A STUDY OF GENETIC CORRELATIONS UNDER FULL-SIB MATING SYSTEM (TWO LOCI CASE)

By

KHIN MOE MOE

THESIS

submitted in partial fulfilment of the
requirements for the degree of

MASTER OF SCIENCE (AGRICULTURAL STATISTICS)

Faculty of Agriculture
Kerala Agricultural University

Department of Statistics
COLLEGE OF VETERINARY AND ANIMAL SCIENCES
Mannuthy - Trichur

1985

CERTIFICATE

Certified that this thesis entitled "A Study of Genetic Correlations under Full-sib Mating System (two loci case)" is a record of research work done independently by Smt. Khin Moe Moe under my guidance and supervision and that it has not previously formed the basis for the award of any degree, fellowship, or associateship to her.

K. C. George

Dr K.C. GEORGE
(CHAIRMAN, ADVISORY BOARD)
PROFESSOR AND HEAD,
DEPARTMENT OF STATISTICS

Mannuthy,
16-9-1985.

DECLARATION

I hereby declare that this thesis entitled "A Study of Genetic Correlations under Full-sib Mating System (two loci case)" is a bonafide record of research work done by me during the course of research and that the thesis had not previously formed the basis for the award to me of any degree, diploma, associateship, fellowship, or other similar title, of any other University or Society.

Mannuthy,
16-9-1985

KHIN MOE MOE

ACKNOWLEDGEMENTS

It gives me great pleasure to express my deep gratitude and special indebtedness to my supervisor Dr K.C. George, Professor and Head, Department of Statistics, for initiating and introducing me into the fascinating field of statistical genetics and kindling my interest for research work in the area of "genetic correlations". I am highly indebted to him for his constant encouragement, inspiring guidance and advice in the inception and completion of this research work and his constructive criticisms and valuable comments on the whole manuscript which have led to considerable improvements in my thesis.

I am indebted to the members of my Advisory Committee, Dr G. Mukundan, Professor and Head, Department of Genetics and Animal Breeding; Dr (Mrs) Sosamma Iype, Associate Professor, Department of Genetics and Animal Breeding and Sri.K.L. Sunny, Assistant Professor, Department of Statistics, for their valuable advice and assistance rendered throughout my research work.

I am equally indebted to Dr M. Krishnan Nair, Dean, College of Veterinary and Animal Sciences, for providing various facilities and help during the course of my study in India.

I am grateful to the Government of Union of Burma and the Institute of Economics, Rangoon for granting me study leave of two years and six months to undergo M.Sc.(Ag.Stats) courses at Kerala Agricultural University.

I am equally grateful to the Government of India and Ministry of Agriculture for providing me the Colombo-plan Fellowship Award in post graduate studies in Agricultural Statistics under Technical Co-operation Schemes between the two Governments.

I also express my deep gratitude to my colleague, U Myint Swe, for his generous help, encouragement and co-operation during my study and stay of two years and six months in India.

I will be failing in my duty if I do not express my gratitude to the staff members of the Department of Statistics for their co-operation and sincere interest in my studies and research work.

I am grateful to Sri. Temsuwati Ao of Trichur Engineering College for his elegant drawings of the graphs.

Lastly, but not the least, I record my heartfelt gratitude and special indebtedness to my mother, Daw Than Myaing, and my husband, U Win Tint, for their strong support, constant encouragement and interest in shaping and brightening my career life.

KHIN MOE MOE

TO MY BELOVED SON,
HTUN WIN MIN FOR HIS
CONSTANT ENCOURAGEMENT
AND MANY MONTHS EVER LOST

CONTENTS

		Page No.
INTRODUCTION	..	1-4
REVIEW OF LITERATURE	..	5-14
MATERIALS AND METHODS	..	15-18
RESULTS	..	19-84
DISCUSSION	..	85-91
SUMMARY	..	92-97
REFERENCES	..	i-ii
APPENDIX	..	i-ii
ABSTRACT	..	..

INTRODUCTION

CHAPTER I

INTRODUCTION

In quantitative genetic studies resemblance between two relatives is used for estimating genetic parameters, such as heritability and genetic correlation. In such cases it is assumed, that the individuals in the population mate at random. The correlation between common relatives, such as full-sib or parent-offspring thus take a simple value of half. But when regular systems of inbreeding is practised, these correlations are increased. The increase in correlation between the relatives depends on the number of generations, during which the inbreeding is practised. Of special interest in quantitative genetics, is the case of correlation between one relative and a number of individuals as the other relative. For instance, the correlation between one parent and several of its offspring is a major determinant in increasing the response to selection. The behaviour of such correlations in inbred populations is not fully known.

The characters may be correlated because of common genetic factors or common environmental factors or both. It is necessary to distinguish these two causes of correlation between characters-genetic and environmental. The genetic cause of correlation may be chiefly due to three different causes: pleiotropy, linkage and heterozygosity. Pleiotropy is simply the property of a gene whereby it affects two or more characters, so that if the gene is segregating it causes simultaneous variation in the character its effect. For example, genes which increase growth rate increase both stature and weight so that they tend to cause correlation between these two characters. Linkage is usually a minor cause of genetic correlation with transitory effect, as crossing over in a freely interbreeding population tends to make the coupling and repulsion heterozygote equally frequent, and once the linkage equilibrium is established the correlation becomes zero. Thus traces of association from this cause may be detectable only for some generations in populations derived from crosses between divergent strains. Sometimes heterozygosity at loci controlling one character may increase the general vigour. Heterotic effects of these genes may influence the expression of the other trait.

Genetic correlation gives an idea about the extent to which the characters are under the control of the same set of genes or have the same physiological basis for their expressions. If the correlation is high then probably pleiotropy is more important, if the correlation is low then we might say that the traits are inherited independently or they are under the control of different sets of genes.

A knowledge of its magnitude and sign helps in judging how the improvement in one character will cause simultaneous change in the other characters. If the genetic correlation is positive, then the selection practised for the improvement of one character will automatically result in the improvement in the others, even though direct selection for its improvement has not been made. If it is negative, then selection for the improvement of one character, if successful, will result in a decline in the others.

In this investigation, an attempt is made to study (i) genetic correlation between full-sib pairs under full-sib mating system, (ii) genetic correlation between parent-offspring pairs under full-sib mating system, (iii) genetic correlation between full-sib pairs under parent-offspring mating system and (iv) genetic correlation between parent-offspring pairs under parent-offspring mating system in the case of two loci with two alleles at each locus with the following objectives.

i) To derive the joint distribution (correlation table) and to find the correlation between full-sib pairs under full-sib mating system in the case of two loci when there is no linkage as well as when there is complete linkage.

ii) To derive the joint distribution (correlation table) and to find the correlation between parent-offspring pairs under full-sib mating system in the case of two loci when there is no linkage as well as when there is complete linkage.

iii) To derive the joint distribution (correlation table) and to find the correlation between full-sib pairs under parent-offspring mating system in the case of two loci when there is no linkage as well as when there is complete linkage.

iv) To derive the joint distribution (correlation table) and to find the correlation between parent-offspring pairs under parent-offspring mating system in the case of two loci when there is no linkage as well as when there is complete linkage.

REVIEW OF LITERATURE

CHAPTER II

REVIEW OF LITERATURE

Generally, there are two different methods for obtaining the genotypic correlation between two relatives. In one of the methods the frequencies of various combinations of the given relatives in a population are first obtained and then the correlation is calculated from such a "correlation table". The correlation between full-sib pairs under different generations of full-sib mating system have been obtained by Li (1955, Population Genetics, page: 119) by this method. The procedure of obtaining the frequencies of uncle-nephew or first cousin combinations is entirely too tedious even with the help of matrix notations (Hogben, 1933). Thus, when the more complex or irregular inbreeding system is practised or for more than one pair of genes, the algebraic methods become cumbersome.

