments.

a) Wald's maximin criterion

This criterion consist in choosing the maximum value among the minimum returns. That is, the decision maker attaches a probability of one to the worst consequence for a given strategy and zero to the other outcomes in that row. Let $E(u_i)$ be the expected utility of his i^{th} strategy (s_i) to the decision maker under Wald's criterion, then $E(u_i) = \min_j P_{ij}$. If $\max_i E(u_i) = E(u_{i*})$, the i^{*th} strategy is optimal to the decision maker. This strategy is for the extreme pessimist who wants to avoid a possible loss in unfavourable conditions.

b) Laplace's principle of insufficient reason

This theory assumes complete ignorance on the part of the decision maker about the state of nature that will prevail. Hence it is assumed that each state of nature is equally probable. Let $E(u_i)$ be the expected utility of the i^{th} strategy to the decision maker under the Laplace's principle. Then $E(u_i) = n^{-1} \sum_{j=1}^{n} P_{ij}$. If $\max_i E(u_i) = E(U_{i*})$ the decision maker will choose the $i*^{th}$ strategy. In effect the estimate obtained through this method protects the farmer from long range risk.

c) Hurwicz 'optimism-pessimism' criterion

This criterion is for the farmer who looks at the best and worst of his outcomes and assigns some weight to both. That is, for the pessimist who is also coatimus about a likely rise sooner or later. According to this criterion, the decision maker assigns a probability of 'a', $0 \leq a \leq 1$ to the best outcome for a given strategy and a probability of '1-a' to the worst outcome in that row. Let $E(ui)$ be the expected utility of the i^{th} strategy to the decision maker under Hurwicz model. Then

$$E(Ui) = a\,(\max_i P_{ij}) + (1-a) \min_i P_{ij}$$

If $\max_i E(ui) = E(Ui*)$, the decision maker will choose the $i*^{th}$ strategy.

d) Savage's regret criterion

The behavioural assumption under this criterion is that the decision maker tries to minimise his 'regret' where regret is defined as the difference between the actual pay off for the i^{th} strategy and the maximum pay off that he would have obtained if he had an advance knowledge of the true state of nature that actually prevailed. Let R be the regret matrix with elements r_{ij}. Then for a given state of nature t_{jo}, $r_{ijo} = P_{ijo} - \max_i P_{ijo}$ clearly $r_{ij} \leq 0$. Let $E(ui)$ be the expected utility of the i^{th} strategy to the decision maker under regret criterion. Then $E(ui) = \min_j r_{ij}$. If $\max_i E(ui) = E(ui^*)$. S_{i*} is optimal to the decision maker under regret criterion.

This criterion focusses on wealthy farmer who are willing to take a risk. It is for the farmer who wants to maximise his long range profit even at the expense of some small losses or set backs at stray periods.

e) Agarwal's excess benefit criterion

This criterion is concerned with the maximisation of additional benefit or surplus. This is suited for those farmers who desire to choose a treatment that will give them an additional benefit in years of unfavourable weather.

Let B be the benefit matrix with elements b_{ij}. For a given state of nature t_{jo}, $b_{ijo} = P_{ijo} - \min_{i} P_{ijo}$. Clearly $b_{ij} \geq 0$. If E(Ui) is the expected utility to the decision maker of his i^{th} strategy under the benefit criterion, then $E(Ui) = \min_{j} b_{ij}$. If $\max_{i} E(Ui) = E(Ui^*)$ then Si^* is the optimal strategy under the benefit theory.

9. Calculation of coefficient of concordance for overall comparison among the different methods.

Let there be k sets of rankings of n treatments and R_{ij} denote the rank of the j^{th} treatment in the i^{th} method. In order to test the hypothesis that the k sets of ranks are independent, a statistic known as Kendall's coefficient of concordance (w) could be calculated from the formula

$$w = \frac{12\, s}{k^2 n\, (n^2-1)}$$

where $s = \sum_{j=1}^{n} \left[R_j - \frac{k(n+1)}{2}\right]^2$ and $R_j = \sum_{i=1}^{k} R_{ij}$

'w' ranges between 0 and 1, with 1 designating perfect concordance and 0 no agreement between the different methods.

The statistic $k(n-1)w$ is expected to follow an approximate chi-square distribution with n-1 degrees of freedom as k becomes large. Hence chi-square test can be used to test the statistical significance of 'w'. A significant 'w' indicates that there is a strong degree of concordance among the rank orders of treatments by the different methods.

B. Fitting of response models

In experiments when one or more quantitative inputs like fertilizers are tested at two or more levels it is often desirable to summarise the available information on crop response pattern by fitting a suitable response surface. The response y may be represented by a suitable function of the levels X_{1u}, X_{2u}, ... X_{ku} of the K factors as $y_u = f(X_{1u}, X_{2u}, ... X_{ku}; \beta) + e_u$ -(1) where u = 1, 2, ...N represent the N observations. X_{iu}, the level of the i^{th} factor in the u^{th} observation (i = 1, 2, ...K) and β is the set of parameters. The residual e_u measures the experimental error of the u^{th} observation. The function 'f' is called the response surface. If there is only one independent variable then relation (1) is called a response curve. Response surfaces enable us to predict responses at varying values of X_{iu} and helps in determining the yield maximising the profit maximising levels of inputs. Several mathematical functions have been used to represent yield fertilizer relationships. In the single variable category the more widely used functions are quadratic, square root polynomial, Nelder's polynomial, Inverse polynomial, Gupta's function, Holliday function etc. Apart from these the mixed model which has not been frequently used for fitting response data and two alternative models are also proposed in this study. The bivariate models considered are quadratic, square root polynomial, resistance or Balmukund functions and transendental function. Multivariate models involving three or more inputs have not been considered in this study.

In the univariate case an empirical comparison of different models was made on the basis of the secondary data gathered from the final reports of completed manurial trials on paddy given in the various issues of the Research Reports of K.A.U. for the past ten years and the various post graduate research theses of the Faculty of Agriculture of K.A.U. A total number of 71 sets of data were available. In the two variate case, very few reports were available in the various issues of the Research Reports of K.A.U. or post graduate dessertations. Hence the results of a long term manurial trial on Jaya variety of rice conducted at Rice Research Station, Karamana during the 18 seasons from 1977-78 to 1986-87 were utilised for the study. There were a total number of 36 sets of data for fitting the two variate response models.

Different mathematical techniques have been used for the estimation of parameters of the fitted models. The predictability of the fitted models could be determined on the basis of the value of coefficient of determination, R^2 and also by the amount of average absolute error. For the linear model, $y = b_0 + \Sigma b_i x_i + e_i$ where y is the response, x_i's are the inputs e_i is the random error and b_i's are the partial regression coefficients, the coefficient of determination, R^2 is calculated as

$$R^2 = \frac{\text{Sum of squares due to regression}}{\text{Total sum of squares}}$$

sum of squares due to regression $= \Sigma b_i S_{xiy}$

where $S_{xiy} = \Sigma x_i y - \dfrac{\Sigma x_i \cdot \Sigma y}{n}$

Total sum of squares $= \Sigma y_i^2 - \dfrac{(\Sigma y_i)^2}{n}$

The non linear functions can be converted into linear functions by employing suitable transformations and the same procedure can be adopted for finding the coefficient of determination. In the case of non linear functions R^2 can also be found out directly as

$$R^2 = \frac{\text{Total sum of squares} - \text{Error sum of squares}}{\text{Total sum of squares}}$$

$$\text{Error sum of squares} = \sum_{i=1}^{n} (yi - \hat{y})^2$$

where $\hat{y}$ is the expected value of y and n is the number of levels. Average absolute error can be estimated by the formula $\Sigma \frac{|y-\hat{y}|}{n}$

The turning points of the functions could be derived and the physical and economic optimum doses of nutrients are estimated by employing the method of calculus. In the case of univariate models, the physical optimum dose is obtained by equating $\frac{df}{dx}$ to zero when $\frac{d^2f}{dx^2} < 0$ and the economic dose is obtained by equating $\frac{df}{dx}$ to the price ratio q/p when $\frac{d^2f}{dx^2} < 0$ Where p is the price per unit quantity of output and q is the price per unit quantity of input. In the two variate category, physical optimum doses are obtained by equating the partial derivatives of the functions with respect to the inputs to zero and solving the resulting equations. Economic optimum doses are obtained by equating the partial derivatives to the respective price ratios, ie. by solving $\frac{df}{dx_1} = \frac{q1}{p}$ and $\frac{df}{dx_2} = \frac{q2}{p}$ where q1 and q2 are the price per unit quantity of nutrient inputs and p is the price per unit quantity of output.

In the case of models which do not permit direct estimation of an economic optimum, profit maximising levels of inputs are estimated from the data on net returns per hectare.

For a univariate model, if π is the net profit,

$$\pi = py - qx \text{ where } y = f(x)$$

Economic optimum dose is obtained by equating $\frac{d\pi}{dx}$ to zero when $\frac{d^2\pi}{dx^2} < 0$. Similarly in the case of two variate models, the net profit π is given by $\pi = py - q_1x_1 - q_2x_2$

Economic optimum doses are obtained by equating the partial derivatives of this function with respect to the inputs to zero and solving the resulting equations.

In case, a dose of zero has been included as a level among the set of levels of the nutrient, considerable difficulties have to be encountered in the estimation of parameters from certain models involving resipreocal and logarithemic terms. In order to circumvent such a situation, it is desirable to raise every dosage by one unit and then transform the estimates back to the original dosage when the process of estimation has been over.

A relation between the polynomial and its first derivative can be derived as below which will be very useful for the estimation of optimum doses for certain complicated models. Let $\frac{x}{y} = P_n(x)$ where y is the response and $P_n(x)$ is a polynomial of degree n in x, the dose of the nutrient

$$\text{ie. } \frac{1}{y} = \frac{P_n(x)}{x}$$

Differentiating with respect to x

$$\frac{-1}{y^2} \quad \frac{dy}{dx} = \frac{xP_n'(x) - P_n(x)}{x^2}$$

$$\frac{dy}{dx} = \frac{y^2}{x^2} \left[P_n(x) - xP_n'(x)\right]$$

$$\frac{dy}{dx} = 0 \Rightarrow P_n(x) - xP_n'(x) = 0$$

This relation can be easily applied for finding the physical optimum in certain type of univariate models.

Details of the various response models considered in the study are given one by one below.

<u>Univariate models</u>

1. Quadratic model

$$y = a + bx + cx^2$$

$$\frac{dy}{dx} = b + 2cx = 0$$

Physical optimum dose, $x = \frac{-b}{2c}$

$$\frac{dy}{dx} = b + 2cx = \frac{q}{p}$$

Economic optimum dose, $x = \frac{(q/p-b)}{2c}$

The constants, a, b and c are estimated using the technique of least squares.

2. Square root polynomial

$$y = a + b\sqrt{x} + cx$$

Put $\sqrt{x} = x'$, then this model became similar to the quadratic model.

$$\frac{dy}{dx} = \frac{b}{2\sqrt{x}} + c = 0$$

physical optimum dose, $x = \left(\frac{-b}{2c}\right)^2$

$$\frac{dy}{dx} = \frac{b}{2\sqrt{x}} + c = \frac{q}{p}$$

Economic optimum dose, $x = \left[\frac{Pb}{2\ (q-pc)}\right]^2$

3. Nelder's polynomial

$$\frac{1}{y} = a + bx$$

Put $\frac{1}{y} = y'$ then $y' = a + bx$

This is the linear regression model and can be fitted by the principle of least squares.

$$\frac{-1}{y^2} \quad \frac{dy}{dx} = b$$

$$\frac{dy}{dx} = -y^2 b = \frac{-b}{(a+bx)^2} = 0$$

This model has no physical optimum dose.

$$\frac{-b}{(a+bx)^2} = \frac{q}{p}$$

$$b^2qx^2 + 2abqx + a^2q + Pb = 0$$

Economic optimum dose,

$$x = \frac{-abq \pm \sqrt{a^2b^2q^2 - b^2q\,(a^2q + Pb)}}{b^2q}$$

4. Inverse polynomial

$$y = \frac{ax}{x+b}$$

$$\frac{1}{y} = \frac{x+b}{ax}$$

$$= \frac{1}{a} + \left(\frac{b}{a}\right) \frac{1}{x}$$

$$= \beta_0 + \beta_1 \left(\frac{1}{x}\right) \text{ where } \beta_0 = \frac{1}{a} \text{ and } \beta_1 = \frac{b}{a}$$

Put $\frac{1}{y} = y'$ and $\frac{1}{x} = x'$

Then $y' = \beta_0 + \beta_1 x'$

This is the linear regression model and can be fitted by the principle of least squares

$$\frac{dy}{dx} = \frac{(x+b)\,a - ax}{(x+b)^2}$$

$$= \frac{ab}{(x+b)^2} = 0$$

This model has no physical optimum dose.

$$\frac{ab}{(x+b)^2} = q/p$$

$$qx^2 + 2bqx + qb^2 - Pab = 0$$

Economic optimum dose,

$$x = \frac{-bq \pm \sqrt{b^2q^2 - q(qb^2-Pab)}}{q}$$

5. Mixed model

$$y = a + b \log x + c\sqrt{x}$$

Put $\log x = x_1$, $\sqrt{x} = x_2$

Then $y = a + bx_1 + cx_2$

This is the regression model and can be fitted by the principle of least squares.

$$\frac{dy}{dx} = \frac{b}{x} + c.\frac{1}{2\sqrt{x}} = 0$$

$$2b + c\sqrt{x} = 0$$

Physical optimum dose, $x = (\frac{-2b}{c})^2$

$$\frac{2b + c\sqrt{x}}{2x} = \frac{q}{p}$$

$$2\,qx - Pc\sqrt{x} - 2\,Pb = 0$$

Economic optimum dose,

$$x = \left[\frac{Pc \pm \sqrt{p^2c^2 + 16\,pqb}}{4q}\right]^2$$

6. Gupta's function

$$y = \beta_0 + \beta_1 x + \beta_2 x^{-1}$$

Put $x = x_1$, $x^{-1} = x_2$

Then $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

This is the regression model and can be fitted by the principle of least squares.

$$\frac{dy}{dx} = \beta_1 + \beta_2 \cdot \frac{-1}{x^2} = 0$$

$$\frac{\beta_2}{x^2} = \beta_1$$

$$\text{Physical optimum dose, } x = \sqrt{\frac{\beta_2}{\beta_1}}$$

$$\beta_1 - \frac{\beta_2}{x^2} = q/p$$

$$\text{Economic optimum dose, } x = \sqrt{\frac{p\ \beta_2}{p\ \beta_1 - q}}$$

7. Holliday function

$$y = \frac{ax}{1 + bx + cx^2}$$

$$\frac{x}{y} = \frac{1 + bx + cx^2}{a}$$

$$= 1/a + (\tfrac{b}{a})\ x + (\tfrac{c}{a})\ x^2$$

$$= \beta_0 + \beta_1 x + \beta_2 x^2$$

where $\beta_0 = \frac{1}{a}$, $\beta_1 = \frac{b}{a}$ and $\beta_2 = \frac{c}{a}$

Put $\frac{x}{y} = y'$, the $y' = \beta_0 + \beta_1 x + \beta_2 x^2$

This is similar to the quadratic model.

$$\frac{x}{y} = \beta_0 + \beta_1 x + \beta_2 x^2 = P_n(x)$$

$$P_n(x) - x\, P_n'(x) = 0$$

ie. $\beta_0 + \beta_1 x + \beta_2 x^2 - x\,(\beta_1 + 2\beta_2 x) = 0$

$$\beta_0 - \beta_2 x^2 = 0$$

Physical optimum dose, $x = \sqrt{\dfrac{\beta_0}{\beta_2}}$

Data on net profit per unit area can be used to find the economic optimum dose.

8. New model -1.

$$y = \beta_0 + \beta_1 \sqrt{x} + \beta_2 x^{-1/2}$$

Put $\sqrt{x} = x_1$, $x^{-1/2} = x_2$

Then $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

This is the regression model and can be fitted by the principle of least squares.

$$\frac{dy}{dx} = \beta_1 \frac{1}{2\sqrt{x}} + \beta_2 \frac{-1}{2} x^{-3/2} = 0$$

$$\beta_1 - \beta_2 x^{-1} = 0$$

Physical optimum dose, $x = \dfrac{\beta_2}{\beta_1}$

Data on net profit per unit area can be used to find the economic optimum dose.