The other method utilizes the concept of path coefficients developed by Wright (1921). By using this method, the correlation coefficients between relatives under different system of mating can be easily worked out. Eventhough this method is very easy for the calculations of the correlation between relatives, it does not give any information about the frequencies of the various combinations of the relatives in the population and also the joint distribution between the two relatives cannot be obtained by this method.

Fisher (1949) developed a generation matrix theory for working out the frequencies of different types of mating under regular systems of inbreeding. The method is simple and flexible for finding the frequencies of different types of mating. The method was first presented in the literature apparently by Bartlett and Haldane. The progress towards homozygosis for sex-linked character was studied by Haldane (1937, 1955), he gave only a general treatment of the subject. Fisher derived the generation matrix for full-sib mating by considering a single locus with two alleles 'A' and 'a'. However, he considered only the mating types and did not make any distinction between the kinds of mating such as AA x AA and aa x aa.

Kempthorne (1955) calculated the correlation of parent-offspring pairs and full-sib pairs in generations of full-sib mating by making use of the generation matrix. By this method he could derive the joint distribution of the pairs of relatives at any generation of a specified system of mating and thus the correlation is worked out directly from the two-way table of the relatives, known as the "correlation table". By using the generation matrix theory for full-sib mating system, he employed kind of matings rather than types viz., mating types AA x AA was considered different from the kind of mating aa x aa. But he did not make any difference between mating types AA x Aa and Aa x AA as well as Aa x aa and aa x Aa.

Horner (1956) worked out the correlation of parent-offspring pairs and full-sib pairs in generation of parent-offspring mating. Korde (1960) worked out the correlation between relatives for a sex-linked character under full-sib mating by making use of generation matrix theory.

The generation matrix method is based on the primitive concepts of the genotype and the results of Mendelian segregation. The method gives the mating types in an arbitrary generation arising from an arbitrary population by a regular system of inbreeding. If the frequencies of mating types are arranged as a column vector, say, $\underline{f}$ and generations are denoted by subscripts in parenthesis, then $\underline{f}^{(n)} = \underset{\sim}{A}\underline{f}^{(n-1)}$ where $\underset{\sim}{A}$ is the generation matrix. Hence it follows that $\underline{f}^{(n)} = \underset{\sim}{A}^{n}\underline{f}^{(o)}$. This shows that frequencies in the n^{th} generation can be worked out if one knows the matrix A as well as the initial vector $\underline{f}^{(o)}$. By this method the joint distribution of pairs of relatives at any generation of a specified system of mating is obtained and thus the correlation is worked out directly from the two-way table of the relatives, known as the "correlation table".

The use of the stochastic matrices in determining the correlation between relatives introduced by Li and Sacks (1954) under the name I.T.O. method, provides an easy alternative, to the laborious method of preparation of correlation tables for various types of relatives and calculation of correlation coefficients therefrom. The stochastic matrices $\underset{\sim}{I}$, $\underset{\sim}{T}$ and $\underset{\sim}{O}$ are matrices of conditional probabilities. From these three basic matrices, the matrix of conditional probabilities for unileneal relatives or bileneal relatives can be worked out. Using them "the correlation table" as well as the correlation between relatives can be easily worked out.

Eventhough Fisher, Haldane and Li derived generation matrix for full-sib mating with sex-linked genes, it is in fact Korde (1960) and George (1974) who made use of this generation matrix technique in studying the inbreeding systems. George (1974) conducted a detailed study of the parent-offspring and full-sib correlations separately under full-sib mating and parent-offspring mating system, both for autosomal as well as sex-linked genes. Two methods, viz., the I.T.O. method, employing stochastic matrices, as well as generation matrix methodology, have been studied. The I.T.O. method applicable for the case of single locus with two alleles has been generalised to the case of single locus with multiple alleles under random mating. Further, he found that in general, the I.T.O. method is not applicable to inbred populations. However, for autosomal genes and in the case of parent-offspring mating system (mating between a fixed sire and his daughter, grand-daughter, great-grand daughter, etc.), the joint distribution of the parent-offspring relationship could be expressed in terms of T and F (suitably defined) matrices. In the case of sex-linked genes the I.T.O. method was found to be applicable in finding the joint distribution and correlation coefficient for brother-brother and father-son relationships, both for the full-sib as well as parent-offspring mating systems. He also developed a general theory for obtaining the correlation between one parent and k offspring as well as the correlation between both the parents and k offspring under a given system of mating both for autosomal as well as sex-linked genes.

George and Narain (1975) developed a general theory for obtaining the correlation between one parent and k offspring, as well as the correlation between both the parents and k offspring under a given system of mating, both for autosomal as well as sex-linked genes. They evolved parent-offspring correlations under full-sib mating and parent-offspring mating system with the help of this theory and the generation matrices for different mating types in case of autosomal as well as sex-linked genes.

In the light of various correlation coefficients obtained under continued full-sib mating or parent-offspring mating, with autosomal or sex-linked genes, it has been found that the correlation increases with increase in the generations of inbreeding, but the mode of increase is different under different systems and depends on whether the case is of autosomal or sex-linked genes. It is also found that in case of autosomal, the increase in correlation is more under full-sib mating than under parent-offspring mating.

George (1979) conducted the study of parent-offspring correlation under half-sib mating system. He evolved the correlation between both the parents and k offspring and between one parent and k offspring in the lines of George and Narain (1975). He performed the tables of correlations of the above two cases when the number of offspring varying between 1 to 10. He has reported that the correlation increases as the number of offspring increases, but the rate of increase of the correlations were more in the case of both the parent and k offspring than that of one parent and k offspring case and the rate of increase is almost nil after the second generation of full-sib mating in the one parent case.

George (1983a) developed the calculation of joint distribution of full-sib pairs and parent-offspring pairs under full-sib and parent-offspring mating systems by the generation matrix technique and also calculated the correlation of these pairs therefrom. He found that the method is tedious in the case of full-sib mating, but it is comparatively easier in the case of parent-offspring mating, as the conditional probability matrix in this case can be easily generated. He calculated the correlation between full-sib pairs and parent-offspring pairs under full-sib and parent-offspring mating systems for ten generations of continued mating of that particular system of mating. Further, he reported that the correlation increases as the number of generation increases and ultimately reaches the limit unity when the number of generation increases indefinitely large. He observed that the parent-offspring correlation is higher in magnitude than that of the full-sib correlation under both the systems of mating. It is also observed that the parent-offspring correlation under parent-offspring mating increases at a rapid rate than all the other three types of correlations even at the first generation of parent-offspring correlations and become almost unity at the tenth generation.

George (1983b) conducted the study of correlation between various full-sib pairs and parent-offspring pairs by evolving joint distribution of relative pairs in case of sex-linked genes. He noted that sister-sister pair correlation is maximum at every generation of full-sib mating followed by mother-son and father-daughter correlation. He also reported that the correlation between mother-son pair and father-daughter pair are identical. He mentioned the important point, that father and sons are uncorrelated under random mating, but as the inbreeding starts the pairs become correlated and the amount of correlation increases as the number of generations of inbreeding increases. He also mentioned that the correlation increases at a rapid rate in the case of mother-daughter pairs than that of brother-brother pairs. Further, he reported that the correlation coefficients of all these types of pairs will be tending to unity as the number of generation of inbreeding increases indefinitely.

All these authors have studied correlation between different relatives under different inbred system only in case of single locus with two alleles. Li, in his book entitled 'Population Genetics (1955)' have established a general formula to determine the correlation coefficient between different relative pairs such as full-sibs, parent and offspring, half-sibs in different generations of the specified mating system, from the corresponding inbreeding coefficient. It is given for full-sib pairs as,

$$m = \frac{1 + 2F' + F''}{2(1+F')} \quad - (1)$$

and for parent-offspring pairs

$$m = \frac{1+2F'}{2\sqrt{2}\,(1+F')} \quad - (2)$$

where

F = inbreeding coefficient in the n^{th} generation

F' = inbreeding coefficient in the $(n-1)^{th}$ generation

F'' = inbreeding coefficient in the $(n-2)^{th}$ generation

and m = correlation coefficient in the n^{th} generation.

The inbreeding coefficient in the different generations of full-sib mating can be obtained using the recurrence relation

$$F = \frac{1}{4}(1+2F^{I}+F^{II}) \quad - (3)$$

and the inbreeding coefficient in the different generations of parent-offspring mating (a fixed sire and his daughter, grand-daughter, great-grand daughter, etc.) can be obtained using the recurrence relation

$$F = \frac{1}{4}(1+2F^{I}) \quad - (4)$$

established by Li, by using path method.

Thus by using the relationships (1), (2) and (3), (4) Li could obtain the correlations between full-sib pairs and parent-offspring pairs in different generations of full-sib mating and parent-offspring mating, directly. As for the numerical values of the correlation coefficient the method is quite simple, but it does not give any information about the absolute frequencies of the full-sib mating types, the parent-offspring mating types and their joint distribution in the different generation of full-sib mating and parent-offspring mating.

In the present investigation an attempt is made to extend the study of genetic correlations under full-sib mating system as well as parent-offspring mating system to two loci case with two alleles at each locus, when there is no linkage. An attempt is also made to extend this theory in case of two loci when there is complete linkage.

MATERIALS AND METHODS

CHAPTER III

MATERIALS AND METHODS

Since the present investigation is of purely theoretical nature, the materials in the sense of numerical data is not required; as such materials are not discussed here and only methods of approach to the problem are discussed in some detail.

In this investigation, an attempt has been made, employing the method of generation matrix, to develop the following correlations.

i) Correlation of full-sib pairs under full-sib mating system.

ii) Correlation of parent-offspring pairs under full-sib mating system.

iii) Correlation of full-sib pairs under parent-offspring mating system.

iv) Correlation of parent-offspring pairs under parent-offspring mating system.

In considering the full-sib mating system, 45 genotypic mating types are brought out from ten classes of phenotypic mating types, from which a generation matrix $\underset{\sim}{A}$ of dimension (45 x 45) is obtained.

In considering the parent-offspring mating system, such as the mating of fixed sire with daughter, grand-daughter, great-grand daughter, etc., 81 genotypic mating types are brought out from ten classes of phenotypic mating types. There is a problem that some parent-offspring pairs cannot be brought out from any of 81 genotypic mating type in any generation. This problem is solved by developing a generation matrix in which the columns of the unavailable parent-offspring pairs having zero elements are eliminated. Therefore, the dimension of generation matrix $\underset{\sim}{A}^*$ for the parent-offspring mating type reduced to (81 x 49), instead of original dimension of (81 x 81).

Denoting the vector of frequencies for n^{th} generation under full-sib mating system by $\underline{U}^{(n)}$, the vector of frequencies for the successive generations under the system of full-sib mating is computed by the following recurrence relation given by

$$\underline{U}^{(n)} = \underset{\sim}{A}\, \underline{U}^{(n-1)} \qquad (3.1)$$

From this recurrence relation, correlation tables for full-sib pairs for any generation under full-sib mating system can be easily worked out. The correlation tables for parent-offspring pairs under any generation of full-sib mating system can also be developed from the recurrence relation

$$\underline{U}^{\prime(n)*} = \underset{\sim}{A}^*\, \underline{U}^{(n-1)} \qquad (3.2)$$

Denoting the vector of frequencies for n^{th} generation under parent-offspring mating system by $\underset{\sim}{u}^{(n)*}$, the vector of frequencies for full-sib pairs for n^{th} generation under parent-offspring mating system can be obtained from the recurrence relation given by

$$\underset{\sim}{u}'^{(n)} = \underset{\sim}{A}\,\underset{\sim}{u}^{(n-1)*} \qquad (3.3)$$

From the above relation, correlation tables for full-sib pairs under parent-offspring mating system can be worked out easily. Similarly, the correlation tables for parent-offspring pairs under parent-offspring mating system can also be directly worked out from the following relation, given by

$$\underset{\sim}{u}^{(n)*} = \underset{\sim}{A}^{*}\underset{\sim}{u}^{(n-1)*} \qquad (3.4)$$

Denoting, in the above correlation tables, sib I by x and sib II by y in the case of full-sib pairs as well as parent by x and offspring by y in the case of parent-offspring pairs and scoring and ordering the genotypes according to their number of dominant genes involved (for example, assigning 4 to AABB, 3 to AABb and AaBB, 2 to AaBb, AAbb and aaBB, 1 to Aabb and aaBb and 0 to aabb), assuming additive genic effect, the simple cross-product correlation is computed with the following equation

$$r = \frac{\Sigma fxy - (\Sigma fx)\,(\Sigma fy)}{\sqrt{\{\Sigma fx^2 - (\Sigma fx)^2\}\{\Sigma fy^2 - (\Sigma fy)^2\}}} \qquad (3.5)$$

Some salient features of method of generation matrix which is used in working out the vector of the frequencies of different mating types for successive generations is briefly discussed as follows:

1. The generation matrix method is based on the primitive concepts of the genotype and the results of Mendelian segregation, and also ignores mutation.

2. The generation matrix method gives the mating types (or whatever else is considered) in an arbitrary generation arising from an arbitrary population by a regular system of inbreeding. We find in fact that, if the frequencies of mating types are arranged as a column matrix $\underset{\sim}{f}$ say, and generations are denoted by superscripts in parentheses, then

$$\underset{\sim}{f}^{(n)} = \underset{\sim}{A}\,\underset{\sim}{f}^{(n-1)}$$

where A is the generation matrix. A typical equation in the totality represented by the one-matrix equation is

$$f_j^{(n)} = a_{j1}f_1^{(n-1)} + a_{j2}f_2^{(n-1)} + \text{------} + a_{jm}f_m^{(n-1)}$$

where $f_r^{(n-1)}$ is the frequency of the r^{th} mating type in generation (n-1).

RESULTS

CHAPTER IV

RESULTS

4.1. Full-sib mating

Full-sibs are those individuals whose parents are in common.

If the two loci case is considered, it has to be taken account of the fact that those loci may be linked. Hence the statistical and probabilistic treatment of the problems may be dealt with by using the two rules.

Consider the case of two loci each with two alleles say (A,a) and (B,b) at each locus.

i) An individual of type AB/ab produces gametes in the following proportions or with the following probabilities:

AB	Ab	aB	ab
$(1-p)/2$	$p/2$	$p/2$	$(1-p)/2$

ii) An individual of type Ab/aB produces gametes in the following proportions:

AB	Ab	aB	ab
$p/2$	$(1-p)/2$	$(1-p)/2$	$p/2$

'p', called the recombination fraction, is the proportion of cross-over gametes. There are on the whole nine genotypes with two loci with two alleles per locus, but because of differences in gamete production between the two types of double heterozygotes, there are in fact ten classes of individuals, with regards to breeding behaviour. To facilitate the presentation, the genotypes shall be specified in usual way, viz., the heterozygotes in the coupling phase by AB/ab and the heterozygotes in the repulsion phase by Ab/aB.

The matrix of genotypes along with their proportions (in coupling phase) under random mating is shown in table 4.1.