9. New model -2

$$y = \frac{ax}{b + c\sqrt{x} + x}$$

$$\frac{x}{y} = \frac{b+c\sqrt{x} + x}{a}$$

$$= (\frac{b}{a}) + (\frac{c}{a})\sqrt{x} + (\frac{1}{a})x$$

$$= \beta_0 + \beta_1 \sqrt{x} + \beta_2 x$$

where $\beta_0 = \frac{b}{a}$, $\beta_1 = \frac{c}{a}$ and $\beta_2 = \frac{1}{a}$

Put $\sqrt{x} = x^1$ then $(\frac{x'}{y})^2 = \beta_0 + \beta_1 x' + \beta_2 (x')^2$

Put $(\frac{x'}{y})^2 = y'$ then $y' = \beta_0 + \beta_1 x' + \beta_2 (x')^2$

This is similar to the quadratic model.

$$\frac{x}{y} = \beta_0 + \beta_1 \sqrt{x} + \beta_2 x = P_n(x)$$

$$P_n(x) - xP_n'(x) = 0$$

ie. $\beta_0 + \beta_1 \sqrt{x} + \beta_2 x - x\left(\beta_1 \frac{1}{2\sqrt{x}} + \beta_2\right) = 0$

$$\beta_0 + \frac{\beta_1}{2}\sqrt{x} = 0$$

Physical optimum dose, $x = \left[\frac{-2\beta_0}{\beta_1}\right]^2$

Data on net profit per unit area can be used to find the economic optimum dose.

Two variate models

10. Quadratic model

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_1^2 + b_4x_2^2 + b_5x_1x_2$$

This can be fitted by least squares.

$$\frac{dy}{dx_1} = b_1 + 2b_3x_1 + b_5x_2 = 0$$

$$\frac{dy}{dx_2} = b_2 + 2b_4x_2 + b_5x_1 = 0$$

Solving these two equations, physical optimum doses are obtained ie.

$$x_1 = \frac{2b_1b_4 - b_2b_5}{b_5^2 - 4b_3b_4}$$

$$\text{and} \quad x_2 = \frac{2b_2b_3 - b_1b_5}{b_5^2 - 4b_3b_4}$$

$$\frac{dy}{dx_1} = b_1 + 2b_3x_1 + b_5x_2 = \frac{q_1}{p}$$

$$\frac{dy}{dx_2} = b_2 + 2b_4x_2 + b_5x_1 = \frac{q_2}{p}$$

Solving these two equations, economic optimum doses are obtained

$$\text{ie.} \quad x_1 = \frac{2Pb_1b_4 - 2q_1b_4 + q_2b_5 - Pb_2b_5}{Pb_5^2 - 4\,Pb_3b_4}$$

$$\text{and} \qquad x_2 = \frac{q_1b_5 - Pb_1b_5 - 2q_2b_3 + 2Pb_2b_3}{Pb_5^2 - 4Pb_3b_4}$$

11. Square root polynomial

$$y = b_0 + b_1x_1 + b_2x_2 + b_3\sqrt{x_1} + b_4\sqrt{x_2} + b_5\sqrt{x_1x_2}$$

Put $\sqrt{x_1} = z_1$ and $\sqrt{x_2} = z_2$

$$y = b_0 + b_1 z_1^2 + b_2z_2^2 + b_3z_1 + b_4z_2 + b_5z_1z_2$$

$$= b_0 + b_3z_1 + b_4z_2 + b_1z_1^2 + b_2z_2^2 + b_5z_1z_2$$

This is similar to the quadratic model.

$$\frac{dy}{dx_1} = b_1 + b_3 \frac{1}{2\sqrt{x_1}} + b_5 \sqrt{x_2} \frac{1}{2\sqrt{x_1}} = 0$$

$$\frac{dy}{dx_2} = b_2 + b_4 \frac{1}{2\sqrt{x_2}} + b_5 \sqrt{x_1} \frac{1}{2\sqrt{x_2}} = 0$$

Solving these two equations, physical optimum doses are obtained.

$$\text{i.e. } x_1 = \left(\frac{2b_2b_3 - b_4b_5}{b_5^2 - 4b_1b_2}\right)^2$$

$$\text{and } x_2 = \left(\frac{2b_1b_4 - b_3b_5}{b_5^2 - 4b_1b_2}\right)^2$$

$$\frac{dy}{dx_1} = \frac{2b_1 \sqrt{x_1} + b_3 + b_5 \sqrt{x_2}}{2\sqrt{x_1}} = \frac{q_1}{p}$$

$$\frac{dy}{dx_2} = \frac{2b_2 \sqrt{x_2} + b_4 + b_5 \sqrt{x_1}}{2\sqrt{x_2}} = \frac{q_2}{p}$$

Solving these two equations, economic optimum doses are obtained.

$$\text{ie.} \quad x_1 = \left(\frac{2p^2b_2b_3 - 2pq_2b_3 - p^2b_4b_5}{p^2b_5^2 - 4p^2b_1b_2 + 4pq_2b_1 + 4pq_1b_2 - 4q_1q_2}\right)^2$$

$$\text{and} \quad x_2 = \left(\frac{2p^2b_1b_4 - 2pq_1b_4 - p^2b_3b_5}{p^2b_5^2 - 4p^2b_1b_2 + 4pq_2b_1 + 4pq_1b_2 - 4q_1q_2}\right)^2$$

12. Transendental function

$$y = ax_1^{b_1} e^{c_1x_1} x_2^{b_2} e^{c_2x_2}$$

$$\log y = \log a + b_1 \log x_1 + c_1x_1 + b_2 \log x_2 + c_2x_2$$

Put $\log y = y'$, $\log x_1 = z_1$ and $\log x_2 = z_2$

then $y' = \log a + b_1 z_1 + c_1x_1 + b_2z_2 + c_2x_2$

This can be fitted by the principle of least squares.

$$\frac{1}{y}\frac{dy}{dx_1} = \frac{b_1}{x_1} + c_1 = 0$$

$$\frac{1}{y}\frac{dy}{dx_2} = \frac{b_2}{x_2} + c_2 = 0$$

Physical optimum dose, $x_1 = \dfrac{-b_1}{c_1}$

and $x_2 = \dfrac{-b_2}{c_2}$

Data on net profit per unit area can be used to find the economic optimum doses.

13. Resistance or Balmukund function

$$y^{-1} = ax_1^{-1} + bx_2^{-1} + cx_1^{-1}x_2^{-1} + d$$

$$\frac{x_1x_2}{y} = ax_2 + bx_1 + c + dx_1x_2$$

$$= c + bx_1 + ax_2 + dx_1x_2$$

Put $\frac{x_1x_2}{y} = y'$

Then $y' = c+bx_1 + ax_2 + dx_1x_2$

This can be fitted by the principle of least squares.

$$\frac{1}{y} = \frac{c + bx_1 + ax_2 + dx_1x_2}{x_1x_2}$$

$$\frac{-1}{y^2}\ \frac{dy}{dx_1} = \frac{x_1x_2\ (b+dx_2)-(c+bx_1+ax_2+dx_1x_2)x_2}{(x_1x_2)^2}$$

$$= \frac{x_2(-c-ax_2)}{(x_1x_2)^2} = 0$$

$$-c-ax_2 = 0$$

$$\frac{-1}{y^2}\ \frac{dy}{dx_2} = \frac{x_1x_2\ (a+dx_1) - (c+bx_1+ax_2+dx_1x_2)x_1}{(x_1x_2)^2}$$

$$= \frac{x_1(-c-bx_1)}{(x_1x_2)^2} = 0$$

$$-c-bx_1 = 0$$

Physical optimum doses, $x_1 = {}^{-c}/{}_{b}$ and $x_2 = {}^{-c}/{}_{a}$

Data on net profit per unit area can be used to find the economic optimum doses.

Results and Discussion

RESULTS AND DISCUSSION

The results obtained are presented in this chapter under two sub headings (A) Analysis of data of long term experiments and (B) Fitting of response models, and discussed thereafter.

A. Analysis of data of long term experiments

1. Method of groups of experiments

The data for each year were analysed separately as in a randomised block design. Homogeneity of error mean squares was tested using Bartlet's test. As the error mean squares were found to be heterogeneous weighted analysis was performed and the resulting analysis of variance is given in Table 1. The significance of interaction effect was tested using chi-square test and the effect was found to be significant. The treatment means for the different years were arranged in a two-way table and the analysis of variance technique was applied to test the significance of various effects. The analysis of variance tablo of the unweighted data is given in Table 2. The treatment mean squares was then tested against the interaction mean square. The overall effect of treatments was found to be significant. The means of treatments were then arranged in descending order of magnitude and the significance of treatment differences was tested using the critical difference. The result obtained is given below

T1 T5 T3 T7 T8 T6 T2 T4

2. Analysis of data as in a split plot design

The data were also analysed as in a split plot design with treatments in main plots and years in sub plots and the resulting analysis of variance is given in Table 3. The treatment effect was found to be highly significant. Treatments were then ranked according to their mean performance and the significance of pairwise differences among the means was tested using the calculated value of the critical difference. The results obtained are summarised below.

T1 T5 T3 T7 T8 T6 T2 T4

3. Principal component analysis

The original data matrix was transformed into a matrix of standardised values. The transformed matrix $Z = (Z_{ij})$

is given in Table 4. From Z, the correlation matrix is obtained and is given in Table 5. The eigen values, eigen vectors and the percentage variation explained by each component vectors are given in Table 6. Since the first principal component explained more than 75 percent of total variation in the data other components were not considered for the analysis. The transformed matrix Z was then multiplied with the eigen vector corresponding to the highest eigen value, and the index value or first principal component score for each treatment was obtained. These are expected to serve as the index of overall performance of the specific treatments in relation to the other treatments in the tested environment. The treatments and their respective index values are as given below.

Treatments	T1	T2	T3	T4	T5	T6	T7	T8
Index values	3.5269	-2.1098	2.4228	-4.1254	3.2945	-2.1344	1.5468	-2.4163

The treatments were then ranked on the basis of the principal component score (index) and their relative standing is as given below.

T1 T5 T3 T7 T2 T6 T8 T4

In the general case, the original 32 x 12 matrix of observations was transformed into a matrix of standardised values. The matrix of standardised scores is given in Table 7. Eigen values and corresponding eigen vectors were generated from this matrix. Then, by multiplying the 32x12 matrix of standardised values with the largest eigen vector of order 12 an index score matrix of order 32x1 is obtained which was rearranged on the form of a two-way table of treatments and replication. The relevant two way table of index scores is given in Table 8. The data were further analysed as in a randomised block design and the results of analysis is given in Table 9. The treatment effect was again found to be significant. Comparisons were also made between pairs of means using the calculated critical difference. The result obtained is as given below.

T5 T1 T3 T7 T6 T8 T2 T4

4. Non parametric method proposed by Bai and Rao (1980)

The observations on different treatments in each block (replication) were ranked and the sums of ranks (R_j) along with the values of the statistic K are presented in Table 10. The statistic K is expected to be distributed as chi-square with 7 degrees of freedom. The K values for each of the different years were found to be significant indicating that in each year the treatment effects were statistically significant. The statistic K calculated for the aggregate data also showed statistical significance. But the K statistic developed for the test of the treatment x year interaction component was found to be non significant. Hence it may be inferred that the treatment differences were apparently consistent with years. The relative performance of different treatments were judged with the help of mean ranks and the calculated value of least significant difference. The results obtained are as given below.

T5 T1 T3 T7 T6 T2 T8 T4

Multiple comparisons among means were also made using Duncan's multiple range test and the result obtained is as given below.

T5 T1 T3 T7 T6 T2 T8 T4

5. Three-way analysis of variance by ranks

The observations in each block were ranked for different treatments and the sums of ranks are given in Table 11. The random variable S and the value of χ^2_r for different years were calculated and are presented in Table 12. The deviation chi-square (175.20) for the overall data is distributed as chi-square with 7 degrees of freedom. Since this was statistically significant it could be concluded that there were significant differences among the treatments in their effects. The heterogeneity chi-square for treatment x year interaction was not found to be statistically significant. Therefore, the hypothesis that treatment effects were invariant under varying seasons (or environments) was not rejected. An analysis of variance of the whole procedures mentioned above is presented in Table 13. The difference in mean ranks of treatments were compared using the relevant critical difference and the result obtained is given below.

T5 T1 T3 T7 T6 T2 T8 T4

The relative performance of different treatments were also judged using Duncan's multiple range test and the result obtained is as given below.

T5 T1 T3 T7 T6 T2 T8 T4

6. Stability analysis proposed by Eberhart and Russell(1966)

The analysis of variance under Eberhart and Russell model is given in Table 14. The linear component of treatment x year interaction was found to be non significant.

Although there had not been any interaction between treatments and years, an attempt was made to estimate the stability parameters for illustrative purpose. Pooled deviation from regression also turned out to be non significant. But deviation from regression for treatment 1 was found to be significant.

The environmental indices, I_j are given in Table 15. Estimates of stability parameters of the various treatments and the relevant 't' and 'F' values are given in Table 16. None of the regression coefficients differed significantly from unity indicating that all the treatments were having more or less average stability. From the 'F' values for testing the residuals the effect due to treatment 1 was found to be significant. The residual variance S^2_{di} for treatment 1 was found to be comparatively higher than those of other treatments indicating that it is relatively less stable than others. All other treatments exhibited average stability with regard to both of the stability parameters.

In order to have a more meaningful comparison among treatments, their relative performance in productivity shall also be taken into account. The treatments 5 and 3 showed average stability with moderately high yield. Treatment 7 had regression coefficient 0.9516 together with a sufficiently high yield. This indicates that the treatment is stable and at the same time it has given high yield. Treatment 1 had yielded a smaller value for regression coefficient (significant) along with very high yield. Hence it can be recommended only under assured better environment and management conditions. The other treatments showed signs of stability with comparatively lower yields. Therefore, treatments 5, 3 and 7 are ideal for adoption ifone is uncertain about the environment and other management conditions.

7. Stability analysis using non parametric measures

The corrected values, x_{ij}^* of each of the different observations are presented in Table 17. The treatments were then ranked on the basis of the corrected values of observations in each year and the ranked data are presented in Table 18. The values of the mean rank $\bar{r}_{i.}$ two non parametric stability parameters $S_i^{(1)}$ and $S_i^{(2)}$ and the statistic $Z_i^{(1)}$ and $Z_i^{(2)}$ for each treatment are given in Table 19. It could be seen that $\sum_{i=1}^{8} Z_i^{(1)} = 18.18$ and $\sum_{i=1}^{8} Z_i^{(2)} = 23.38$. Since the value of these statistics exceeded the critical values of chi-squares, it could be concluded that the treatments differed significantly among themselves with regard to these phenotypic stability. On comparing each of $Z_i^{(1)}$ and $Z_i^{(2)}$ values with the tabled value of chi-square with 1 degrees of freedom at 1 percent level, the effect due to treatment 1 was found to be significant. All the other treatments showed almost equal stability.

An examination of the values of $S_i^{(1)}$ and $S_i^{(2)}$ revealed that treatments 5 and 7 showed relatively higher stability. These treatments also have recorded relatively higher yield when compared to other treatments. The treatments 8 and 6 were found to be more stable than the remaining treatments. But these treatments showed low productivity. Among all the treatments, treatment 1 gave the maximum yield. But it was found to be less stable than other treatments because of the significance of $Z_i^{(1)}$ statistic . Hence this treatment cannot be recommended for general adoption. It can be recommended only on assured better conditions of environment. Treatments 5 and 7 is expected to produce a good response even when the environments are not favourable.

8. Analysis based on the principles of game theory

The costs of the different treatments were calculated and using these values and corresponding yield in term of money value, the pay off matrix was formed. The pay off matrix of the experiment $P = (P_{ij})$ is given in Table 20. The results obtained through the application of different decision criteria are given below.

a) Wald's maximin criterion

In this criterion $E(Ui) = \min P_{ij}$, where $E(Ui)$ is the expected utility of the i^{th} strategy. The minimum pay off value for each strategy was obtained from the pay off matrix and the maximum of the minimum pay off values determined for each treatment. The minimum pay off values obtained are tabulated below.

Category/Treatment	E(Ui)
T1	10254
T2	-4722
T3	5655
T4	12633
T5	13292
T6	3848
T7	9458
T8	14708

$\max_{i} E(Ui) = 14708$

Thus the 8th treatment was optimal to the decision maker. The strategies were then ranked on the basis of minimum pay off values and the rank order is given below.

T8 T5 T4 T1 T7 T3 T6 T2

b) Laplace's principle of insufficient reason

In this criterion, $E(Ui) = n^{-1} \sum_{j=1}^{s} P_{ij}$. The average pay off value for the different strategies are tabulated below.

Strategy/Treatment	E(Ui)
T1	15397.25
T2	927
T3	9156.5
T4	16310.5
T5	16804.25
T6	8476
T7	13251.75
T8	17371

$\max_i E(Ui) = 17371$

Thus the 8th treatment was optimal to the decision maker. The treatments were then ranked in the following order.

T8, T5, T4, T1, T7, T3, T6 T2

c) Hurwicz 'optimism-pessimism' criterion

According to this criterion, the decision maker assigns a probability of 'a' ($o \leq a \leq 1$) to the best outcome for a given strategy and a probability of (1-a) to the worst outcome in that row. Here 'a' is taken to be 0.8, $E(Ui) = a\ (\max_j P_{ij}) + (1-a) \min_j P_{ij}$. The maximum pay off value, minimum pay off value and expected utility for each strategy are given below.