Table 4.1. Genotype matrix

♀ \ ♂	AB $(1-p)/2$	Ab $p/2$	aB $p/2$	ab $(1-p)/2$
AB $(1-p)/2$	AABB $(1-p^2/4$	AABb $p(1-p)/4$	AaBB $p(1-p)/4$	AaBb $(1-p)^2/4$
Ab $p/2$	AABb $p(1-p)/4$	AAbb $p/4$	AaBb $p^2/4$	Aabb $p(1-p)/4$
aB $p/2$	AaBB $p(1-p)/4$	AaBb $p^2/4$	aaBB $p^2/4$	aaBb $p(1-p)/4$
ab $(1-p)/2$	AaBb $(1-p)^2/4$	Aabb $p(1-p)/4$	aaBb $p(1-p)/4$	aabb $(1-p)^2/4$

From this table, the frequencies of nine genotypes can be obtained as follows:

AABB	$(1-p)^2/4$	AaBb	$(2p^2-2p+1)/2$	aaBB	$p^2/4$
AABb	$p(1-p)/2$	AAbb	$p^2/4$	aaBb	$p(1-p)/2$
AaBB	$p(1-p)/2$	Aabb	$p(1-p)/2$	aabb	$(1-p)^2/4$

In the case of repulsion phase, the frequencies of genotypes of AABB, AAbb, aaBB and aabb will change as $p^2/4$, $(1-p)^2/4$, $(1-p)^2/4$ and $p^2/4$ respectively and others remain unchanged. But for finding the genetic correlations there will be no difference in coupling phase and repulsion phase. Hence the frequencies of genotype in coupling phase are used here.

Now the ten classes of phenotypic mating and the various genotypic mating under each class can be considered as follows:

I. <u>AB x AB</u>

AB can be of the genotypes: AABB, AABb, AaBB and AaBb. Consider the mating type (i) AABB x AABB. Each AABB produces AB gamete with proportion 1. Thus their offspring will be of genotype AABB with proportion 1. This can be shown as follows:

$$
\text{i)}\quad
\begin{array}{ccccc}
\text{AABB} & & \text{x} & & \text{AABB} \\
\downarrow & & & & \downarrow \\
\text{AB} & & & & \text{AB} \\
1 & & & & 1 \\
& \searrow & & \swarrow & \\
& & \text{AABB} & & \\
& & 1 & &
\end{array}
$$

Thus full-sib pair producedin this case will be (AABB,AABB) with proportion 1.

Then consider the mating type (ii) AABB x AABb. AABB produces gamete AB with proportion 1 and AABb produces gametes AB and Ab with proportion 1/2 each. Hence their offsprings are of genotypes AABB and AABb with proportion 1/2 each. This can be shown as follows:

ii) AABB x AABb

AB   AB , Ab

1   1/2   1/2

AABB   AABb

1/2   1/2

Thus full-sib pairs are (AABB,AABB) (AABB,AABb) and (AABb,AABb) with proportion 1/4, 1/2 and 1/4 respectively.

In the same way, the other genotypic mating types can be obtained as follows:

iii) AABB x AaBB

AB   AB , aB

1   1/2   1/2

AABB   AaBB

1/2   1/2

Thus full-sib pairs along with their proportions are as follows:

(AABB,AABB)	(AABB,AaBB)	(AaBB,AaBB)
1/4	1/2	1/4

iv) AABB x AaBb

AB   AB, Ab, aB, ab

1   1/4 1/4 1/4 1/4

	AB(1/4)	Ab(1/4)	aB(1/4)	ab(1/4)
AB	AABB	AABb	AaBB	AaBb
1	1/4	1/4	1/4	1/4

Thus full-sib pairs along with their proportions are as follows:

(AABB,AABB)	(AABB,AABb)	(AABB,AaBB)	(AABB,AaBb)	(AABb,AABb)
1/16	1/8	1/8	1/8	1/16
(AABb,AaBB)	(AABb,AaBb)	(AaBB,AaBB)	(AaBB,AaBb)	(AaBb,AaBb)
1/8	1/8	1/16	1/8	1/16

v) AABb x AABb

AB , Ab   AB , Ab

1/2   1/2   1/2   1/2

	AB(1/2)	Ab(1/2)
AB	AABB	AABb
1/2	1/4	1/4
Ab	AABb	AAbb
1/2	1/4	1/4

Offsprings along with their proportions are AABB(1/4), AABb(1/2) and AAbb(1/4).

Thus full-sib pairs along with their proportions are as follows:

(AABB,AABB)	(AABB,AABb)	(AABB,AAbb)	(AABb,AABb)	(AABb,AAbb)
$1/16$	$1/4$	$1/8$	$1/4$	$1/4$
(AAbb,AAbb)				
$1/16$				

vi) AABb x AaBB

AB , Ab   AB , aB
$1/2$ $1/2$ $1/2$ $1/2$

	AB($1/2$)	Ab($1/2$)
AB ($1/2$)	AABB $1/4$	AABb $1/4$
aB ($1/2$)	AaBB $1/4$	AaBb $1/4$

Offsprings are AABB, AABb, AaBB and AaBb with proportion $1/4$ each.

Thus full-sib pairs along with their proportions are as follows:

(AABB,AABB)	(AABB,AABb)	(AABB,AaBB)	(AABB,AaBb)	(AABb,AABb)
$1/16$	$1/8$	$1/8$	$1/8$	$1/16$
(AABb,AaBB)	(AABb,AaBb)	(AaBB,AaBB)	(AaBB,AaBb)	(AaBb,AaBb)
$1/8$	$1/8$	$1/16$	$1/8$	$1/16$

vii) AABb x AaBb

AB , Ab   AB, Ab, aB, ab
$1/2$ $1/2$ $1/4$ $1/4$ $1/4$ $1/4$

	AB(1/4)	Ab(1/4)	aB(1/4)	ab(1/4)
AB 1/2	AABB 1/8	AABb 1/8	AaBB 1/8	AaBb 1/8
Ab 1/2	AABb 1/8	AAbb 1/8	AaBb 1/8	Aabb 1/8

Offsprings along with their proportions are as follows:

AABB	AABb	AaBB	AaBb	AAbb	Aabb
1/8	1/4	1/8	1/4	1/8	1/8

Thus full-sib pairs along with their proportions are as follows:

(AABB,AABB) 1/64	(AABB,AABb) 1/16	(AABB,AaBB) 1/32	(AABB,AaBb) 1/16	(AABB,AAbb) 1/32
(AABB,Aabb) 1/32	(AABb,AABb) 1/16	(AABb,AaBB) 1/16	(AABb,AaBb) 1/8	(AABb,AAbb) 1/16
(AABb,Aabb) 1/16	(AaBB,AaBB) 1/64	(AaBB,AaBb) 1/16	(AaBB,AAbb) 1/32	(AaBB,Aabb) 1/32
(AaBb,AaBb) 1/16	(AaBb,AAbb) 1/16	(AaBb,Aabb) 1/16	(AAbb,AAbb) 1/64	(AAbb,Aabb) 1/32
(Aabb,Aabb) 1/64				

viii) AaBB x AaBB

$$\begin{array}{ccc} AaBB & \times & AaBB \\ \downarrow & & \downarrow \\ AB\;,\;aB & & AB\;,\;aB \\ 1/2\quad 1/2 & & 1/2\quad 1/2 \end{array}$$

	AB(1/2)	aB(1/2)
AB(1/2) 1/4	AABB 1/4	AaBB 1/4
aB(1/2)	AaBB 1/4	aaBB 1/4

Offsprings are AABB, AaBB and aaBB with proportions 1/4, 1/2 and 1/4 respectively.