Treatments	$\max_j P_{ij}$	$\min_j P_{ij}$	E(Ui)
T1	2026	10254	18231.6
T2	3999	-4722	2254.8
T3	12177	5655	10872.6
T4	18855	12633	17610.6
T5	19151	13292	11979.2
T6	10760	3648	9377.6
T7	15419	9458	14226.8
T8	19475	14708	18521.6

$\max_i E(Ui) = 18521.6$

Thus it could be seen that the 8th treatment was optimal to the decision maker. The treatments were then ranked as given below.

T8, T1, T5, T4, T7, T3, T6, T2

d) Savage's regret criterion

The regret matrix R was calculated and is given in Table 21. In this criterion $E(Ui) = \min_{i} r_{ij}$.

Treatment	E(Ui)
T1	-6399
T2	-19430
T3	-10998
T4	-3947
T5	-2362
T6	-10697
T7	-6073
T8	-2020

$\max_{i} E(Ui) = -2020$

The maximum value of E(Ui) has been recorded against the 8th treatment and hence it could be regarded as optimal to the decision maker. The strategies were ranked on the basis of expected utility and the rank order is

T8, T5, T4, T7, T1, T6, T3, T2

e) Agarwal's excess benefit criterion

The benefit matrix B was calculated and is given in Table 22. In this criterion $E(Ui) = \min_{j} b_{ij}$. Expected utility corresponding to each treatment is as given below.

Treatment	E (Ui)
T1	11055
T2	0
T3	6456
T4	12234
T5	1176[illegible]
T6	4493
T7	10508
T8	13598

$$\max_{i} E(U_i) = 13598$$

Thus the 8th treatment was found to be optimal to the decision maker. The strategies were then ranked and the rank order is

T8, T4, T5, T1, T7, T3, T6, T2

9. Overall comparison of the different methods

Analysis based on groups of experiments, split plot design and method of principal component make use of F test for testing the significance of treatment effects and hence the relative efficiencies of these three methods can be empirically compared on the basis of the relative magnitude of the resulting F ratios. The F values for testing the overall treatment effects as obtained in the three methods are given below.

Methods	F values
Group of experiments	24.49**
Split plot analysis	27.83**
Principal component analysis	27.89**

Principal component analysis has recorded the maximum F value for detecting real treatment effect than the other two methods. Thus it could be inferred that principal component analysis would be more efficient and sensitive in detecting the real treatment differences when compared to the other methods. Principal component analysis have certain other distinct advantages over the other methods. The usual assumption of independence of error terms does not seem to be valid in the case of repeated trials on the same site. Principal component analysis takes care of this difficulty by way of generating a new composite variable from the yearly responses and thus the resulting analysis of the data is expected to be in better conformity to the underlying assumptions of independence of error terms. It is well known that the first principal component is that linear compound which explains the maximum amount of variation in the experimental data than any other linear component not excluding simple aggregate values of treatment responses over the whole period. Due to this fact the sensitivity of the F test in detecting true treatment differences is expected to be more in principal component analysis than the other methods. This can be very well evident from the amount of percentage variation explained by the treatments in the three methods. The percentage variation explained by overall treatment differences in the three methods were found to be 20.63 percent in analysis of groups of experiments, 21.94 percent in split plot analysis and 90.29 percent in principal compound analysis. Thus principal component analysis provides greater predictability for overall treatment comparisons than the other methods. This may be the case with similar sets of data generated from other experiments of long term nature. In fact, principal component analysis does not require an underlying statistical model to explain the error structure. Thus it may be concluded that principal component analysis should be preferred to ordinary methods for the analysis of data from long term manurial trials. But in case the first principal component fails to explain an adequately high percentage variation (say more than 60 percent) two or more components may have to be used for the resulting analysis. The treatments may be grouped into several homogeneous groups on the basis of the plotted points of component scores of the two generated variables on a two dimensional chart. D^2 analysis based on the values of each of the generated variables may also be attempted and clusters formed using canonical analysis or other methods so as to get a clear configuration of the set of treatments into a few homogeneous clusters.

The split plot analysis seems to be more sensitive than the method of groups of experiments because in that we are using two different types of error, for reducing the risk of drawing invalid inferences. But the method suffers from a number of drawbacks when viewed from a logical stand point and cannot be recommended for general adoption. Split plot design requires the random arrangement of set of sub plot treatments within each main plot and that cannot be expected in the case of trials repeated over several seasons. In this analysis the assumptions of independence of error terms does not seem to be wholly valid.

Analysis of groups of experiments makes use of the assumption of independence of error terms. Therefore, the results obtaining from it may be faulty and unrealistic to some extend. Further, in such types of data analysis no general test appears to be available for overall treatment comparisons when error variances are heterogeneous and interaction effect is absent. Hence principal component analysis can be considered as a better alternative to analysis of groups of experiments and split plot analysis.

It is also interesting to make an empirical comparison between the two non parametric methods of data analysis with an objective of choosing a better method for general adoption This can be achieved by comparing the chi-square values for testing the effects of treatments and interaction in both methods. Although the chi-square values for the method proposed by Rai and Rao (1980) are higher than that for the new extended Friedman's analysis of variance by ranks the difference is negligibly small. The chi-square values for testing the effects of treatments and interaction are 200.97 and 71.36 in the method proposed by Rai and Rao (1980) and 175.20 and 63.71 in the extended Friedman's analysis.

The newly developed procedure as an extension of Friedman's two way analysis of variance shows certain distinct advantages over the method proposed by Rai and Rao (1980). It is entirely distribution free, in the real sense of the term. But the method proposed by Rai and Rao (1980) make use of the assumption that the sampling distribution of the means of ranks is approximately normal. The method is applicable only for cases when the number of replications per experiment is four or more. The amount of information lost in the process will be more when there are only a few treatments. Therefore, the newly developed method is a better non parametric alternative to the analysis of data of long term menurial trials over the existing methods.

Analysis based on principle of game theory suggest specific recommendations for farmers with varying decision environments. Ranking of treatments obtained by applying various criteria are given below:

Wald's maximin criterion	T8	T5	T4	T1	T7	T3	T6	T2
Laplace's principle of insufficient reason	T8	T5	T4	T1	T7	T3	T6	T2
Hurwicz 'optimissm-pessimism' criterion	T8	T1	T5	T4	T7	T3	T6	T2
Savage's regret criterion	T8	T5	T4	T7	T1	T6	T3	T2
Agarwal's excess benefit criterion	T8	T4	T5	T1	T7	T3	T6	T2

It could be seen that treatment 8 (Ammonium sulphate to supply 45 kg N/ha + Super phosphate to supply 45 kg P_2O_5/ha + MOP to supply 45 kg K_2O/ha) was the best strategy under all the different decision criteria. This recommendation could definitely fit in to the requirements of a broad spectrum of farmers. It is suitable not only for a wealthy farmer aiming at huge profit but also for a subsistenc farmer who wants to avoid a possible loss. It could be expected that such a strategy would ensure in the long run maximum net revenue and at the same time maximum protection from likely losses in years of disaster.

One drawback of game theory approach is that one cannot predict likely responses of the crop at intermediate levels not tried in the experiment and hence the realised optimum is only an estimate in the descrete sense of the term. Estimation of the optimal point on a continuous regime can be attempted by fitting response surface models. Further there is no known method of testing the significance of the difference between the performance of the strategies under various criteria. Comparisons are based entirely on the mean values which are subjected to change at different environments. Thus the reliability of the result could not be assessed statistically. However they gave a better understanding of the problem and help in giving specific recommendations to different types of farmers with varying requirements.

Kendall's coefficient of concordance (w) was calculated to measure the degree of overall agreement among the different methods in detecting the true rank order among the set of treatments. A significant 'w' indicates that there is a strong degree of concordance among the rank orders of treatments by the different approaches and in that situation a composite ranking on the basis of the rank sums appeared to be feasible.

The extend of mutual concordance among the rank orders of treatments in the analysis of data as groups of experiments/split plot analysis, the principal component analysis and the non parametric method was examined. Since the rank orders of treatments in the analysis of data as groups of experiments and those of split plot analysis were same, the common rank order of treatments alone was taken into consideration. The same was the case with the ordering of treatments in the two non parametric methods discussed in this study. The two way lay out necessary for calculating 'w' is presented in table 23. The corresponding chi-square value was also calculated.

The coefficient of concordance 'w' was calculated to be equal to 0.9629 and the corresponding chi-square value (20.22) was found to be significant at one percent level. The result indicated that there was almost perfect agreement or concordance among the different methods of analysis of data viz. method of groups of experiments/split plot analysis, principal component analysis and the non parametric methods with regard to the rank orders of treatments.

In all tho above five methods the ranking of treatments was done based on their yield performances. Since stability analysis was done with a different objective no attempt was made to make an empirical comparison between the ordering of treatments according to phenotypic stability and those according to the other procedures referred above. Analysis based on principle of game theory is also performed to meet with a different objective and hence that method also was not considered in calculating the value of 'w'.

Danford et al.(1960) used split plot analysis and multivariate analyses such as likelihood criterion and Hotellings T^2 for a given data to test the significance of treatment effects and found that the univariate and multivariate procedures gave the same result. The results obtained in the present study are also in agreement with the findings of Danford et al.

The results obtained in this study by the application of method of groups of experiments and that by the principal component analysis were same. This is also in agreement with the findings of Cole and Grizzle (1966). They used method of groups of experiments and likelihood criterion to test the significance of treatment effects and found that the univariate and multivariate procedures have the same scope, power and flexibility.

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Krishnan *et al*. (1982) have reported that stability analysis can be considered as an alternative to the method of groups of experiments. The results obtained in this study are not in agreement with their findings. In this study treatment 1 was found to be the least stable treatment but at the same time the most productive among the lot. Thus the selection of treatments solely in accordance with their interaction with environment alone need not indicate a subtle treatment. As reported by Rawlo and Das (1978) it could be always better to select the more stable and high yielding treatments. Therefore, conclusion drawn from stability analysis is not sufficient to draw valid inference.

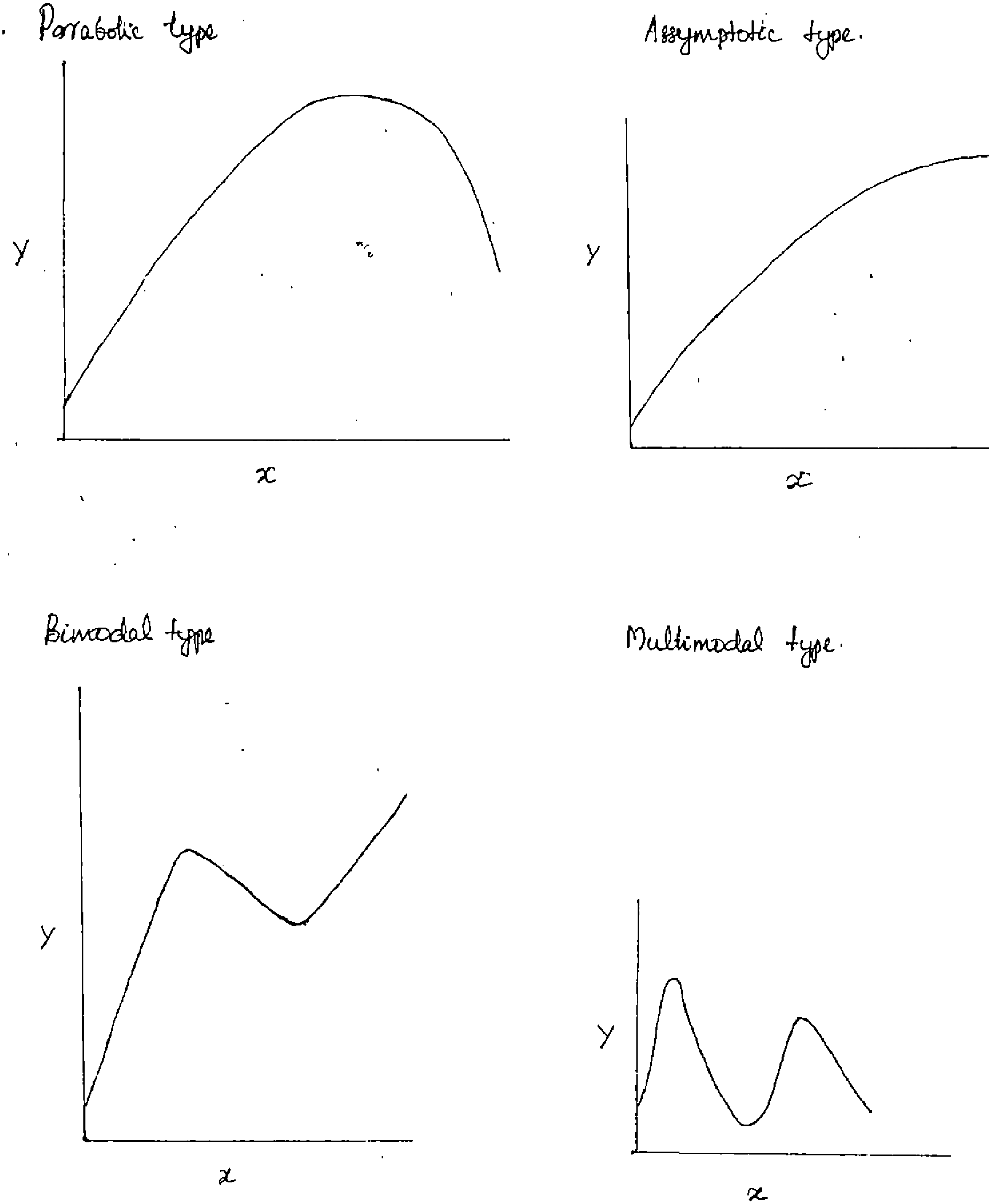
B. Fitting of response models

A number of univariate and two variate models were fitted using the data given in appendices I and II and their relative efficiencies evaluated on the basis of the observed values of the coefficient of determination and average absolute error. As a preliminary step in detecting the nature of the response model suited to any particular data the observed values, in the univariate case, were plotted graphically and based on the shape of the graph the data were broadly classified as belonging to one or the other of four mutually exclusive categories (1) parabolic type

(2) Assymptotic (extremely assymetric parabolic) type (3) Distorted parabolic (bimodal) type (4) Multimodal or irregular type. Specimen graphs representing the above four categories of data are given in Figure 1. Of the 71 sets of data considered in this study, the number of data sets that fell into each of the above four categories were 39, 4, 25 and 3 respectively. The values of coefficient of determination and average absolute error corresponding to each of the fitted models were calculated and are presented in Tables 24, 25, 26 and 27 respectively as per the particular class of response curve. The mean and range of R^2 values and those of average absolute error for each of the different models under the four categories of response were also determined and are presented in the tables. The sets of data for which a model failed to locate either a physical or an economic optimum on computational grounds were not considered in this study for fitting the specific model. The position corresponding to such entries in the table has been left as blank.

Among the tested models majority of the models showed high degree of predictability in representing the parabolic response pattern. The square root polynomial ranked first with a multiple correlation coefficient of 0.8977. It was

Figure 1. Graphical representation of the nature of response of different types of data.



Y - Yield, x - Dose of nutrient.

closely followed by the quadratic model (R^2 = 0.8916). Thus a parabolic response pattern could be well represented by either a quadratic or square root polynomial response function with slight advantage for square root function over the usual quadratic function. The percentage variation explained by other models, especially newly proposed model, model-1 (R^2 = 0.8746), Gupta's function (R^2 = 0.8661) and mixed model (R^2 = 0.8470) were also relatively high. The values of average absolute error as obtained from these models are also not very high. Thus these three models can also be recommended along with the quadratic and square root polynomial for representing the parabolic response. However, the quadratic model had certain distinct advantages over others. Fitting quadratic function is simple as only linear estimation is involved and the usual technique of analysis of variance and tests of significance can be easily applied with this function. The standard errors of estimated optimum is expected to be smaller in the case of quadratic function as compared to that in other functions. It was further observed from the empirical data that the quadratic function gave more realistic estimates of optimum requirements of nutrients and expected optimum value of the response than other models.

A critical examination of the shapes of the different curves drawn to each set of data revealed another interesting observation. For almost all the curves with atleast slight amount of assymetry about the anticipated optimum the square root model was found to be a better fit than the ordinary quadratic. It goes without saying that the square root polynomial also shared all the distinct advantages of the quadratic polynomial as it is obtained just by fitting a quadratic polynomial to the transformed data obtained by taking square roots of the original observations.

Thus in fertilizer trials where the yield fertilizer relationship is expected to be represented by an assymmetric parabola the square root polynomial is best suited in predicting the optimum response and optimum level of nutrients. Since most of the fertilizer trials belong to this category, as a general recommendation the square root polynomial is to be preferred over quadratic polynomial in fitting the response pattern unless data exhibit specialised patterns or wide distortions from the normal modalities.