Thus full-sib pairs along with their proportions are as follows:

(AABB,AABB)	(AABB,AaBB)	(AABB,aaBB)	(AaBB,AaBB)	(AaBB,aaBB)
1/16	1/4	1/8	1/4	1/4
(aaBB,aaBB)				
1/16				

ix) AaBB × AaBb

AaBB → AB , aB (1/2, 1/2)

AaBb → AB, Ab, aB, ab (1/4, 1/4, 1/4, 1/4)

	AB(1/4)	Ab(1/4)	aB(1/4)	ab(1/4)
AB(1/2)	AABB 1/8	AABb 1/8	AaBB 1/8	AaBb 1/8
aB(1/2)	AaBB 1/8	AaBb 1/8	aaBB 1/8	aaBb 1/8

Offsprings along with their proportions are as follows:

AABB,	AABb,	AaBB,	AaBb,	aaBB,	aaBb
1/8	1/8	1/4	1/4	1/8	1/8

Thus full-sib pairs along with their proportions are as follows:

(AABB,AABB)	(AABB,AABb)	(AABB,AaBB)	(AABB,AaBb)	(AABB,aaBB)
1/64	1/32	1/16	1/16	1/32
(AABB,aaBb)	(AABb,AABb)	(AABb,AaBB)	(AABb,AaBb)	(AABb,aaBB)
1/32	1/64	1/16	1/16	1/32
(AABb,aaBb)	(AaBB,AaBB)	(AaBB,AaBb)	(AaBB,aaBB)	(AaBB,aaBb)
1/32	1/16	1/8	1/16	1/16

(AaBb,AaBb)	(AaBb,aaBB)	(AaBb,aaBb)	(aaBB,aaBB)	(aaBB,aaBb)
1/16	1/16	1/16	1/64	1/32
(aaBb,aaBb)				
1/64				

x) AaBb x AaBb

AB, Ab, aB, ab   AB, Ab, aB, ab

1/4 1/4 1/4 1/4   1/4 1/4 1/4 1/4

	AB(1/4)	Ab(1/4)	aB(1/4)	ab(1/4)
AB(1/4)	AABB 1/16	AABb 1/16	AaBB 1/16	AaBb 1/16
Ab(1/4)	AABb 1/16	AAbb 1/16	AaBb 1/16	Aabb 1/16
aB(1/4)	AaBB 1/16	AaBb 1/16	aaBB 1/16	aaBb 1/16
ab(1/4)	AaBb 1/16	Aabb 1/16	aaBb 1/16	aabb 1/16

Offsprings along with their proportions are as follows:

AABB,	AABb,	AaBB,	AaBb,	AAbb,	Aabb,	aaBB,	aaBb,	aabb
1/16	1/8	1/8	1/4	1/16	1/8	1/16	1/8	1/16

Thus full-sib pairs along with their proportions are as follows:

(AABB,AABB)	(AABB,AABb)	(AABB,AaBB)	(AABB,AaBb)	(AABB,AAbb)
1/256	1/64	1/64	1/32	1/128
(AABB,Aabb)	(AABB,aaBB)	(AABB,aaBb)	(AABB,aabb)	(AABb,AABb)
1/64	1/128	1/64	1/128	1/64
(AABb,AaBB)	(AABb,AaBb)	(AABb,AAbb)	(AABb,Aabb)	(AABb,aaBB)
1/32	1/16	1/64	1/32	1/64

(AABb,aaBb)	(AABb,aabb)	(AaBB,AaBB)	(AaBB,AaBb)	(AaBB,AAbb)
1/32	1/64	1/64	1/16	1/64
(AaBB,Aabb)	(AaBB,aaBB)	(AaBB,aaBb)	(AaBB,aabb)	(AaBb,AaBb)
1/32	1/64	1/32	1/64	1/16
(AaBb,AAbb)	(AaBb,Aabb)	(AaBb,aaBB)	(AaBb,aaBb)	(AaBb,aabb)
1/32	1/16	1/32	1/16	1/32
(AAbb,AAbb)	(AAbb,Aabb)	(AAbb,aaBB)	(AAbb,aaBb)	(AAbb,aabb)
1/256	1/64	1/128	1/64	1/128
(Aabb,Aabb)	(Aabb,aaBB)	(Aabb,aaBb)	(Aabb,aabb)	(aaBB,aaBB)
1/64	1/64	1/32	1/64	1/256
(aaBB,aaBb)	(aaBB,aabb)	(aaBb,aaBb)	(aaBb,aabb)	(aabb,aabb)
1/64	1/128	1/64	1/64	1/256

In a similar manner, the other genotypic mating types under the nine classes of phenotypic mating can be obtained as follows:

II. <u>AB x Ab</u>

i) AABB x AAbb

Full-sib pair along with its proportion is (AABb,AABb).
1

ii) AABB x Aabb

Full-sib pairs	(AABb,AABb)	(AABb,AaBb)	(AaBb,AaBb)
Proportions	1/4	1/2	1/4

iii) AABb x AAbb

Full-sib pairs	(AABb,AABb)	(AABb,AAbb)	(AAbb,AAbb)
Proportions	1/4	1/2	1/4

iv) AABb x Aabb

Full-sib pairs	(AABb,AABb)	(AABb,AAbb)	(AABb,AaBb)
Proportions	1/16	1/8	1/8

(AABb,Aabb) (AAbb,AAbb) (AAbb,AaBb) (AAbb,Aabb)
1/8 1/16 1/8 1/8

(AaBb,AaBb) (AaBb,Aabb) (Aabb,Aabb)
1/16 1/8 1/16

v) AaBB x AAbb

Full-sib pairs	(AABb,AABb)	(AABb,AaBb)	(AaBb,AaBb)
Proportions	1/4	1/2	1/4

vi) AaBB x Aabb

Full-sib pairs	(AABb,AABb)	(AABb,AaBb)	(AAB,aaBb)
Proportions	1/16	1/4	1/8

(AaBb,AaBb) (AaBb,aaBb) (aaBb,aaBb)
1/4 1/4 1/16

vii) AaBb x AAbb

Full-sib pairs	(AABb,AABb)	(AABb,AAbb)	(AABb,AaBb)
Proportions	1/16	1/8	1/8

(AABb,Aabb) (AAbb,Aabb) (AAbb,AaBb) (AAbb,Aabb)
1/8 1/16 1/8 1/8

(AaBb,AaBb) (AaBb,Aabb) (Aabb,Aabb)
1/16 1/8 1/16

viii) AaBb x Aabb

Full-sib pairs	(AABb,AABb)	(AABb,AaBb)	(AABb,AAbb)
Proportions	1/64	1/16	1/32

(AABb,Aabb)	(AABb,aaBb)	(AABb,aabb)	(AaBb,AaBb)	(AaBb,AAbb)
1/16	1/32	1/32	1/16	1/16
(AaBb,Aabb)	(AaBb,aaBb)	(AaBb,aabb)	(AAbb,AAbb)	(AAbb,Aabb)
1/8	1/16	1/16	1/64	1/16

(AAbb,aaBb)	(AAbb,aabb)	(Aabb,Aabb)	(Aabb,aaBb)	(Aabb, aabb)
1/32	1/32	1/16	1/16	1/16