Johnson (1953) emphasised that in the case of single input, quadratic and square root polynomials were better than other forms with some preference to the square root quadratic attributed to its non symmetrical and flatter shape in xy plane. The findings obtained in this study also agree with the above results.

In the second category of curves with assymptotic tendency the newly proposed model (model-2) was found to be the most efficient. This model possessed an extremely high R^2 value (0.9991) and least average absolute error (6.39). Square root polynomial model and mixed model were also found to be as efficient as new model 2. Quadratic polynomial also gave relatively good fit to the data. But the estimates of optimum doses observed from these three models in the most cases, fell beyond the range of the inputs tried in the experiment and hence were not useful for making general recommendation. The new model gave more realistic estimates with such highly assymptotic data. Thus the newly developed model combined the qualities of an asymptotic growth curve like the Mitscherlich's and those of an ordinary polynomial and hence is better suited for general adoption in response curve technique.

In the case of curves showing bimodal tendancy the cubic polynomial is considered to be ideal. But if bimodal tendency is not very much pronounced, other models could also be used satisfactorily to represent the response pattern. Such models possess the added advantage that they require lesser number of parameters to be estimated and hence may provide estimates with lesser standard error. Among the different models, square root polynomial topped all others in predictability. But the model explained only 60.75 percent of the total variability in the response of the curve to nutrient input. New model-1, mixed model and Gupta's function also gave relatively good fit to the data. In this case also, the quadratic function was found to be inferior to the square root function in representing the response pattern. Thus it can be inferred that the square root function is better suited in representing the yield-fertilizer relationship in response curve studies than the ordinary quadratic as it is more stable and is not affected by minor distortions in the data. This is also in agreement with the findings of Johnson (1953).

In the fourth category, the newly proposed model, model-1 gave the maximum predictability than all other models. The average variation explained by this model was found to be 37.22 percent which was the highest among that of the tested models. Gupta's function and mixed model gave relatively high average predictability and relatively lesser average absolute error when compared to the other models. Thus these models are not inefficient in representing the type of data under this category.

The percentage number of cases included within specified ranges of coefficient of determination under each model are presented in Table 28. Quadratic function, square root polynomial, Nelder's polynomial and new model-2 have been fitted to all sets of data. The other models failed to locate either a physical optimum or an economic optimum for certain sets of data and no attempt was made to represent such data sets by the relevant model. The percentage of data which were actually utilised for fitting each of the different models are also given in Table 28. Mixed model and inverse polynomial could fit to more sets of data than new model-1, Holliday function and Gupta's function. The percentage number of cases included in the range 0.98-1 of R^2 were found to be more for mixed model (26.76 percent), square root polynomial (25.35 percent) and quadratic model (23.94 percent). Thus in general these three models gave better fit to the data than other models. Holliday function, Nelder's polynomial and new model-2 lagged behind the other models in general adaptability as more than 50 percent of the R^2 values attributed to these models were below 0.5.

Mean values of coefficient of determination and average absolute error for the entire sets of data under each of the fitted models are given in Table 29. An overall comparison among the models can be made using these values. Among the different models square root polynomial topped all others with an average R^2 value of 0.7736 and minimum average absolute error of 82.12. New model-1, mixed model, quadratic model and Gupta's function also gave better performances. Therefore, these models can be used for general adoption in fertilizer trials.

Nelder (1966) compared the goodness of fit of ordinary and inverse polynomial models and found that inverse polynomial models were better than others. But in this study the performance of inverse polynomial was found to be inferior to that of ordinary polynomials.

Clarke (1968) compared inverse polynomial surfaces of linear and quadratic type and found that the latter often succeeds even in cases where a maximum was not reached. But in this study inverse polynomial surfaces of quadratic type were found to be inferior to the others by considering Holliday function and new model-2 as quadratic type inverse polynomials.

The percentage cases of estimates on physical and economic optima which fell into the specific ranges of nutrients tried in the trial under each model are given in Table 30. In the case of new model-2, mixed model, quadratic model and square root polynomial model more than 75 percent estimates on physical and economic optima fell in the specific ranges of nutrients. Thus these models produced estimates on physical and economic optima with greater practical value than the other models. No estimate on physical optimum could be estimated from Nelder's polynomial or inverse polynomial. This has been indicated in the table by using a blank entry. The economic optimum values obtained for Nelder's polynomial were far above the doses tried. The estimated optimum values as estimated from quadratic and square root polynomial were found to be appreciably closer. Gupta's function failed to give optimum values in 36.12 percent cases, Holliday function in 33.80 percent cases, New model-1 in 30.99 percent cases, inverse polynomial in 12.68 percent cases and mixed model in 1.41 percent cases.

All the models except Nelder's polynomial and inverse polynomial have two independent constants. Nelder's polynomial and inverse polynomial have only one independant constant. Even then these two polynomials have explained as much variation as the other functions for most of the data.

In fitting bivariate response models no effort was made to classify the pattern of response as perceived by the shape of the observed surface. This is largely due to the complexity of the problem and the lack of sufficient data to represent the varying categories of response.

The value of coefficient of determination and average absolute error corresponding to each of the fitted models were calculated and are presented in Table 31. The mean and range of R^2 values and those of average absolute error for each of the different models were also determined and are presented in the same table.

The four response functions were fitted to each of the available data set and their relative efficiencies compared. It was found that all the functions are useful in representing response surface. Among the tested models, the resistance function was found to be the most efficient. This function, on the average explained as much as 99.35 percent variation in yield differences. The percentage variation explained by the square root polynomial (83.80 percent) and quadratic response model (83.66 percent) were also relatively high. The average absolute error was found to be smaller with these functions. Thus in the two variable case also the square root polynomial model was found to be slightly better than the quadratic function in representing the response surface. Transendental function was found to be less efficient in describing the response pattern when compared to the other response function.

The percentage number of cases included within specified ranges of coefficient of determination under each model are presented in Table 32. The percentage of data which were actually utilised for fitting each of the different models are also given in the same table. In the two variate case, all the 36 sets of data could be utilised for fitting each of the different models. The minimum value of coefficient of determination for the resistance function was found to be as large as 0.95. Thus for this model, none of the cases fell outside the highest range of R^2 (0.95 to 1) whereas the percentage number of cases fell within that range under the quadratic and square root polynomial models was found to be equal to 16.67. Transendental function gave relatively lower R^2 values.

The estimates of physical and economic optima under each model along with their means and standard deviation are presented in Table 33. Certain models failed to locate a positive physical or economic optimum value for some data sets. The position corresponding to such entries in the table has been left as blank. The percentage number of cases of estimates on physical and economic optima under each model which lie in the specific ranges of nutrients tried in the experiment are given in Table 34. Among the different models, quadratic function square root polynomial and transendental function had about half of their estimates on optimum within the stipulated interval. In the case of resistance function about one half of the estimates of physical optimum doses were within the specific ranges while all of the estimates of economic optimum were distributed within the range.

The standard deviation of the optimum doses were very high for the square root polynomial model. Quadratic model also have relatively high standard deviation for the optimum doses when compared to transendental function. With regard to resistance function, the standard deviation of the physical optimum doses were comparatively high whereas that of economic optimum doses were very small. Thus resistance function gave relatively more stable estimates than the other functions.

The quadratic surface and square root polynomial model removed more than 80 percent of the yield variation in about 72 percent of the experiments whereas transendental function removed the same variation in about 50 percent of the experiments. But in the case of resistance function all the experiments removed more than 95 percent of the yield variation. Thus resistance function gave uniformly better fit than other models.

Among the bivariate models, quadratic polynomial model and square root polynomial model are based on five independent constants. At the same time the transendental function was four and the resistance function has only three independent constant. It is a fact that the percentage variation explained by a model is positively related to the number of independent constants. In this study it has been observed that the resistance function has yielded comparatively higher values of R^2 even with lesser number of parameters. The estimated standard error of the estimates from this model were also relatively lesser than those obtained from the other models. Thus the estimates on optimum response obtained for different sets of data under this model were more realistic and stable. Therefore the resistance function can well be recommended for representing the response pattern and estimating the optimum level of nutrients in multifactor experiments.

The above result is in agreement with the findings of Abraham and Rao (1966). They have pointed out that resistance function would give uniformly better fit to sets of data when nutrient interaction was present. In most of the experiments we can expect a significant interaction between the constituent factors. According to them quadratic models are better suited for general adoption in fitting the response surface of fertilizer trials. The results of the present study are not in quite agreement with these findings. Here the square root polynomial function was found to give slightly better results than the ordinary quadratic polynomial, although both are equally efficient in describing the response surface. More than 50 percent of the estimates on optimum doses were included in the stipulated range for this function whereas the quadratic function yielded only 38.90 percent of estimates in the specified range. Although the transendental function was in general less efficient in describing the response pattern than other models, it was found to be highly efficient in locating the physical and economic optimum. The estimates obtained through this function were more realistic and exhibited comparatively lesser standard errors. The standard deviation of estimates on physical optimum was least for the transendental function. Also the square root polynomial model can be adjudged to be superior to ordinary quadratic polynomial model in representing the response pattern and estimating the optimum level of nutrients. Thus it is essential to have a proper rethinking of the existing practice of estimating optimum doses from response surface models by employing the ordinary quadratic polynomial model.

Tables

Table 1. Analysis of variance of the data in the case of weighted analysis in groups of experiments

Source	d.f.	S.S.
Treatments	7	663.7578
Years	11	2244.3595
Interaction (Treatment x year)	77	307.0703
Total	95	3215.1875

Table 2. Analysis of variance of the data in the case of unweighted analysis in groups of experiments

Source	df	S.S.	M.S.	F
Treatment	7	344.6172	49.2310	24.4857**
Year	11	823.0001	74.8182	37.2118**
Interaction (Treatment x year)	17	154.8164	2.0106	
Pooled error	252		2.7671	

** Significant at 1% level

Table 3. Analysis of variance of the data as in the case of split plot design

Source	df	SS	MS	F
Replications	3	21.1621	7.0540	0.9993
Main plot (Treatments)	7	1375.1194	196.4456	27.8292**
Error (a)	21	148.2385	7.0590	
Sub plot (years)	11	3289.6315	299.0574	97.2808**
Interaction	77	622.6273	8.0861	2.6303**
Error (b)	264	811.5799	3.0742	
Total	383	6268.3587		

** Significant at 1% level

Table 4. Matrix of standardised values for the original 8 x 12 matrix

-1.029	0.484	-0.036	1.131	0	1.164	1.448	1.768	1.404	1.720	1.189	1.502
-1.304	-1.051	0.222	-1.044	-0.234	-0.395	-1.374	-1.026	0.367	-0.418	-0.331	-1.012
-0.180	0.748	0.144	0.779	1.030	0.535	0.842	0.707	0.981	0.667	0.958	0.702
-0.060	-1.028	-2.020	-0.847	-1.570	-1.357	-0.948	-0.951	-1.054	-1.241	-1.660	-1.297
1.209	1.429	1.335	1.489	1.336	1.227	0.739	0.769	-0.128	0.804	0.794	0.781
-0.137	-0.786	0.114	-0.476	0.401	-0.773	-0.874	-0.691	-1.126	-0.916	-0.612	-0.441
1.647	0.937	0.824	-0.117	0.780	0.535	0.287	0.011	0.623	0.008	0.432	0.464
-0.129	-0.711	-0.607	-0.921	-0.930	-0.962	-0.132	-0.589	-1.071	-0.631	-0.781	-0.718

Table 5. Correlation matrix from the matrix of standardised values

1.0007	0.6100	0.4110	0.2714	0.4693	0.2681	0.2443	0.0810	-0.1247	-0.0112	0.1416	0.2291
	0.9998	0.7079	0.8179	0.8836	0.9042	0.8364	0.7874	0.5941	0.7674	0.8460	0.8666
		0.9992	0.5638	0.8703	0.7518	0.4118	0.4351	0.4522	0.5438	0.7422	0.6087
			0.9999	0.7530	0.9051	0.8765	0.9203	0.5887	0.8808	0.8550	0.9113
				0.9734	0.8505	0.6150	0.9093	0.6112	0.6705	0.8586	0.7394
					1.0000	0.8367	0.8882	0.7928	0.9355	0.9619	0.9367
						1.0003	0.9600	0.6519	0.8976	0.8436	0.9399
							0.9988	0.7313	0.9635	0.8851	0.9653
								0.9998	0.8333	0.8472	0.7506
									1.0003	0.9397	0.9390
										0.9997	0.9431
											1.0000

Table 6. Eigen values and corresponding eigen vectors.

Eigen vectors (Principal component)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
	0.0957	-0.7015	-0.3475	-0.3650	0.1374	0.2225	-0.0855	-0.0939	0.1629	0.0375	-0.1434	0.3344
	0.3080	-0.2601	-0.1533	-0.1635	-0.1940	-0.0339	0.2430	0.4469	-0.1487	-0.4506	0.1768	-0.4830
	0.2363	-0.3508	0.5399	0.3736	0.4224	0.1118	0.0539	-0.1636	-0.1549	-0.2324	0.2824	0.1079
	0.3081	0.0099	-0.2378	0.4015	-0.5768	0.1295	-0.2122	-0.3150	0.2478	-0.1825	0.2761	0.1501
	0.2827	-0.2928	0.3024	-0.0049	-0.4128	-0.5059	-0.0037	0.1269	-0.2363	0.4507	-0.1050	0.1742
	0.3272	-0.0077	0.1081	0.0220	-0.0681	0.5307	0.1617	-0.2609	-0.0462	0.4264	-0.2864	-0.4854
	0.2999	0.1186	-0.4003	-0.0586	0.3245	-0.4475	0.3857	-0.4298	-0.0185	0.1543	0.2537	-0.0743
	0.3073	0.2269	-0.2665	0.1146	0.0956	0.1817	-0.1578	0.0079	-0.6546	-0.2349	-0.4140	0.2722
	0.2592	0.3001	0.3423	-0.7169	-0.1156	0.1246	-0.1347	-0.2149	-0.0909	-0.1426	0.2639	0.1572
	0.3126	0.2518	-0.0043	0.0825	0.0661	0.2530	0.4618	0.4849	0.2775	0.1667	0.0697	0.4549
	0.3239	0.0738	0.1997	-0.0059	0.1193	-0.3083	-0.0806	-0.0632	0.5356	-0.3276	-0.5126	-0.0888
	0.3227	0.0826	-0.1325	0.0366	0.3330	-0.0316	-0.6698	0.3344	0.1073	0.2983	0.2494	-0.1890
Eigen Values	9.0592	1.5754	0.8021	0.2956	0.1319	0.0719	0.0369	.000077	•000019	-.000009	-000057	-0.000088
%variation explained by eigen vector	75.49	13.13	6.68	2.46	1.10	0.60	0.31	0.0006	0.0002	0.00008	0.0005	0.0007