(aaBb,aaBb)	(aaBb,aabb)	(aabb,aabb)
1/64	1/32	1/64

III. AB x aB

i) AABB x aaBB

Full-sib pair	(AaBB,AaBB)
Proportion	1

ii) AABB x aaBb

Full-sib pairs	(AaBB,AaBB)	(AaBB,AaBb)	(AaBb,AaBb)
Proportions	1/4	1/2	1/4

iii) AABb x aaBB

Full-sib pairs	(AaBB,AaBB)	(AaBB,AaBb)	(AaBb,AaBb)
Proportions	1/4	1/2	1/4

iv) AABb x aaBb

Full-sib pairs	(AaBB,AaBB)	(AaBB,AaBb)	(AaBB,Aabb)
Proportions	1/16	1/4	1/8

(AaBb,AaBb)	(AaBb,Aabb)	(Aabb,Aabb)
1/4	1/4	1/16

v) AaBB x aaBB

Full-sib pairs	(AaBB,AaBB)	(AaBB,aaBB)	(aaBB,aaBB)
Proportions	1/4	1/2	1/4

vi) AaBB x aaBb

Full-sib pairs	(AaBB,AaBB)	(AaBB,AaBb)	(AaBB,aaBB)
Proportions	1/16	1/8	1/8

(AaBB,aaBb)	(AaBb,AaBb)	(AaBb,aaBB)	(AaBb,aaBb)
1/8	1/16	1/8	1/8

(aaBB,aaBB) (aaBB,aaBb) (aaBb,aaBb)
1/16 1/8 1/16

vii) AaBb x aaBB

Full-sib pairs	(AaBB,AaBB)	(AaBB,AaBb)	(AaBB,aaBB)
Proportions	1/16	1/8	1/8

(AaBB,aaBb)	(AaBb,AaBb)	(AaBb,aaBB)	(AaBb,aaBb)
1/8	1/16	1/8	1/8

(aaBB,aaBB)	(aaBB,aaBb)	(aaBb,aaBb)
1/16	1/8	1/16

viii) AaBb x aaBb

Full-sib pairs	(AaBB,AaBB)	(AaBB,AaBb)	(AaBB,Aabb)
Proportions	1/64	1/16	1/32

(AaBB,aaBB)	(AaBB,aaBb)	(AaBB,aabb)	(AaBb,AaBb)	(AaBb,Aabb)
1/32	1/16	1/32	1/16	1/16
(AaBb,aaBB)	(AaBb,aaBb)	(AaBb,aabb)	(Aabb,Aabb)	(Aabb,aaBB)
1/16	1/8	1/16	1/64	1/32
(Aabb,aaBb)	(Aabb,aabb)	(aaBB,aaBB)	(aaBB,aaBb)	(aaBB,aabb)
1/16	1/32	1/64	1/16	1/32
(aaBb,aaBb)	(aaBb,aabb)	(aabb,aabb)		
1/16	1/16	1/64		

IV. <u>AB x ab</u>

i) AABB x aabb

Full-sib pair (AaBb,AaBb)

Proportion 1

ii) AABb x aabb

Full-sib pairs	(AaBb,AaBb)	(AaBb,Aabb)	(Aabb,Aabb)
Proportions	1/4	1/2	1/4

iii) AaBB x aabb

Full-sib pairs	(AaBb,AaBb)	(AaBb,aaBb)	(aaBb,aaBb)
Proportions	1/4	1/2	1/4

iv) AaBb x aabb

Full-sib pairs	(AaBb,AaBb)	(AaBb,Aabb)	(AaBb,aaBb)
Proportions	1/16	1/8	1/8

(AaBb,aabb)	(Aabb,Aabb)	(Aabb,aaBb)	(Aabb,aabb)
1/8	1/16	1/8	1/8

(aaBb,aaBb)	(aaBb,aabb)	(aabb,aabb)
1/16	1/8	1/16

V. <u>Ab x aB</u>

i) AAbb x aaBB

Full-sib pair (AaBb,AaBb)

Proportion 1

ii) AAbb x aaBb

Full-sib pairs	(AaBb,AaBb)	(AaBb,Aabb)	(Aabb,Aabb)
Proportions	1/4	1/2	1/4

iii) Aabb x aaBB

Full-sib pairs	(AaBb,AaBb)	(AaBb,aaBb)	(aaBb,aaBb)
Proportions	1/4	1/2	1/4

iv) Aabb x aaBb

Full-sib pairs	(AaBb,AaBb)	(AaBb,Aabb)	(AaBb,aaBb)
Proportions	1/16	1/8	1/8

(AaBb,aabb)	(Aabb,Aabb)	(Aabb,aaBb)	(Aabb,aabb)
1/8	1/16	1/8	1/8

(aaBb,aaBb)	(aaBb,aabb)	(aabb,aabb)
1/16	1/8	1/16

VI. Ab x Ab

i) AAbb x AAbb

Full-sib pair	(AAbb,AAbb)
Proportion	1

ii) AAbb x Aabb

Full-sib pairs	(AAbb,AAbb)	(AAbb,Aabb)	(Aabb,Aabb)
Proportions	1/4	1/2	1/4

iii) Aabb x Aabb

Full-sib pairs	(AAbb,AAbb)	(AAbb,Aabb)	(AAbb,aabb)
Proportions	1/16	1/4	1/8

(Aabb,Aabb)	(Aabb,aabb)	(aabb,aabb)
1/4	1/4	1/16

VII. Ab x ab

i) AAbb x aabb

Full-sib pair	(Aabb,Aabb)
Proportion	1

ii) Aabb x aabb

Full-sib pairs	(Aabb,Aabb)	(Aabb,aabb)	(aabb,aabb)
Proportions	1/4	1/2	1/4

VIII. aB x aB

i) aaBB x aaBB

Full-sib pair	(aaBB,aaBB)
Proportion	1

ii) aaBB x aaBb

Full-sib pairs	(aaBB,aaBB)	(aaBB,aaBb)	(aaBb,aaBb)
Proportions	1/4	1/2	1/4

iii) aaBb x aaBb

Full-sib pairs (aaBB,aaBB) (aaBB,aaBb) (aaBB,aabb)
Proportions 1/16 1/4 1/8

(aaBb,aaBb) (aaBb,aabb) (aabb,aabb)
1/4 1/4 1/16

IX. aB x ab

i) aaBB x aabb

Full-sib pair (aaBb,aaBb)
Proportion 1

ii) aaBb x aabb

Full-sib pairs (aaBb,aaBb) (aaBb,aabb) (aabb,aabb)
Proportions 1/4 1/2 1/4

X. ab x ab

i) aabb x aabb

Full-sib pair (aabb,aabb)

Proportion 1

In the mating type mentioned above, the reciprocal crosses as well as reciprocal full-sib pairs are not separately considered, because in the case of full-sib mating the frequencies for full-sib pairs of direct crosses and reciprocal crosses are the same in all generations. By taking only direct crosses and multiplying these frequencies by two and the equilibrium proportion is obtained. Thus fortyfive mating types as well as fortyfive full-sib pairs are obtained.

From the above full-sib mating type, the generation matrix $\underset{\sim}{A}$ for full-sib mating can be obtained.

Denoting the vector of frequencies for the n^{th} generation as $\underset{\sim}{U}^{(n)}$ the recurrence relation for the vector of frequencies is given by

$$\underset{\sim}{U}^{(n)} = \underset{\sim}{A}\, \underset{\sim}{U}^{(n-1)} \qquad (4.1.1)$$

where $\underset{\sim}{A}'$ is given in appendix I.

4.1.1. Full-sib correlation

Consider the case of two loci each with two alleles say (A,a) and (B,b) with proportions for AB as $(1-p)/2$, for Ab as $p/2$, for aB as $p/2$ and for ab as $(1-p)/2$.