Table 7. Matrix of standardised values for the original 32 x 12 matrix

-1.601	-1.187	-0.242	1.158	-0.446	0.337	1.172	1.519	-0.395	1.173	0.908	1.657
0.496	0.527	-0.101	1.336	-0.075	1.857	-1.658	1.896	1.580	1.989	1.159	1.544
-1.339	0.675	0.774	0.849	-0.241	1.490	1.602	1.272	0.963	0.505	1.361	1.554
-1.036	0.915	-0.366	0.344	0.914	0.431	0.962	1.873	1.174	1.377	1.076	1.297
-1.175	-1.002	-1.083	-0.529	-0.248	-0.625	-1.077	-1.397	0.747	-0.823	-0.579	-1.034
-1.629	-0.514	-1.219	-1.890	-0.766	-0.443	-1.305	-0.314	0.088	-0.391	-0.279	-0.764
-1.306	-1.524	-1.010	-0.934	-0.119	-0.639	-1.089	-1.125	0.324	-0.313	-0.364	-1.111
0.069	-0.400	-0.067	0.095	0.293	0.554	-1.155	-0.906	0.190	-0.012	0	-0.956
0.088	0.482	0.389	1.067	0.198	0.625	1.040	0.627	1.900	0.100	0.947	-0.271
-0.567	0.983	0.644	0.317	1.256	0.342	0.852	0.859	0.325	-0.283	1.308	0.917
-0.284	0.777	0.025	-0.976	1.434	0.536	0.393	0.684	-0.396	1.264	0.260	0.305
0.259	0.203	0.157	1.342	0.632	0.369	1.219	0.424	1.598	1.342	1.172	0.889
-0.555	-1.373	-1.714	-1.441	-1.635	-1.106	-1.188	-1.234	-1.411	-1.594	-1.896	-1.344
0.289	-1.165	-1.113	-0.362	-0.713	-1.004	0.463	-0.515	-1.099	-0.561	-1.415	-1.234
0.482	-0.143	-1.402	0.042	-1.712	-1.446	-1.533	-1.133	-0.560	-0.862	-1.455	-1.314
-0.531	-1.058	-2.116	-0.753	-1.399	-1.231	0.353	-0.575	-0.246	-1.321	-1.508	-1.090
0.652	0.946	1.283	0.793	1.635	1.684	1.119	0.559	0.347	0.586	0.969	0.982
0.319	1.178	1.017	0.826	0.883	0.679	0.605	0.505	-0.472	0.731	0.770	0.573
1.102	0.982	1.559	1.952	1.026	0.903	0.714	0.863	1.360	1.309	1.002	0.658
1.649	1.518	1.054	1.043	1.196	1.169	0.513	0.930	-0.634	0.309	0.232	1.007
0.813	0.668	0.967	0.520	0.743	-0.048	-0.817	0.215	0.374	0.024	-0.282	-0.181
-0.272	-0.579	0.272	-0.645	-1.032	-1.061	-0.244	-1.002	-0.497	-1.046	-0.600	-0.629
0.673	-1.319	-0.260	-0.552	-0.282	-1.006	-0.398	-0.829	-1.901	-1.126	-1.139	-0.526
-1.415	-0.782	0.306	-0.753	-1.004	-0.615	-1.347	-1.012	-1.095	-0.868	-0.262	-0.249
1.134	0.482	0.284	-0.620	0.496	0.481	-0.233	0.314	-0.321	0.446	0.375	0.473
1.818	0.853	1.390	-0.136	1.309	0.342	1.063	-0.829	1.201	0.397	0.018	0.454
1.135	0.879	0.810	0.170	0.576	0.352	0.047	0.802	0.174	-1.103	0.655	0.822
0.922	0.751	0.979	0.344	0.406	0.862	0.802	-0.318	0.165	0.015	0.598	0.226
0.611	0.946	0.126	-0.939	-0.743	-1.347	-0.021	-0.606	-0.568	-0.486	-0.436	-0.292
-0.460	-1.296	-0.900	0.543	-0.873	-0.724	0.216	-0.603	-1.126	-0.842	-0.955	-0.861
-0.482	-0.348	-0.510	-0.552	-0.691	-0.198	0.269	-0.573	0.040	0.335	-0.316	-0.382
0.069	-1.167	0.082	-1.652	-1.060	-1.538	-0.641	-0.416	-1.160	-0.842	-1.310	-1.138

Table 8. Two way table of data generated through principal component analysis

Treatments	Replications				Total
	R1	R2	R3	R4	
T1	1.8505	3.1743	3.1647	2.9675	11.1570
T2	-2.4517	-2.4703	-2.5261	-0.7269	-8.1750
T3	2.0856	2.2002	1.3013	2.8334	8.4205
T4	-4.7964	-2.5719	-3.3912	-3.5168	-14.2763
T5	3.1458	2.2371	3.7412	2.6884	11.8125
T6	0.6423	-2.1477	-2.7397	-2.4570	-6.7021
T7	0.7876	1.8573	1.3394	1.5106	5.4949
T8	-1.2876	-2.2350	-0.8989	-3.3208	-7.7423
Total	-0.0239	0.0440	-0.0093	-0.0216	-0.0108

Table 9. Analysis of variance of the data generated through principal component analysis

Source	d.f.	S.S.	M.S.	F
Replication	3	0.0003789	0.0001263	
Treatment	7	185.1543	26.4506	27.89**
Error	21	19.9177	0.9485	
Total	31	205.0724		

** Significant at 1% level

Table 10. Sums of ranks and values of K obtained in Bei and Rao's method

Sums of ranks (R_j)

Year	T1	T2	T3	T4	T5	T6	T7	T8	K
				Treatments					
1973	25	26	20	20	9	18	5	21	18.0372*
1974	17	24	14	27	4	22	12	24	19.8460**
1975	21	28	14	31	5	14	9	22	27.3668**
1976	8	25	15	25	8	24	19	20	16.5140*
1977	17	20	10	29	7	22	11	28	22.6068**
1978	10	20	13	29	6	26	14	27	25.0820**
1979	12	28	9	25	11	26	14	22	18.7988**
1980	4	26	11	26	10	27	18	22	25.3676**
1981	11	12	11	27	17	24	14	28	17.4660**
1983	6	22	12	28	8	28	17	23	24.7964**
1985	8	23	8	32	10	24	14	25	27.8428**
1987	4	26	14	31	10	20	13	26	28.6044**
Total	143	280	151	330	105	275	159	285	200.9726

K Statistic for treatment x year interaction

= 272.3288-200.9728 = 71.356 with 77 d.f.

* Significant at 5% level

** Significant at 1% level

Table 11. Rank sums of treatments in different years

Sums of ranks (R_j)

Year	Treatments T1	T2	T3	T4	T5	T6	T7	T8	Total
1973	25	26	20	20	9	18	5	21	144
1974	17	24	14	27	4	22	12	24	144
1975	21	28	14	31	5	14	9	22	144
1976	8	25	15	25	8	24	19	20	144
1977	17	20	10	29	7	22	11	28	144
1978	10	20	13	29	6	26	13	27	144
1979	12	28	9	25	11	26	14	19	144
1980	4	26	11	26	10	27	19	22	144
1981	11	12	11	27	17	24	14	28	144
1983	6	22	12	28	8	28	17	23	144
1985	8	23	8	32	10	24	14	25	144
1987	4	26	14	31	10	20	13	26	144
Total	143	280	151	330	105	275	159	285	1728

Table 12. Values of random variable 's' and χ^2_r for different years

Years	S	χ^2_r	d.f.
1973	380	15.8333	7
1974	418	17.4167	7
1975	576	24	7
1976	348	14.5	7
1977	476	19.8333	7
1978	528	22	7
1979	396	16.5	7
1980	534	22.25	7
1981	368	15.3333	7
1983	522	21.75	7
1985	586	24.4167	7
1987	602	25.0833	7
Total		238.9166	84

Table 13. Analysis of variance and chi-square values in the case of extended Friedman's analysis.

Source	d.f.	S.S.	χ^2
Treatment	7	1051.2083	175.2017**
Year	11	0	
Replication	3	0	
Treatment x year interaction	77	382.29	63.7149
Residual	285	582.5017	
Total	383	2016	

** Significant at 1% level

Table 14. Analysis of variance under Eberhart and Russell Model

Source	d.f.	S.S.	M.S.	F
Total	95	1322.469		
Treatments	7	344.652	49.236	26.3519**
Year + (Treatment x year)	88	977.816		
Year (linear)	1	823.026		
Treatment x year (linear)	7	5.323	0.760	0.4069
Pooled deviation	80	149.468	1.8684	0.6752
Treatment 1	10	58.040	5.804	2.0975*
Treatment 2	10	17.267	1.7267	0.6240
Treatment 3	10	13.661	1.3661	0.4937
Treatment 4	10	25.229	2.5229	0.9117
Treatment 5	10	8.540	0.8540	0.3086
Treatment 6	10	11.590	1.159	0.4189
Treatment 7	10	5.696	0.5696	0.2058
Treatment 8	10	9.446	0.9446	0.3414
Pooled error	252	697.308	2.767	

** Significant at 1% level

* Significant at 5% level

Table 15. Environmental indices (I_j) under Eberhart and Russell model

Year	I_j
1973	-1.9445
1974	1.5943
1975	0.9443
1976	1.5630
1977	2.1536
1978	3.2099
1979	-4.3770
1980	2.5927
1981	0.6418
1983	-2.1876
1985	2.2877
1987	-6.4779

Table 16. Stability parameters b_i and S_{di}^2 and their corresponding 't' and 'F' statistics

Treatment	b_i	S_{di}^2	t(i)	F(i)
1	0.9310	3.0369	-0.5120	2.0975*
2	1.1916	-1.0404	1.4221	0.6240
3	1.0130	-1.4010	0.0966	0.4937
4	0.9696	-0.2442	-0.2259	0.9117
5	1.0091	-1.9131	0.0676	0.3086
6	1.0149	-1.6081	0.1109	0.4188
7	0.9516	-2.1975	-0.3594	0.2058
8	0.9193	-1.8255	-0.5986	0.3414

* Significant at 5% level

Table 17. Corrected x_{ij} values for stability analysis using non parametric measures

Treatment	Years 1	2	3	4	5	6	7	8	9	10	11	12
1	5069	6517	6241	6806	6497	7372	6032	8393	6971	6523	7742	7238
2	6208	7196	6906	7082	7606	7808	4901	6809	7541	6271	7421	6601
3	5616	6892	6602	6932	7275	7329	5807	7605	6984	5965	7790	6601
4	6698	7504	6878	7481	7262	7632	5558	7185	6944	6021	6371	5672
5	5946	7027	7027	7134	7316	7566	5613	7562	6176	6040	7522	6476
6	6498	7249	7457	7255	7451	7545	5241	7068	6522	5863	7060	6195
7	6367	7135	7082	6718	7331	7506	5519	7084	6935	5790	7431	6505
8	6535	7277	7169	7095	7229	7485	5896	7207	6592	6088	6924	5907

Table 18. Ranks of treatments in each year based on the corrected x_{ij} values

Treat-ments	Years 1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	1	2	1	2	8	8	6	8	7	8
2	4	5	4	4	8	8	1	1	8	7	4	1
3	2	2	2	3	4	1	6	7	7	3	8	7
4	8	8	3	8	3	7	4	4	5	4	1	2
5	3	3	5	6	6	6	5	6	1	5	6	5
6	6	6	8	7	7	5	2	2	2	2	3	4
7	5	4	6	1	6	4	3	3	4	1	5	6
8	7	7	7	5	2	3	7	5	3	6	2	3

Table 19. The values of stability parameters $S_i^{(1)}$ and $S_i^{(2)}$ and statistics $Z_i^{(1)}$ and $Z_i^{(2)}$ for each treatment

Treat-ment	$\bar{r}_{i.}$	$S_i^{(1)}$	$Z_i^{(1)}$	$S_i^{(2)}$	$Z_i^{(2)}$
1	4.417	3.682	6.980**	10.811	14.265**
2	4.583	3.136	1.634	7.356	2.046
3	4.333	2.909	0.504	6.242	0.454
4	4.750	2.864	0.356	6.023	0.275
5	4.667	1.636	6.109	2.424	3.684
6	5.400	2.667	0.011	5.182	0.002
7	4.000	2.030	2.210	3.091	2.151
8	4.750	2.379	0.379	4.205	0.504
$S^{(m)} = \sum_{i=1}^{8} Z_i^{(m)}$			18.18*		23.38*
Ho : $E\left(S_i^{(m)}\right)$		2.625		5.25	
$Var\left(S_i^{(m)}\right)$		0.16		2.168	

* Significant at 5% level
** Significant at 1% level

Table 20. Pay off matrix for the analysis based on game theory

Strategies or treatment

T1	10254	14598	13770	15465	14538	17163	13143	20226	15960	14616	18273	16761
T2	-801	2163	1293	1821	3393	3999	-4722	1002	3198	-612	2838	-2448
T3	5655	9483	8613	9603	10632	10794	6228	11622	9759	6702	12177	8610
T4	16653	18471	16593	18402	17745	18855	12633	17514	16791	14022	15072	12975
T5	14291	17534	17534	17855	18401	19151	13292	19139	14981	14573	19019	15881
T6	7619	9872	10496	9890	10478	10760	3818	9329	7691	5714	9305	6710
T7	12002	14306	14147	13055	14894	15419	9458	14153	13706	10271	15194	12416
T8	16625	18851	18527	18305	18707	19475	14708	18641	16796	15284	17792	14771

Table 21. Regret matrix in Savage's regret criterion

$$
\begin{bmatrix}
-6399 & -4253 & -4757 & -2937 & -4169 & -2312 & -1565 & 0 & -836 & -668 & -746 & 0 \\
-17454 & -16688 & -17234 & -16581 & -15314 & -15476 & -19430 & -19224 & -13598 & -15896 & -16181 & -19209 \\
-10998 & -9368 & -9914 & -8799 & -8075 & -8681 & -8430 & -8604 & -7037 & -8582 & -6842 & -8151 \\
0 & -380 & -1934 & 0 & -962 & -620 & -2075 & -2712 & -5 & -1262 & -3947 & -3786 \\
-2362 & -1317 & -993 & -547 & -306 & -325 & -1416 & -1087 & -1815 & -711 & 0 & -880 \\
-9034 & -8979 & -8031 & -8512 & -8229 & -8715 & -10860 & -10897 & -9105 & -9570 & -9714 & -10051 \\
-4651 & -4545 & -4380 & -5347 & -3813 & -4056 & -5250 & -6073 & -3090 & -5013 & -3825 & -4345 \\
-28 & 0 & 0 & -97 & 0 & 0 & 0 & -1585 & 0 & 0 & -1227 & -2020
\end{bmatrix}
$$

Table 22. Benefit matrix in Agarwal's excess benefit criterion

$$
\begin{bmatrix}
11055 & 12435 & 12477 & 13644 & 11145 & 13164 & 17865 & 19224 & 12762 & 15228 & 15435 & 19209 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6456 & 7320 & 7320 & 7782 & 7239 & 6795 & 10950 & 10620 & 6561 & 7314 & 9339 & 11058 \\
17454 & 16308 & 15300 & 16581 & 14352 & 14856 & 17355 & 16512 & 13593 & 14634 & 12234 & 15423 \\
15092 & 15371 & 16241 & 16043 & 15008 & 15152 & 18014 & 18137 & 11783 & 15185 & 16181 & 18329 \\
8420 & 7709 & 9303 & 8069 & 7085 & 6771 & 8570 & 8327 & 4493 & 6326 & 6467 & 9158 \\
12803 & 12143 & 12854 & 11234 & 11501 & 11420 & 14180 & 13151 & 10808 & 10883 & 12356 & 14864 \\
17426 & 16688 & 17234 & 16484 & 15314 & 15476 & 19330 & 17639 & 13598 & 15896 & 14954 & 17189
\end{bmatrix}
$$

Table 24. The values of coefficient of determination and average absolute error for different models corresponding to different sets of data under category I along with their mean and range of variations.