$\underset{\sim}{U}^{(0)}$, the vector of the frequencies of the fortyfive mating types under random mating from equilibrium population would be,

$$U^{(0)} = \begin{bmatrix} (1-p)^4/16 \\ p(1-p)^3/4 \\ p(1-p)^3/4 \\ \vdots \\ (1-p)^2(2p^2-2p+1)/4 \\ p^2(1-p)^2/4 \\ p^2(1-p)^2/2 \\ p(1-p)(2p^2-2p+1)/2 \\ p^2(1-p)^2/4 \\ p(1-p)(2p^2-2p+1)/2 \\ (2p^2-2p+1)^2/4 \end{bmatrix}$$

$p^2(1-p)^2/8$
$p(1-p)^3/4$
$p^3(1-p)/4$
$p^2(1-p)^2/2$
$p^3(1-p)/4$
$p^2(1-p)^2/2$
$p^2(2p^2-2p+1)/4$
$p(1-p)(2p^2-2p+1)/2$
$p^2(1-p)^2/8$
$p(1-p)^3/4$
$p^3(1-p)/4$
$p^2(1-p)^2/2$
$p^3(1-p)/4$
$p^2(1-p)^2/2$
$p^2(2p^2-2p+1)/4$
$p(1-p)(2p^2-2p+1)/2$
$(1-p)^4/8$
$p(1-p)^3/4$
$p(1-p)^3/4$
$(1-p)^2(2p^2-2p+1)/4$
$p^4/8$
$p^3(1-p)/4$
$p^3(1-p)/4$
$p^2(1-p)^2/2$
$p^4/16$
$p^3(1-p)/4$

$$\begin{bmatrix} p^2(1-p)^2/4 \\ p^2(1-p)^2/8 \\ p(1-p)^3/4 \\ p^4/16 \\ p^3(1-p)/4 \\ p^2(1-p)^2/4 \\ p^2(1-p)^2/8 \\ p(1-p)^3/4 \\ (1-p)^4/16 \end{bmatrix}$$

Now the vector of frequencies of the full-sib pairs from this fortyfive matings can be obtained as

$$\begin{bmatrix} \frac{1}{1024}(4p^4-40p^3+136p^2-180p+81) \\ \frac{1}{256}(-4p^4+24p^3-36p^2+8p+9) \\ \frac{1}{256}(-4p^4+24p^3-36p^2+8p+9) \\ \frac{1}{128}(4p^4-16p^3+20p^2-12p+5) \\ \frac{1}{256}(4p^4-8p^3-20p^2+24p+5) \\ \frac{1}{128}(4p^4-16p^3+20p^2-12p+5) \\ \frac{1}{64}(-4p^4+8p^3-8p^2+4p+3) \\ \frac{1}{256}(4p^4-8p^3-20p^2+24p+5) \\ \frac{1}{64}(-4p^4+8p^3-8p^2+4p+3) \\ \frac{1}{64}(4p^4-8p^3+24p^2-20p+11) \\ \frac{1}{512}(4p^4-8p^3+4p+1) \\ \frac{1}{256}(-4p^4+8p^3-4p^2+1) \end{bmatrix}$$

$$\underline{u}^{(1)} = \left| \begin{array}{l}
\frac{1}{256}\,(-4p^4-8p^3+12p^2+8p+1) \\
\frac{1}{128}\,(4p^4-4p^2+4p+1) \\
\frac{1}{256}\,(-4p^4+8p^3-4p^2+1) \\
\frac{1}{128}\,(4p^4-8p^3+4p^2+1) \\
\frac{1}{128}\,(4p^4-4p^2+4p+1) \\
\frac{1}{64}\,(-4p^4+8p^3-8p^2+4p+3) \\
\frac{1}{512}\,(4p^4-8p^3+4p+1) \\
\frac{1}{256}\,(-4p^4+8p^3-4p+1) \\
\frac{1}{256}\,(-4p^4+8p^3-4p^2+1) \\
\frac{1}{128}\,(4p^4-8p^3+4p^2+1) \\
\frac{1}{256}\,(-4p^4-8p^3+12p^2+8p+1) \\
\frac{1}{128}\,(4p^4-4p^2+4p+1) \\
\frac{1}{128}\,(4p^4-4p^2+4p+1) \\
\frac{1}{64}\,(-4p^4+8p^3-8p^2+4p+3) \\
\frac{1}{512}\,(4p^4-8p^3+8p^2-4p+1) \\
\frac{1}{256}\,(-4p^4+8p^3-4p^2+1) \\
\frac{1}{256}\,(-4p^4+8p^3-4p^2+1) \\
\frac{1}{128}\,(4p^4-16p^3+20p^2-12p+5) \\
\frac{1}{512}\,(4p^4-8p^3+8p^2-4p+1) \\
\frac{1}{256}\,(-4p^4+8p^3-4p^2+1) \\
\frac{1}{256}\,(-4p^4+8p^3-4p^2+1)
\end{array} \right|$$

$$\left|\begin{array}{l}
\frac{1}{128}\,(4p^4-16p^3+20p^2-12p+5) \\
\frac{1}{1024}\,(4p^4+24p^3+40p^2+12p+1) \\
\frac{1}{256}\,(-4p^4-8p^3+12p^2+8p+1) \\
\frac{1}{256}\,(4p^4-8p^3-20p^2+24p+5) \\
\frac{1}{512}\,(4p^4-8p^3+4p+1) \\
\frac{1}{256}\,(-4p^4+24p^3-36p^2+8p+9) \\
\frac{1}{1024}\,(4p^4+24p^3+40p^2+12p+1) \\
\frac{1}{256}\,(-4p^4-8p^3+12p^2+8p+1) \\
\frac{1}{256}\,(4p^4-8p^3-20p^2+24p+5) \\
\frac{1}{512}\,(4p^4-8p^3+4p+1) \\
\frac{1}{256}\,(-4p^4+24p^3-36p^2+8p+9) \\
\frac{1}{1024}\,(4p^4-40p^3+136p^2-180p+81)
\end{array}\right|$$

The column vector of the frequencies of the full-sib pairs after the first generation of full-sib mating can be obtained as in the previous case as,

$$\underline{U}^{(2)} = \underset{\sim}{A}\, \underline{U}^{(1)} \qquad (4.1.2)$$

$$\left[\begin{array}{l}
\frac{1}{16384}\,(4p^4-136p^3+1144p^2-2612p+1739) \\
\frac{1}{4096}\,(-4p^4+72p^3-232p^2+68p+161) \\
\frac{1}{4096}\,(-4p^4+72p^3-232p^2+68p+161) \\
\frac{1}{2048}\,(4p^4-40p^3+72p^2-52p+43) \\
\frac{1}{4096}\,(4p^4-8p^3-160p^2+164p+131)
\end{array}\right.$$

$$
\underline{u}^{(2)} = \begin{vmatrix}
\frac{1}{2048}\,(4p^4-40p^3+72p^2-52p+43) \\
\frac{1}{1024}\,(-4p^4+8p^3+16p^2-20p+41) \\
\frac{1}{4096}\,(4p^4-8p^3-160p^2+164p+131) \\
\frac{1}{1024}\,(-4p^4+8p^3+16p^2-20p+41) \\
\frac{1}{1024}\,(4p^4-8p^3+40p^2-36p+51) \\
\frac{1}{8192}\,(4p^4-8p^3-88p^2+92p+43) \\
\frac{1}{4096}\,(-4p^4+8p^3+8p^2-12p+17) \\
\frac{1}{4096}\,(-4p^4-56p^3-40p^2+196p+65) \\
\frac{1}{2048}\,(4p^4+24p^3-24p^2+12p+27) \\
\frac{1}{4096}\,(-4p^4+8p^3+8p^2-12p+17) \\
\frac{1}{2048}\,(4p^4-8p^3+16p^2-12p+19) \\
\frac{1}{2048}\,(4p^4+24p^3-24p^2+12p+27) \\
\frac{1}{1024}\,(-4p^4+8p^3+16p^2-20p+41) \\
\frac{1}{8192}\,(4p^4-8p^3-88p^2+92p+43) \\
\frac{1}{4096}\,(-4p^4+8p^3+8p^2-12p+17) \\
\frac{1}{4096}\,(-4p^4+8p^3+8p^2-12p+17) \\
\frac{1}{2048}\,(4p^4-8p^3+16p^2-12p+19) \\
\frac{1}{4096}\,(-4p^4-56p^3-40p^2+196p+65) \\
\frac{1}{2048}\,(4p^4+24p^3-24p^2+12p+27) \\
\frac{1}{2048}\,(4p^4+24p^3-24p^2+12p+27)
\end{vmatrix}
$$