Table No.	Quadratic model		Square root polynomial		Nelder's polynomial		Inverse polynomial		Mixed model		Gupta's function		Holliday function		New Model I		New model 2	
	R^2	A.A.E.	R^2	A.A.E.	R^2	A.A.E.	R^2	A.A.E.	R^2	A.A.E.	R^2	A.A.E.	R^2	A.A.E.	R^2	A.A.E.	R^2	A.A.E.
1	0.99	14.50	0.998	6.19	0.9459	33.28	0.8244	53.68	0.9964	7.64	-		-	-	-	-	0.8197	49.19
3	0.9031	160.40	0.6617	285.42	0.0846	545.13	0.0500	468.59	0.5579	319.18	0.5655	311.28	0.0004	1006.05	0.5185	328.85	0.0002	1033.17
4	0.9895	34.21	0.8507	123.12	0.1365	359.30	0.0403	311.49	0.7670	150.43	0.7765	144.92	0.0011	627.97	0.7330	158.99	0.0026	614.77
5	0.9998	2.01	0.9073	41.11	0.1048	151.68	0.6265	82.55	0.8514	50.84	0.8443	51.25	0.0008	263.23	0.8273	54.15	0.0065	201.39
6	0.9772	42.67	0.9669	49.74	0.9325	93.69	0.8959	102.21	0.9686	47.70	-	-	-	-	0.9692	47.22	0.9097	84.71
8	1.0000	0.005	0.9967	21.26	0.8676	183.62	0.9815	65.10	0.9960	23.34	-	-	-	-	-	-	0.9908	38.36
10	0.9977	10.88	0.9990	6.73	0.7919	125.06	0.9975	11.99	0.9976	10.04	-	-	0.9136	50.65	0.9974	11.31	0.9875	25.07
12	1.0000	0.80	0.9902	22.05	0.8536	108.39	0.9032	78.60	0.9896	22.12	-	-	-	-	-	-	0.9373	47.57
19	0.9886	33.61	0.9809	31.31	0.9898	32.71	-	-	0.5293	133.87	-	-	-	-	-	-	0.9603	64.77
22	1.0000	2.36	0.9605	14.80	0.1367	116.69	-	-	0.6270	50.21	0.9719	18.07	0.9574	22.50	0.9791	16.26	0.8993	37.18
23	0.9815	12.62	1.0000	19.13	0.4614	73.59	-	-	0.2558	70.01	0.9370	21.77	0.9014	26.88	0.9402	21.46	0.8404	30.99
24	0.5613	60.16	0.7583	42.94	0.3944	67.31	-	-	0.8401	34.34	0.8239	35.27	-	-	0.8670	30.88	0.1261	69.93
25	0.8232	67.52	0.5744	100.78	0.2008	143.23	0.5219	105.56	0.5076	106.42	0.4838	106.77	0.0016	379.09	0.4813	107.87	0.0076	297.86
26	0.9706	14.09	0.9442	18.67	0.9267	20.90	0.4350	57.80	0.9178	22.25	0.9359	19.26	0.0008	146.88	0.9089	23.14	0.0085	122.79
27	0.9791	13.84	0.9922	8.23	0.9405	21.47	0.8007	40.49	0.9892	9.32	-	-	0.6884	35.27	-	-	0.5163	52.67
28	0.8037	101.74	0.5374	150.24	0.1408	208.80	0.4969	156.90	0.4628	158.96	0.4414	158.82	0.0003	535.65	0.4337	161.16	0.0005	436.94
30	0.8058	88.64	0.9125	57.22	0.6999	111.09	-	-	0.9558	40.04	0.9436	44.22	-	-	0.9679	33.60	0.9776	23.22
32	0.9582	78.90	0.8365	148.63	0.5612	349.28	0.9131	156.94	0.8060	158.33	-	-	0.0034	365.48	0.7933	161.34	0.4094	301.44
35	0.9997	8.10	0.9767	65.70	0.6732	353.15	0.9762	84.14	0.9665	76.93	-	-	0.1693	312.41	-	-	0.7776	191.86
36	0.9841	78.99	0.9768	90.89	0.9266	255.42	0.7957	351.51	0.9844	72.64	-	-	-	-	0.9862	67.60	0.9526	123.47
38	0.6547	226.15	0.9376	90.38	0.0192	357.51	0.3404	272.55	0.9847	40.79	0.9861	37.73	0.9832	48.65	0.9926	27.83	0.0192	299.31
44	0.5503	111.04	0.4631	109.17	0.3202	128.81	0.4292	110.64	0.4374	111.74	-	-	0.0174	324.02	-	-	0.0076	265.46
46	0.9245	53.48	0.9722	27.46	0.4005	154.67	0.9423	43.85	0.9513	36.28	0.9357	39.76	0.0281	189.02	0.9384	39.93	0.0432	157.61
47	0.9620	28.34	0.8294	59.76	0.2608	134.76	0.7397	74.57	0.7595	71.37	0.7283	74.70	0.0047	355.13	0.7287	74.96	0.0261	292.74
48	0.9657	75.15	0.9015	117.46	0.7091	291.81	0.8839	139.30	0.8917	124.59	-	-	-	-	-	-	0.1325	300.57
49	0.9604	95.03	0.9817	53.41	0.2638	505.98	0.8838	180.51	0.9580	94.00	0.9538	97.71	0.0056	341.15	0.9464	107.86	0.2579	349.45
50	0.9478	108.92	0.9893	43.72	0.9725	445.98	0.9821	70.93	0.9836	52.18	0.9799	55.75	0.7281	177.78	0.9795	56.83	0.8396	158.63
52	1.0000	0.10	0.9581	18.76	0.5454	69.75	-	-	-	-	-	-	0.0178	194.70	-	-	0.0065	91.75
53	0.8677	44.50	0.9957	7.62	0.0132	125.67	-	-	0.9996	2.58	0.9990	3.52	0.9917	7.32	0.9963	6.85	0.5869	60.90
55	0.7745	43.70	0.9638	16.66	0.0479	88.36	0.5897	51.31	0.9892	9.02	0.9909	8.03	0.6127	31.52	0.9953	5.75	0.9810	10.88
56	0.7076	112.90	0.8441	78.48	0.5082	131.88	0.0391	189.20	0.8977	62.44	0.8843	65.18	-	-	0.9160	55.79	0.6577	104.40
57	0.5015	158.23	0.7671	103.09	0.1258	190.76	0.0641	191.95	0.8549	79.35	0.8429	81.29	-	-	0.8804	71.14	0.0009	195.67
58	0.8059	109.70	0.9708	40.47	0.1835	229.63	0.7927	106.90	0.9908	22.38	0.9935	18.47	0.8101	77.24	0.9960	14.35	0.9925	18.61
59	0.8927	31.20	0.9983	3.58	0.1104	95.30	0.7038	45.90	0.9983	3.57	0.9971	4.69	0.8370	26.90	0.9942	6.62	0.7153	43.97
61	0.7306	49.40	0.8840	30.87	0.4548	64.29	0.0142	85.35	0.9334	23.00	0.9235	24.16	-	-	0.9493	19.76	0.8746	28.21
62	0.9232	62.70	0.9999	2.15	0.1485	228.68	0.7600	105.71	0.9900	21.04	0.9885	22.21	0.0078	175.44	0.9827	27.29	0.0723	173.61
63	0.9681	26.40	0.7785	66.23	0.0003	162.50	0.2852	114.00	0.6839	77.23	0.6869	75.69	0.0037	444.33	0.6495	80.30	0.0041	339.03
64	0.9992	3.70	0.9546	26.49	0.4047	117.40	0.8999	38.23	0.9308	32.23	0.9184	34.30	0.0063	141.62	0.9175	34.62	0.2640	101.62
65	0.9212	39.40	0.9995	2.91	0.0261	148.66	0.2206	108.80	0.9858	15.55	0.9860	15.23	0.1485	92.58	0.9746	20.65	0.0955	118.26
Mean	0.8916	56.57	0.8977	56.48	0.4432	180.16	0.6197	126.79	0.8470	64.32	0.8661	60.39	0.3275	237.20	0.8746	62.48	0.4540	178.44
Range	0.4985	226.15	0.5369	283.27	0.9895	524.23	0.9833	456.60	0.7438	316.60	0.5576	307.76	0.9914	998.73	0.5637	323.10	0.9923	1022.29

Table 23. Ranks of treatments as obtained in groups of experiments, principal component analysis and Rai and Rao's method

Methods	Treatments							
	1	2	3	4	5	6	7	8
1	1	7	3	8	2	6	4	5
2	2	7	3	8	1	5	4	6
3	2	6	3	8	1	5	4	7
Total (R_j)	5	20	9	24	4	16	12	18

Table 24. The values of coefficient of determination and average absolute error for different models corresponding to different sets of data under category I along with their meansand range of variations.

Table No.	Quadratic model		Square root polynomial		Nelder's polynomial		Inverse polynomial		Mixed model		Gupta's function		Holliday function		New Model I		New model 2	
	R^2	A.A.E.	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E.	R^2	A.A.E.	R^2	A.A.E	R^2	A.A.E.	R^2	A.A.E	R^2	A.A.E
1	0.99	14.50	0.998	6.19	0.9459	33.28	0.8244	53.68	0.9964	7.64	-		-	-	-	-	0.8197	49.19
3	0.9031	160.40	0.6617	285.42	0.0846	545.13	0.0500	468.59	0.5579	319.18	0.5655	311.28	0.0004	1006.05	0.5185	328.85	0.0002	1033.17
4	0.9895	34.21	0.8507	123.12	0.1365	359.30	0.0403	311.49	0.7670	150.43	0.7765	144.92	0.0011	627.97	0.7330	158.99	0.0026	614.77
5	0.9998	2.01	0.9073	41.11	0.1048	151.68	0.6265	82.55	0.8514	50.84	0.8443	51.25	0.0008	263.23	0.8273	54.15	0.0065	201.39
6	0.9772	42.67	0.9669	49.74	0.9325	93.69	0.8959	102.21	0.9686	47.70	-	-	-	-	0.9692	47.22	0.9097	84.71
8	1.0000	0.005	0.9967	21.26	0.8676	183.62	0.9815	65.10	0.9960	23.34	-	-	-	-	-	-	0.9908	38.36
10	0.9977	10.88	0.9990	6.73	0.7919	125.06	0.9975	11.99	0.9976	10.04	-	-	0.9136	50.65	0.9974	11.31	0.9875	25.07
12	1.0000	0.80	0.9902	22.05	0.8536	108.39	0.9032	78.60	0.9896	22.12	-	-	-	-	-	-	0.9373	47.57
19	0.9886	33.61	0.9809	31.31	0.9898	32.71	-	-	0.3293	133.87	-	-	-	-	-	-	0.9603	64.77
22	1.0000	2.36	0.9605	14.80	0.1367	116.69	-	-	0.6270	50.21	0.9719	18.07	0.9574	22.50	0.9791	16.26	0.8993	37.18
23	0.9815	12.62	1.0000	19.13	0.4614	73.59	-	-	0.2558	70.01	0.9370	21.77	0.9014	26.88	0.9402	21.46	0.8404	30.99
24	0.5613	60.16	0.7583	42.94	0.3944	67.31	-	-	0.8401	34.34	0.8239	35.27	-	-	0.8670	30.88	0.1261	69.93
25	0.8232	67.52	0.5744	100.78	0.2008	143.23	0.5219	105.56	0.5076	106.42	0.4838	106.77	0.0016	379.09	0.4813	107.87	0.0076	297.86
26	0.9706	14.09	0.9442	18.67	0.9267	20.90	0.4350	57.80	0.9178	22.25	0.9359	19.26	0.0008	146.88	0.9089	23.14	0.0085	122.79
27	0.9791	13.84	0.9922	8.23	0.9405	21.47	0.8007	40.49	0.9892	9.32	-	-	0.6884	35.27	-	-	0.5163	52.67
28	0.8037	101.74	0.5374	150.24	0.1408	208.80	0.4969	156.90	0.4628	158.96	0.4414	158.82	0.0003	535.65	0.4337	161.16	0.0005	436.94
30	0.8058	88.64	0.9125	57.22	0.6999	111.09	-	-	0.9558	40.04	0.9436	44.22	-	-	0.9679	33.60	0.9776	23.22
32	0.9582	78.90	0.8365	148.63	0.5612	349.28	0.9131	156.94	0.8060	158.33	-	-	0.0034	365.48	0.7933	161.34	0.4094	301.44
35	0.9997	8.10	0.9767	65.70	0.6732	353.15	0.9762	84.14	0.9665	76.93	-	-	0.1693	312.41	-	-	0.7776	191.86
36	0.9841	78.99	0.9768	90.89	0.9266	255.42	0.7957	351.51	0.9844	72.64	-	-	-	-	0.9862	67.60	0.9526	123.47
38	0.6547	226.15	0.9376	90.38	0.0192	357.51	0.3404	272.55	0.9847	40.79	0.9861	37.73	0.9832	48.65	0.9926	27.83	0.0192	299.31
44	0.5505	111.04	0.4631	109.17	0.3202	128.81	0.4292	110.64	0.4374	111.74	-	-	0.0174	324.02	-	-	0.0076	265.46
46	0.9245	53.48	0.9722	27.46	0.4005	154.67	0.9423	43.85	0.9513	36.28	0.9357	39.76	0.0281	189.02	0.9384	39.93	0.0432	157.61
47	0.9620	28.34	0.8294	59.76	0.2608	134.76	0.7397	74.57	0.7595	71.37	0.7283	74.70	0.0047	355.13	0.7287	74.96	0.0261	292.74
48	0.9657	75.15	0.9015	117.46	0.7091	291.81	0.8839	139.30	0.8917	124.59	-	-	-	-	-	-	0.1325	300.57
49	0.9604	95.03	0.9817	53.41	0.2638	505.98	0.8838	180.51	0.9580	94.00	0.9538	97.71	0.0056	341.15	0.9464	107.86	0.2579	349.45
50	0.9478	108.92	0.9893	43.72	0.9725	445.98	0.9821	70.93	0.9836	52.18	0.9799	55.75	0.7281	177.78	0.9795	56.83	0.8396	158.63
52	1.0000	0.10	0.9581	18.76	0.5454	69.75	-	-	-	-	-	-	0.0178	194.70	-	-	0.0065	91.75
53	0.8677	44.50	0.9957	7.62	0.0132	125.67	-	-	0.9996	2.58	0.9990	3.52	0.9917	7.32	0.9963	6.85	0.5869	60.90
55	0.7745	43.70	0.9638	16.66	0.0479	88.36	0.5897	51.31	0.9892	9.02	0.9909	8.03	0.6127	31.52	0.9953	5.75	0.9810	10.88
56	0.7076	112.90	0.8441	78.48	0.5082	131.88	0.0391	189.20	0.8977	62.44	0.8843	65.18	-	-	0.9160	55.79	0.6577	104.40
57	0.5015	158.23	0.7671	103.09	0.1258	190.76	0.0641	191.95	0.8549	79.35	0.8429	81.29	-	-	0.8804	71.14	0.0009	195.67
58	0.8059	109.70	0.9708	40.47	0.1835	229.63	0.7927	106.90	0.9908	22.38	0.9935	18.47	0.8101	77.24	0.9960	14.35	0.9925	18.61
59	0.8927	31.20	0.9983	3.58	0.1104	95.30	0.7038	45.90	0.9983	3.57	0.9971	4.69	0.8370	26.90	0.9942	6.62	0.7153	43.97
61	0.7306	49.40	0.8840	30.87	0.4548	64.29	0.0142	85.35	0.9334	23.00	0.9235	24.16	-	-	0.9493	19.76	0.8746	28.21
62	0.9232	62.70	0.9999	2.15	0.1485	228.68	0.7600	105.71	0.9900	21.04	0.9885	22.21	0.0078	175.44	0.9827	27.29	0.0723	173.61
63	0.9681	26.40	0.7785	66.23	0.0003	162.50	0.2852	114.00	0.6839	77.23	0.6869	75.69	0.0037	444.33	0.6495	80.30	0.0041	339.03
64	0.9992	3.70	0.9546	26.49	0.4047	117.40	0.8999	38.23	0.9308	32.23	0.9184	34.30	0.0063	141.62	0.9175	34.62	0.2640	101.62
65	0.9212	39.40	0.9995	2.91	0.0261	148.66	0.2206	108.80	0.9858	15.55	0.9860	15.23	0.1485	92.58	0.9746	20.65	0.0955	118.26
Mean	0.8916	56.57	0.8977	56.48	0.4432	180.16	0.6197	126.79	0.8470	64.32	0.8661	60.39	0.3275	237.20	0.8746	62.48	0.4540	178.44
Range	0.4985	226.15	0.5369	283.27	0.9895	524.23	0.9833	456.60	0.7438	316.60	0.5576	307.76	0.9914	998.73	0.5637	323.10	0.9923	1022.29

Table 25. The values of coefficient of determination and average absolute error for different models corresponding to different sets of data under category II along with their means and range of variations.

Table No.	Quadratic model		Square root polynomial		Nelder's polynomial		Inverse polynomial		Mixed model		Gupta's function		Holliday function		New model 1		New model 2	
	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E
7	0.9994	7.93	0.9997	4.27	0.9251	106.31	0.9537	75.85	0.9998	3.84	-	-	-	-	-	-	0.9999	2.05
9	0.9964	21.998	1.0000	2.37	0.8772	157.55	0.9822	54.56	1.0000	0.491	-	-	-	-	-	-	0.9996	7.64
11	0.9945	31.10	0.9994	9.87	0.8403	202.27	0.9296	126.43	0.9993	10.83	-	-	-	-	-	-	0.9999	1.62
29	0.9963	15.45	0.9975	12.42	0.8304	126.85	0.9166	79.39	0.9969	13.11	-	-	-	-	-	-	0.9968	14.24
Mean	0.9966	19.12	0.9991	7.23	0.8683	148.25	0.9455	84.06	0.9990	7.06	-	-	-	-	-	-	0.9991	6.39
Range	0.0049	23.17	0.0025	10.05	0.0947	95.96	0.0656	71.87	0.0031	12.62	-	-	-	-	-	-	0.0031	12.62

Table 26. The values of coefficient of determination and average absolute error for different models corresponding to different sets of data under category III along with their means and range of variation

Table No.	Quadratic model		Square root polynomial		Nelder's polynomial		Inverse polynomial		Mixed model		Gupta's function		Holliday function		New Model 1		New model 2	
	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E
2	0.7462	105.30	0.8475	77.72	0.6575	117.42	0.8939	66.82	0.8629	72.07	-	-	0.0002	654.51	-	-	0.0037	257.50
13	0.6343	90.80	0.7839	66.46	0.5192	100.12	0.8615	56.84	0.8145	60.22	0.8331	56.21	0.0012	621.12	0.8283	57.21	0.0168	201.28
14	0.8777	54.80	0.9206	41.93	0.8310	60.09	0.8637	59.40	0.9203	41.18	-	-	0.0067	393.93	-	-	0.2061	161.07
15	0.1507	139.60	0.2674	123.45	0.1520	139.10	0.3240	115.87	0.3151	116.78	0.3340	113.26	0	6258.36	0.3384	113.29	0.0002	596.46
16	0.3827	181.64	0.4046	188.27	0.3865	182.43	0.3536	179.49	0.2890	165.79	0.3761	181.96	0.3572	187.41	0.3440	174.81	0.3674	188.84
17	0.5202	165.74	0.5295	156.82	0.3989	104.97	0.2874	190.36	0.3156	152.39	0.5817	152.80	0.5317	167.12	0.5251	141.07	0.5774	155.26
18	0.7205	148.23	0.6614	139.47	0.7000	148.41	-	-	0.3549	193.26	0.7144	147.18	-	-	0.7720	155.75	0.6454	172.67
20	0.1403	136.45	0.1863	132.57	0.0631	136.62	0.1093	128.24	0.1206	142.30	0.2400	126.43	0.1517	133.66	0.2183	132.46	0.1949	137.16
21	0.1368	146.33	0.1419	153.80	0.1382	155.28	0.1230	151.64	0.0027	147.32	0.1337	155.52	0.0606	146.74	0.1686	159.61	0.0603	153.81
31	0.9095	89.43	0.9292	76.06	0.8732	95.70	0.8180	142.87	0.9208	79.00	-	-	0.0004	523.60	-	-	0.0056	314.30
33	0.9033	84.20	0.9338	49.30	0.7309	144.13	0.9730	58.70	0.9699	43.78	-	-	0.0017	204.78	-	-	0.7413	121.09
34	0.6020	140.50	0.7822	98.96	0.4691	164.87	0.8874	84.12	0.8221	87.47	0.8383	82.05	0.0001	2676.61	0.8385	82.28	0.0652	275.87
37	0.6923	95.998	0.7317	90.55	0.6861	92.91	0.6643	82.25	0.7263	89.90	-	-	0.0004	1667.84	-	-	0.0001	689.31
39	0.8392	77.14	0.7758	91.32	0.7034	108.28	0.1839	161.52	0.7321	95.66	0.7546	90.92	0	10095.51	0.7179	95.79	0.0003	1152.21
40	0.2539	588.15	0.2786	588.06	0.2246	675.33	0.0004	736.35	0.3659	532.17	0.3371	538.39	-	-	0.4021	505.77	0.0011	1011.80
41	0.0828	292.12	0.3985	242.10	0.0033	260.32	0.2139	208.04	0.5438	206.89	0.5453	203.16	0.0002	1511.67	0.5970	191.57	0.	2657.49
42	0.4571	204.03	0.4456	178.89	0.4710	188.67	0.1289	203.44	0.4955	162.41	0.4656	164.18	0.0005	730.60	0.5110	157.08	0.0041	468.66
43	0.2934	177.60	0.1790	197.88	0.1717	195.13	0.1461	188.95	0.1593	196.85	-	-	0.0001	1678.37	-	-	0	48692.77
51	0.9315	121.98	0.9841	56.86	0.6565	334.34	0.9890	60.08	0.9901	42.45	-	-	-	-	-	-	0.8869	118.80
54	0.6608	67.60	0.8702	39.82	0.2179	92.59	-	-	0.9095	32.61	0.9205	30.04	-	-	0.9259	29.13	0.3065	77.88
60	0.6314	159.04	0.6508	147.27	0.6295	158.84	0.6130	141.09	0.6419	146.31	-	-	0.0003	1231.97	-	-	0.0056	489.69
66	0.4889	112.80	0.7528	74.70	0.1677	135.88	0.6857	84.99	0.8136	63.63	0.8257	60.45	-	-	0.8392	58.31	0.1323	140.90
67	0.4315	162.998	0.3848	161.45	0.3627	166.04	0.3100	158.37	0.3646	160.97	-	-	0.0002	3354.28	-	-	0.0016	626.26
68	0.6456	81.30	0.8614	48.41	0.2774	109.31	0.8373	54.52	0.9094	38.19	0.9168	36.06	-	-	0.9248	34.38	0.0756	120.23
69	0.2730	60.70	0.4861	48.58	0.1814	60.60	0.0024	63.31	0.5926	42.20	0.5706	42.70	-	-	0.6275	39.89	0.0413	175.13
Mean	0.5362	147.38	0.6075	130.82	0.4269	167.50	0.4850	146.84	0.5981	124.47	0.5867	136.33	0.0618	1785.45	0.5986	133.03	0.1736	2367.34
Range	0.8487	533.35	0.8422	548.24	0.8699	615.24	0.9886	681.83	0.9874	499.56	0.7868	508.35	0.5317	9961.85	0.7573	476.64	0.8869	48614.89

Table 27. Thev values of coefficient of determination and average absolute error for different models corresponding to different sets of data under category IV along with their means and range of variations

Table No.	Quadratic model		Square root polynomial		Nelder's polynomial		Inverse polynomial		Mixed model		Gupta's function		Holliday function		New Model 1		New model 2	
	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E	R^2	A.A.E
45	0.4149	105.45	0.4732	97.49	0.4000	105.92	0.0413	137.20	0.5569	89.59	0.5275	88.89	-	-	0.5888	85.87	0.0004	236.25
70	0.1369	95.28	0.1706	92.24	0.0976	102.30	0.0185	102.10	0.1965	90.74	0.2244	87.28	0.0006	223.46	0.2273	88.04	0.0007	211.69
71	0.0329	137.96	0.0884	138.37	0.0083	139.24	0.1589	112.18	0.2050	134.19	0.2782	122.95	0.0002	1167.02	0.3005	125.11	0.0001	709.89
Mean	0.1949	112.90	0.2441	109.37	0.1686	115.82	0.0729	117.16	0.3195	104.84	0.3434	99.71	0.0004	695.24	0.3722	99.67	0.0004	385.94
Range	0.3820	42.68	0.3848	46.13	0.3917	36.94	0.1404	35.10	0.3604	44.60	0.3031	35.67	0.0004	943.56	0.3615	39.24	0.0006	498.20

Table 28. The percentage number of cases included under each model within the specified ranges of coefficient of determination

Range of R^2	Different models: Quadratic model	Square root polynomial	Nelder's polynomial	Inverse polynomial	Mixed model	Gupta's function	Holliday function	New model 1	New model 2
0.98-1	23.94	25.35	1.408	7.042	26.761	9.859	2.817	11.268	11.268
0.95-0.98	11.27	14.08	-	4.225	8.451	4.225	1.408	7.042	4.225
0.9-0.95	11.27	9.86	8.451	7.042	9.859	11.268	2.817	12.676	2.817
0.8-0.9	11.27	11.27	9.859	16.90	14.085	9.859	2.817	8.451	8.451
0.7-0.8	7.04	11.27	7.042	7.042	5.634	5.634	1.408	7.042	4.225
0.6-0.7	9.86	4.225	8.451	5.634	4.225	1.408	2.817	2.817	2.817
0.5-0.6	5.63	4.225	5.634	2.817	8.451	7.042	1.408	7.042	4.225
Below 0.5	19.72	19.72	59.155	36.62	21.127	14.085	50.704	12.766	61.972
Total	100.00	100.00	100.00	87.32	98.593	63.380	66.196	69.014	100.00

Table 29. Mean values of coefficient of determination and average absolute error for the different models

Models	R^2	A.A.E.
Quadratic model	0.7429	88.81
Square root polynomial	0.7736	82.12
Nelder's polynomial	0.4498	171.18
Inverse polynomial	0.5655	131.00
Mixed model	0.7442	84.26
Gupta's model	0.7319	90.01
Holliday function	0.2118	849.64
New model 1	0.7538	87.79
New model 2	0.3668	948.25

Table 30. Percentage number of cases of estimates on physical and economic optima under each model which lie in the specified ranges of nutrients

Models	Physical optimum dose	Economic optimum dose
Quadratic model	80.28	84.51
Square root polynomial	76.06	77.46
Nelder's polynomial	-	8.45
Inverse polynomial	-	83.10
Mixed model	85.92	91.55
Gupta's function	60.56	50.56
Holliday function	63.38	64.79
New model 1	63.38	66.20
New model 2	95.77	95.77

Table 31. The values of coefficient of determination and average absolute error for different bivariate models corresponding to different sets of data along with their means and range of variation.

Table No.	Quadratic model		Square root polynomial		Transcendental function		Resistance function	
	R^2	A.A.B	R^2	A.A.B	R^2	A.A.B	R^2	A.A.B
1.	0.3830	234.97	0.8613	493.64	0.811	223.68	0.992	2579.83
2.	0.9631	484.44	0.9626	712.07	0.969	118.68	0.998	954.71
3.	0.9611	225.30	0.9683	118.23	0.950	149.32	0.999	628.91
4.	0.8196	574.11	0.8110	377.95	0.755	256.56	0.993	1049.68
5.	0.6998	324.35	0.7389	1142.06	0.636	248.72	0.998	908.58
6.	0.1199	180.31	0.1229	227.74	0.126	168.87	0.999	527.43
7.	0.4798	858.94	0.5479	971.72	0.389	191.10	0.995	840.83
8.	0.9716	399.39	0.9795	488.61	0.952	60.03	0.999	533.03
9.	0.8071	158.41	0.7980	384.20	0.777	175.99	0.997	1093.63
10.	0.9039	308.33	0.8730	311.85	0.648	159.65	0.999	685.68
11.	0.8066	314.50	0.8203	737.72	0.793	146.33	0.998	1342.32
12.	0.8934	456.28	0.8640	152.95	0.676	134.17	0.999	694.72
13.	0.9158	699.11	0.9018	381.45	0.414	282.13	0.997	2937.12
14.	0.8692	512.67	0.8451	456.09	0.850	133.46	0.996	1932.98
15.	0.7786	373.39	0.7963	261.93	0.731	145.75	0.992	667.15
16.	0.9351	551.39	0.9313	1110.93	0.915	140.37	0.999	364.74
17.	0.9540	122.09	0.9476	162.22	0.938	117.77	0.998	862.52
18.	0.8455	721.89	0.8327	768.86	0.814	191.46	0.995	8778.44
19.	0.6936	548.17	0.7145	287.01	0.283	280.28	0.996	737.49
20.	0.9279	289.44	0.9283	514.25	0.935	96.35	0.998	398.92
21.	0.8832	726.28	0.9341	643.39	0.761	143.58	0.999	630.85
22.	0.6925	1365.50	0.6240	836.63	0.368	321.29	0.980	5952.25
23.	0.5750	273.12	0.6140	257.58	0.562	286.47	0.952	1065.67
24.	0.9465	741.67	0.9466	435.49	0.875	160.24	0.969	1044.76
25.	0.9173	188.19	0.9199	617.28	0.896	216.92	0.999	662.39
26.	0.9194	314.33	0.9220	796.79	0.927	166.96	0.997	606.93
27.	0.9850	1200.83	0.9800	1439.72	0.918	134.88	0.999	1166.68
28.	0.9608	800.00	0.9723	2106.00	0.917	156.74	0.997	1636.05
29.	0.9329	239.56	0.9298	862.27	0.915	125.55	0.997	397.09
30.	0.7292	387.33	0.7114	1027.58	0.730	218.68	0.984	1460.28
31.	0.6987	226.98	0.6623	891.58	0.631	210.57	0.976	1055.51
32.	0.9260	162.33	0.9512	612.71	0.836	199.03	0.996	560.68
33.	0.9239	1036.00	0.9461	704.75	0.741	225.03	0.997	532.78
34.	0.9307	969.45	0.9305	984.08	0.933	97.38	0.996	1976.39
35.	0.9349	1122.72	0.9484	724.90	0.708	234.36	0.996	427.33
36.	0.9342	487.61	0.9297	953.86	0.938	84.49	0.995	3208.85
Mean	0.8366	516.09	0.8380	665.45	0.751	177.86	0.9935	1413.98
Range	0.8651	1243.41	0.8571	1987.77	0.843	261.26	0.047	8413.70

Table 32. The percentage number of cases included under each model within the specified ranges of coefficient of determination

Range of R^2	Different models			
	Quadratic model	Square root polynomial	Transendental function	Resistance function
0.95-1	16.667	10.667	8.333	100
0.9-0.95	36.111	36.111	25.000	-
0.8-0.9	22.222	19.444	16.667	-
0.7-0.8	5.555	13.889	22.222	-
0.6-0.7	11.111	8.333	11.111	-
0.5-0.6	2.778	2.778	2.778	-
Below 0.5	2.555	2.778	13.889	-
Total	100.00	100.00	100.00	100.00

Table 33. Physical and economic optimum doses of the different sets of data under each of the tested bivariate models along with their means and standard deviation

Table No.	Quadratic model				Square root polynomial				Transcendental function				Resistance function			
	Physical optimum		Economic optimum		Physical optimum		Economic optimum		Physical optimum		Economic optimum		Physical optimum		Economic optimum	
	X_1	X_2	X_1	X_2	X_1	X_2	X_1	X_2	X_1	X_2	X_1	X_2	X_1	X_2	X_1	X_2
1.	-	-	-	-	4.42	0.995	5.11	0.629	-	10.40	-	8.95	108.73	8.80	81.92	38.96
2.	132.82	28.90	128.92	35.39	176.80	13.91	162.82	17.68	185.37	13.92	168.28	16.92	33.05	2.79	83.63	40.01
3.	194.86	505.86	140.81	183.25	415.64	570.75	148.45	100.57	140.29	21.77	133.21	-	72.56	-	83.23	39.84
4.	30.47	64.40	30.37	78.51	45.69	24.85	45.76	40.53	45.41	11.36	47.01	18.82	49.22	4.55	83.36	38.60
5.	89.05	48.22	86.13	51.40	85.56	28.98	81.87	33.73	85.46	21.59	82.44	25.92	24.36	5.89	82.23	40.99
6.	91.03	55.21	107.20	81.68	87.31	43.04	141.87	805.90	82.69	32.73	101.46	-	79.60	56.83	79.35	40.29
7.	83.72	36.05	76.13	42.10	74.96	17.82	68.16	23.01	79.99	16.18	68.31	18.62	177.18	767.90	78.16	40.17
8.	107.51	36.86	103.38	46.62	110.06	16.19	103.22	26.11	112.63	18.70	103.34	26.84	544.91	-	81.48	39.59
9.	111.81	1.52	103.76	9.93	108.42	6.73	98.70	7.85	104.28	6.05	93.74	7.22	216.04	347.75	80.96	41.64
10.	111.35	50.71	99.63	52.23	130.52	34.61	108.14	35.70	156.85	9.42	121.81	13.11	127.38	86.91	81.54	40.62
11.	89.40	58.41	85.77	51.89	85.30	51.33	80.46	34.80	80.81	47.41	77.37	30.82	69.26	-	79.62	41.93
12.	177.55	-	137.96	-	128.02	11.26	109.52	9.63	112.34	-	92.23	-	91.88	51.86	81.33	41.76
13.	24.70	-	41.52	5.40	50.12	2.38	54.09	5.81	100.42	-	88.06	123.40	54.77	9.44	78.82	40.15
14.	107.26	22.84	104.56	33.62	114.71	10.64	106.89	16.80	119.04	14.35	109.08	19.95	53.30	8.73	81.77	40.47
15.	76.03	-	74.10	-	81.03	3.05	74.30	5.04	91.95	4.72	81.72	5.94	-	-	79.29	42.36
16.	113.86	67.35	107.56	62.84	120.49	89.81	108.68	64.46	108.56	92.13	100.54	59.21	38.14	3.75	82.85	44.98
17.	107.25	215.04	101.90	182.17	82.51	5.03	79.29	8.83	85.82	-	82.83	-	24.72	5.05	81.07	42.16
18.	0.931	72.21	9.01	64.55	30.85	56.23	32.38	40.49	25.48	38.91	29.81	26.59	62.88	-	81.99	41.78
19.	-	-	34.74	-	51.34	0.005	57.54	3.00	80.81	-	75.50	39.71	58.47	5.61	77.11	41.45
20.	240.36	80.24	210.23	70.63	4422.85	1488.19	3366.74	304.19	-	-	-	140.24	-	-	82.40	41.72
21.	74.94	62.93	73.02	56.66	67.02	74.15	67.82	46.40	72.86	66.99	69.81	36.23	55.20	-	80.54	41.49
22.	68.37	46.80	67.85	41.82	67.86	23.26	66.80	19.44	65.03	16.46	62.90	14.10	25.19	11.28	79.90	39.32
23.	37.83	-	-	-	47.84	0.242	42.07	0.980	39.70	3.99	37.68	7.18	23.05	-	79.15	43.36
24.	33.64	108.93	33.37	98.84	11.45	1317.79	36.04	227.62	55.18	673.46	53.91	48.37	79.77	4.26	76.31	41.24
25.	108.99	100.29	105.35	91.45	0.263	8453.58	194.04	2440.19	94.76	-	91.93	-	-	-	81.68	43.36
26.	1280.33	251.84	1144.23	226.39	7.69	34.99	5.92	17.76	-	-	-	-	304.87	-	82.57	42.83
27.	106.29	54.62	99.86	52.52	112.97	43.65	102.49	37.79	119.98	53.41	106.76	41.52	75.88	-	83.26	45.10
28.	23.82	44.63	35.16	44.27	40.43	28.28	46.03	26.63	43.07	34.96	47.71	30.86	2473.88	1.90	79.96	42.86
29.	138.99	83.01	128.65	75.48	224.44	288.53	163.76	139.43	132.86	688.23	118.27	100.94	36.03	2.98	81.16	43.10
30.	-	43.64	-	44.98	19.62	51.26	29.68	40.42	17.98	63.95	31.84	43.70	67.91	14.70	80.75	43.28
31.	-	35.16	35.39	44.99	14.27	27.11	37.30	33.00	107.64	75.53	50.06	47.34	66.06	17.01	78.69	44.14
32	181.76	147.89	146.77	122.25	18.92	8.04	15.07	10.27	66.58	-	57.49	-	11.97	6.44	76.96	43.53
33.	55.44	90.45	51.71	86.31	208.37	18.22	482.07	473.48	99.25	-	87.54	370.74	385.86	15.60	82.61	43.05
34.	54.82	60.67	61.51	56.25	55.83	64.14	60.02	46.91	55.25	61.81	59.24	41.74	63.03	9.17	81.41	42.17
35.	42.64	98.38	39.90	93.98	228.87	53.76	1072.39	2357.88	89.72	-	80.79	181.90	129.17	14.51	82.50	43.12
36.	48.47	58.47	57.15	54.35	51.72	58.72	57.12	43.27	52.32	57.61	57.16	39.25	65.29	9.11	81.19	42.22
Mean	129.57	87.72	117.08	72.35	210.67	361.71	208.69	209.62	88.19	82.96	80.90	54.69	176.05	58.91	80.85	41.77
SD	213.39	93.16	186.38	47.90	716.49	1405.91	563.30	552.89	36.26	174.28	30.04	72.57	420.78	159.88	1.8638	1.6067

X_1 - Nitrogen in kg/ha X_2 - P_2O_5 in kg/ha

Table 34. Percentage number of cases of estimates on physical and economic optima under each model which lie in the specified ranges of nutrients

Models	Physical optimum dose	Economic optimum dose
Quadratic model	38.89	38.89
Square root polynomial	52.78	58.33
Transendental function	44.44	50.00
Resistance function	36.11	100.00

Summary

SUMMARY

Investigations were made to suggest suitable methods of analysis of data from long term manurial trials with fixed set of treatments utilising the secondary data on grain yield of the permanent manurial experiment on paddy at the Regional Agricultural Research Station, Pattambi. The nature of the relationship between the doses of fertilizers and crop yield was also examined empirically with a view to suggest suitable mathematical models to represent the proposed pattern of relationship.

The statistical techniques evaluated for the analysis of data from long term experiments included the analysis of data as in groups of experiments, analysis of the split plot design, the principal component analysis, stability analysis, non parametric procedures and analysis based on the principle of game theory. In addition to the conventional method of stability analysis proposed by Eberhart and Russell, a non parametric variant of the method proposed by Nassar and Huhn has been also discussed. A new non parametric method of analysis of data of long term trials was also developed. This new method consisted in extending the ordinary Friedman two way analysis of variance

for ranked data to the case of three way classification with years as the additional factor. Empirical comparisons was also made between the newly proposed method and the non parametric procedure for long term trials developed by Rai and Rao.

One of the basic assumptions underlying the analysis of variance technique namely independence of error terms is not satisfied in experiments of repeatative nature and hence the classical method of treating them as special cases of 'groups of experiments' does not seem to be logically sound. Analysis of data from groups of experiments introduces added difficulties in the sense that no general test for overall treatment comparison appear to be available in cases where error variances are heterogeneous and interaction effect is absent. Principal component analysis is expected to obviate these difficulties in the sense that it does not require any underlying statistical model to explain the error structure. The results of analysis of data pertaining to this study revealed that principal component analysis would be atleast as efficient as the other two methods viz. groups of experiments and split plot analysis in detecting the true treatment differences. Therefore the method of principal component analysis could be recommended as a better alternative for the analysis of data on long term trials with a fixed set of treatments.

An empirical comparison between two non parametric methods of data analysis viz. non parametric method proposed by Rai and Rao and extended Friedman's analysis (newly proposed method) was also made. The method proposed by Rai and Rao is based on the assumption that the sampling distribution of the means of the ranks is approximately normal. The method is applicable only for cases when the number of replication per experiment is four or more. The amount of information lost in the process will be more when there are only a few treatments. But the newly developed procedure is entirely distribution free and it utilises none of the usual assumptions required for the analysis of variance. Thus the newly developed extended two way analysis of variance by ranks can be considered asanother viable alternative for the analysis of data of long term trials.

The non parametric analysis of stability proposed by Nassar and Huhn has certain distinct advantages over the method of analysis of stability proposed by Eberhart and Russell) as it is entirely distribution free and can be fitted to any type of data. But treatments cannot ordinarily be recommended on the basis of their stability of the performance alone as high yielding treatments need not be stable. In this study the most high yielding treatment (T1 - cattle manure at 1800 kg/ha to supply 90 kg N/ha) was found to be the least stable. Thus it is more logical to take into account the yield variation also in making recommendation of treatments on the basis of the results of stability analysis.

Analysis based on the principle of game theory is useful to suggest specific recommendations to different types of farmers with varying decision environments. Comparisons among the different decision making criteria viz. Wald's maximin criterion, Laplace's principle of insufficient reason, Hurwicz 'optimism-pessimism' criterion, Savage's regret criterion and Agarwal's excess benefit criterion showed that there was almost perfect agreement in the results obtained through the various criteria.

Kendall's coefficient of concordance was calculated for judging the overall agreement among the selected methods of analyses in detecting the true rank order of treatments. The analyses based on decision theory and stability were excluded from the process of finding concordance due to logical reasons. It was found that there was almost perfect agreement among the different methods with regard to the rank orders of treatments.

Different mathematical functions were used to describe the response pattern of fertilizers on crop yield and their efficiencies compared on the basis of secondary data gathered from various fertilizer trials conducted in Kerala Agricultural University. The univariate models selected for the study consisted of the ordinary quadratic polynomial, square root polynomial, Nelder's polynomial, Inverse polynomial, mixed model, Gupta's function and Holliday function. Two new models were also developed for describing the response pattern for certain types of trivial data.

In the single variable category, each of the observed data were plotted graphically and based on the shape of the graph the data were broadly classified as belonging to one or the other of four mutually exclusive categories.
(1) Parabolic type (2) assymptotic type (3) bimodal type (4) multi model type. Different models were compared based on the values of coefficient of determination (R^2) and average absolute error. It was found that parabolic response pattern could well be represented by a quadratic or square root polynomial response function with slight preference to square root function over the usual quadratic function. The quadratic function is to be preferred in cases where there would be symmetry on either side of the anticipated optimum. In the

case of moderately assymmetric response curves the square root polynomial was found to be more efficient than the ordinary quadratic. The newly proposed model (model-2) was found to be the most efficient in describing the response pattern of an assymptotic nature. In the case of curves showing bimodal tendancy square root polynomial was found to be satisfactory in representing the response pattern. In the fourth category, the newly proposed model (model-1) gave the maximum predictability than all other models.

Among the different models, mixed model, square root polynomial and quadratic model showed relatively high R^2 values. An overall comparison among the different models were made using the mean values of coefficient of determination and average absolute error.square root polynomial model, new model-1, mixed model, quadratic polynomial model and Gupta's function gave better performances than others.

Mixed model, quadratic model, and square root polynomial model had more than 75 percent estimates on physical and economic optima within the specific ranges of nutrients. Thus these models produced estimates on physical and economic optima with greater practical value than the other models. Gupta's function, Holliday function and New model-1 failed to give optimum values for about one third of the data set.

In the bivariate case the different models considered are quadratic function, square root polynomial, transendental function and resistance function. The four response functions were fitted to each of the available data set and their relative efficiencies were compared. Among the tested models, resistance function gave very high R^2 values compared to other models.

All the different models had about half of their estimates on optimum within the stipulated interval. In the case of resistance function all of the estimates of economic optimum were distributed in the range of nutrients covered in the experiment. Resistance function has yielded comparatively higher values of R^2 even with lesser number of parameters. The estimated standard error of the estimates from this model were relatively lesser than those obtained from the other models. The estimates obtained for different sets of data under this model were more realistic and stable. Therefore the resistance function can well be recommended for representing the response pattern and estimating the optimum level of nutrients in multifactor experiments. Although the transendental function was in general less efficient in describing the response pattern than other functions, it was found to be highly efficient in locating the physical and economic optimum. Thus in experiments where the sole objective is to find the optimum dose and the resulting response transendental function can also be used.

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Appendices

Appendix I. Grain yield of paddy corresponding to graded doses of fertilizer in different experiments involving a single nutrient.

Serial number of sets of data (n)

Dose (x)	1	2	3	4	5
0	2930	2930	2930	2930	2930
29	3154	3447	3594	3467	3271
58	3252	3301	3984	3467	3320
87	3369	3545	2496	2588	3057

x/n	6	7	8	9	10
0	3052	3052	3052	3052	3052
58	3549	3735	3935	3906	3638
87	3857	3906	4126	3028	3687
116	3857	4028	4150	4126	3662

x/n	11	12	x/n	13	14	15
0	2689	2227	0	2373	2373	2373
40	3443	2624	29	2783	2686	2734
80	3749	2842	58	2617	2666	2373
120	3918	2889	87	2783	2861	2686

x/n	16	17	18	19	20	21	22	23
45	3136	2692	3303	3254	3264	2366	2445	1913
60	2968	2380	3323	2861	3668	1972	2633	1972
75	3658	3037	2642	2544	3293	2435	2603	1952
90	3382	3007	2841	2268	3500	2287	2327	1725

x/n	24	25	26	27	28	29	30	31
0	2833	2200	2833	2200	2833	2200	2833	2200
27	2996	2400	2986	2350	3103	2650	3040	2700
54	2770	2700	2923	2400	3534	2850	2635	2700
87	2725	2350	3085	2500	2995	2950	2428	3150

x/n	32	33	34
0	1444	1444	1444
30	2148	2062	2086
60	2654	2049	1807
90	2173	2247	2012

x/n	35	36
0	2036	1715
50	2990	2323
100	3342	3178
150	3173	3490

x/n	37	38	39	40	41	42	43	44	45	46	47
0	3500	4064	3500	4064	3500	4064	4500	4064	3607	3607	3607
29	3900	5246	3500	2757	4420	3942	3940	4173	3370	4086	3891
58	3700	4757	3700	5273	3760	4201	3380	4634	3860	4236	4120
87	3900	4513	3610	4718	3500	4824	3640	4281	3759	4113	3998
116	4100	4158	4160	4566	3850	4336	4020	4377	3819	4065	3849

x/n	48	49	50	51
0	2930	2930	2930	2930
58	3646	4265	4265	4199
87	3972	4460	4460	4134
116	4297	4199	4265	4265

x/n	52	53	54	55	56
0	2514	2514	2514	3503	3504
20	2299	2155	2155	3785	3249
40	2227	2270	2299	3672	3743
60	2299	2414	2270	3601	3856

x/n	57
0	3503
40	4054
80	3503
120	3432

x/n	58	59	60	61
0	3781	3313	3781	3313
20	4548	3995	4282	3173
40	4313	3531	4013	3387
60	4173	3433	4563	3461

x/n	62	63
0	3781	3313
40	4438	3589
80	4328	3672
120	4078	3298

x/n	64	65	66	67
0	2613	3037	2613	3037
20	2931	3390	3107	3392
40	3001	3249	2825	3037
60	2860	3008	2995	3002

x/n	68	69
0	2613	3037
40	3037	3178
80	2860	2966
120	2895	3008

x/n	70	71
0	3888	3445
7.5	5142	3980
15	4874	3566
30	4847	3659
45	5061	3459
90	4834	3632

Appendix II. Grain yield of paddy corresponding to graded doses of nitrogen and phosphorous in the long term fertilizer experiment conducted at CRS, Karamana

Dose of N (X1)	Dose of P (X2)	Serial number of sets of data (n)											
		1	2	3	4	5	6	7	8	9	10	11	12
40	0	6044	5350	4322	4594	4767	5267	4022	3750	5672	6028	3294	3517
80	0	6461	6222	5467	4489	6672	5694	4211	4256	6422	6394	4056	4117
120	0	6883	7117	5517	5833	5917	5156	3550	4267	6300	7078	3294	4133
40	40	5811	5244	3933	4694	4928	5444	3422	3356	6128	6206	3611	3994
80	40	6806	6389	5439	4461	5561	4900	3433	4111	6139	6656	4183	4106
120	40	6772	6606	5989	5694	5250	5439	4039	4250	6783	6700	4350	4428
40	80	5350	5350	4206	4006	5422	5428	3933	3644	6578	6628	3778	4322
80	80	5806	6772	5217	5467	5461	5139	4194	4217	7211	6994	4211	4422
120	80	7156	7222	5900	6006	5733	5272	4000	4367	6756	6683	4044	4211

Contd.......

Dose of N (X1)	Dose of P (X2)	Serial number of sets of data (n)											
		13	14	15	16	17	18	19	20	21	22	23	24
40	0	4967	4456	2656	2178	4906	5022	3211	2906	6017	6344	2711	2928
80	0	4778	5283	3017	3133	5989	5822	3572	3161	6844	6356	2944	2933
120	0	4389	5128	2583	2811	5394	6050	2539	3600	6506	6794	2694	2494
40	40	5006	4478	2783	3344	5572	5406	2983	3456	6978	6828	3917	3722
80	40	5033	5083	2806	3583	6111	6044	3506	3639	7200	7022	3144	3356
120	40	5378	4994	3350	3911	6089	6878	3617	4128	6633	6494	2206	2561
40	80	4478	4539	3094	3239	5694	5933	3117	3389	6961	6683	3594	3772
80	80	5628	5017	3650	3772	6644	5700	3289	4139	7111	7267	3328	3817
120	80	5761	5683	3450	4117	6572	6450	3878	4128	6778	5417	3439	2383

Contd......

Dose of N (X1)	Dose of P (X2)	Serial number of sets of data (n)											
		25	26	27	28	29	30	31	32	33	34	35	36
40	0	3944	3861	2278	2844	3739	3650	2806	3206	3606	4061	3472	3933
80	0	5361	4639	3044	2617	4444	4089	3194	2650	4222	4072	4083	4061
120	0	4517	4778	3128	2756	4144	3939	2594	2583	4711	3639	4533	4406
40	40	4444	4711	3433	3683	4383	4522	3539	3378	4900	4844	4750	4689
80	40	5711	4889	3922	3728	5044	4917	3717	3894	4917	5039	4772	4861
120	40	5944	6156	4139	4544	5239	4600	3478	3661	4872	5100	4711	4961
40	80	4944	4778	3683	3194	4622	4589	3583	3794	5006	4828	4922	4656
80	80	5778	5683	3833	3839	4906	4217	3161	4189	5400	4756	5256	4572
120	80	6306	6033	3878	4161	5594	5350	4183	4106	4894	5461	4617	5311

STATISTICAL INVESTIGATIONS ON THE ANALYSIS OF DATA OF LONG TERM MANURIAL TRIALS ON PADDY

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ABSTRACT OF THE THESIS

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ABSTRACT

The suitability of different statistical techniques for the analysis of data of long term fertilizer trials was examined with the help of secondary data gathered from the permanent manurial experiment in paddy at Regional Agricultural Research Station, Pattambi and certain new methods with distinct advantages over the existing methods were suggested for the same. The relative efficiencies of various mathematical functions in representing the yield-fertilizer relationship and in estimating the optimum level of the applied nutrient were also evaluated on the basis of secondary data gathered from the various fertilizer experiments on paddy conducted at the various rice research stations under the KAU during the last ten years. Two new mathematical functions were also developed to represent the response pattern for certain types of trivial data.

The methods evaluated for the analysis of data of long term trials include method of groups of experiments, split plot analysis, principal component analysis, non parametric method proposed by Rai and Rao, stability analysis proposed by Eberhart and Russell, non parametric stability analysis proposed by Nassar and Huhn and analysis based on principle of game theory. A new non parametric method as an extension of Friedman's two way analysis of variance by ranks was also developed for the analysis of such data. This method was found to be almost as powerful as the method proposed by Rai and Rao and hence can be regarded as an improvement over the existing methods as it is free from any stringent assumptions on the nature of the underlying universe. Principal component

analysis was also found to be empirically atleast as efficient as the method of groups of experiments/split plot analysis and can be adjudged to be a better alternative to the solution of the same problem on the grounds of theoretical and statistical validity.

The univariate models used to describe the response pattern of fertilizers on crop yield include quadratic polynomial, square root polynomial, Nelder's polynomial, inverse polynomial, mixed model, Gupta's function and Holliday function. The square root polynomial was found to be better then the ordinary quadratic polynomial in representing the response pattern of a parabolic nature. The newly developed model $y = \frac{ax}{b+c\sqrt{x}+x}$ where y is the response, x is the input and a, b, c are constants, was found to be the most efficient in describing the response pattern of an assymptotic nature. In representing the multimodal response, the new model $y = \beta_0 + \beta_1\sqrt{x} + \beta_2 x^{-\frac{1}{2}}$ where β_0, β_1 and β_2 are constants, gave the maximum predictability than all other models.

The bivariate models selected for the study consisted of quadratic function, square root polynomial, transendental function and resistance function. The resistance function was found to be the most efficient in representing the response surface in multifactor experiments. The estimates of optimum levels obtained through the use of this function was found to be realistic and relatively more stable.