$$
\left.\begin{array}{l}
\frac{1}{1024}\,(-4p^4+8p^3+16p^2-20p+41) \\
\frac{1}{8192}\,(4p^4-8p^3+24p^2-20p+11) \\
\frac{1}{4096}\,(-4p^4+8p^3+8p^2-12p+17) \\
\frac{1}{4096}\,(-4p^4+8p^3+8p^2-12p+17) \\
\frac{1}{2048}\,(4p^4-40p^3+72p^2-52p+43) \\
\frac{1}{8192}\,(4p^4-8p^3+24p^2-20p+11) \\
\frac{1}{4096}\,(-4p^4+8p^3+8p^2-12p+17) \\
\frac{1}{4096}\,(-4p^4+8p^3+8p^2-12p+17) \\
\frac{1}{2048}(4p^4-40p^3+72p^2-52p+43) \\
\frac{1}{16384}\,(4p^4+120p^3+760p^2+716p+139) \\
\frac{1}{4096}\,(-4p^4-56p^3-40p^2+196p+65) \\
\frac{1}{4096}\,(4p^4-8p^3-160p^2+164p+131) \\
\frac{1}{8192}\,(4p^4-8p^3-88p^2+92p+43) \\
\frac{1}{4096}\,(-4p^4+72p^3-232p^2+68p+161) \\
\frac{1}{16384}\,(4p^4+120p^3+760p^2+716p+139) \\
\frac{1}{4096}(-4p^4-56p^3-40p^2+196p+65) \\
\frac{1}{4096}(4p^4-8p^3-160p^2+164p+131) \\
\frac{1}{8192}\,(4p^4-8p^3-88p^2+92p+43) \\
\frac{1}{4096}\,(-4p^4+72p^3-232p^2+68p+161) \\
\frac{1}{16384}\,(4p^4-136p^3+1144p^2-2612p+1739)
\end{array}\right]
$$

Hence the joint distribution (correlation table) of full-sib pairs after the first generation of full-sib mating is written as in table 4.2.

From this table, the correlation coefficient between full-sib pairs after the first generation of full-sib mating can be directly worked out by assuming additive genic effects.

Taking sib I as x, sib II as y and scoring the genotypes according to the number of dominant genes present such as AABB as 4, AABb,AaBB as 3, AAbb,aaBB, AaBb as 2, Aabb,aaBb as 1 and aabb as 0 and by using simple correlation coefficient formula the correlation coefficient can be obtained.

Here, the correlation coefficient can be of two values depending upon the value of p. p is ranging from 0 to 0.5. When there is complete linkage p takes the value 0 and when there is no linkage p takes the value 0.5.

The correlation coefficient between full-sib pairs in the initial population is worked out from the vector $\underline{U}^{(1)}$, in the similar manner as mentioned above, is $\frac{2-2p}{3-2p}$. Thus it can be seen that when there is no linkage, the value of correlation coefficient is 0.5 and when there is complete linkage it will be 0.6667.

The correlation coefficient between full-sib pairs after the first generation of full-sib mating ${}_{s}r^{(1)}_{F-s}$ is obtained as

$${}_{s}r^{(1)}_{F-s} = \frac{5-4p}{7-4p} \qquad (4.1.3)$$

Thus in case of complete linkage, the value of correlation coefficient is 0.7143 and in case of no linkage, the value of correlation coefficient is 0.6.

Similarly the correlation table for full-sib pairs, after the 2nd, 3rd .... etc., generations of full-sib mating can be worked out assuming additive genic effects. Correlation coefficients of full-sib pairs upto ten generations of full-sib mating in general and in case of complete linkage as well as in case of no linkage, thus worked out, are given in table 4.3.

The above correlations have been graphically represented and shown in Fig.1, curve (1) and (2).

4.1.2. Parent-offspring correlation

The joint distribution of parent and offspring after the first generation of full-sib mating with two loci A and B with two alleles A,a and B,b at each locus with proportions of AB as (1-p)/2, Ab as p/2, aB as p/2 and ab as (1-p)/2 can be obtained by pairing one of the parents with an offspring obtained from the respective mating, out of the 45 types of full-sib matings, whose frequencies are given by the elements of vector $\underline{U}^{(1)}$, in table 4.4.

The correlation coefficient ${}_{s}r^{(1)}_{p-0}$, between parent and offspring after the first generation of full-sib mating is obtained directly from the correlation table, by considering

Table 4.3

Correlation coefficient of full-sib pairs in ten generations under full-sib mating system

Generation	Correlation coefficient	Value of correlation coefficient	
		Complete linkage (p=0)	No linkage (p=1/2)
0	$\frac{2-2p}{3-2p}$	0.6667	0.5
1	$\frac{5-4p}{7-4p}$	0.7143	0.6
2	$\frac{12-8p}{15-8p}$	0.8	0.7273
3	$\frac{27-16p}{32-16p}$	0.8438	0.7917
4	$\frac{59-32p}{67-32p}$	0.8806	0.8431
5	$\frac{126-64p}{139-64p}$	0.9065	0.8785
6	$\frac{265-128p}{286-128p}$	0.9266	0.9054
7	$\frac{551-256p}{585-256p}$	0.9419	0.9256
8	$\frac{1136-512p}{1191-512p}$	0.9538	0.9412
9	$\frac{2327-1024p}{2416-1024p}$	0.9632	0.9533
10	$\frac{4743-2048p}{4887-2048p}$	0.9705	0.9627

as in case of full-sib pairs, assuming the additive genic effect as

$$ {}_{s}r^{(1)}_{p-0} = \frac{5-4p}{\sqrt{(6-4p)(7-4p)}} \qquad (4.1.4) $$

Thus the value of correlation coefficient ${}_{s}r^{(1)}_{p-0}$ is 0.7715 when there is complete linkage and it is 0.6708 when there is no linkage.

Here also it can be seen that under the random mating population the correlation coefficient is 0.5 when there is no linkage and it is 0.5773 when there is linkage.

In a similar manner, the joint distribution and the correlation coefficient for parent and offspring pairs after the second generation of full-sib mating can be obtained by using the vector of frequencies $\underline{U}^{(2)}$ of the second generation of full-sib mating as in table 4.5.

Similarly, the joint distribution and the correlation coefficient for parent and offspring for the 3rd, 4th ...... etc., generations of full-sib mating can be worked out. The correlation coefficients between parent and offspring upto ten generations of full-sib mating in general and in case of complete linkage as well as in case of no linkage, thus worked out, are given in table 4.6.

The above correlations have been graphically represented and are shown in Fig.1, curve (3) and (4).

Table 4.6

Correlation coefficient between parent and offspring in ten generations of full-sib mating

Generation	Correlation coefficient	Value of correlation coefficient	
		Complete linkage (p=0)	No linkage (p=1/2)
0	$\frac{1-p}{\sqrt{(1-p)(3-2p)}}$	0.5773	0.5
1	$\frac{5-4p}{\sqrt{(6-4p)(7-4p)}}$	0.7715	0.6708
2	$\frac{12-8p}{\sqrt{(14-8p)(15-8p)}}$	0.8281	0.7628
3	$\frac{27-16p}{\sqrt{(30-16p(32-16p)}}$	0.8714	0.8269
4	$\frac{59-32p}{\sqrt{(64-32p)(67-32p)}}$	0.9009	0.8691
5	$\frac{126-64p}{\sqrt{(134-64p)(139-64p)}}$	0.9232	0.8998
6	$\frac{265-128p}{\sqrt{(278-128p)(286-128p)}}$	0.9398	0.9222
7	$\frac{551-256p}{\sqrt{(572-256p)(585-256p)}}$	0.9525	0.9391
8	$\frac{1136-512p}{\sqrt{(1170-512p)(1191-512p)}}$	0.9623	0.9519
9	$\frac{2327-1024p}{\sqrt{(2382-1024p)(2416-1024p)}}$	0.9700	0.9619
10	$\frac{4743-2048p}{\sqrt{(4832-2048p)(4887-2048p)}}$	0.9757	0.9697

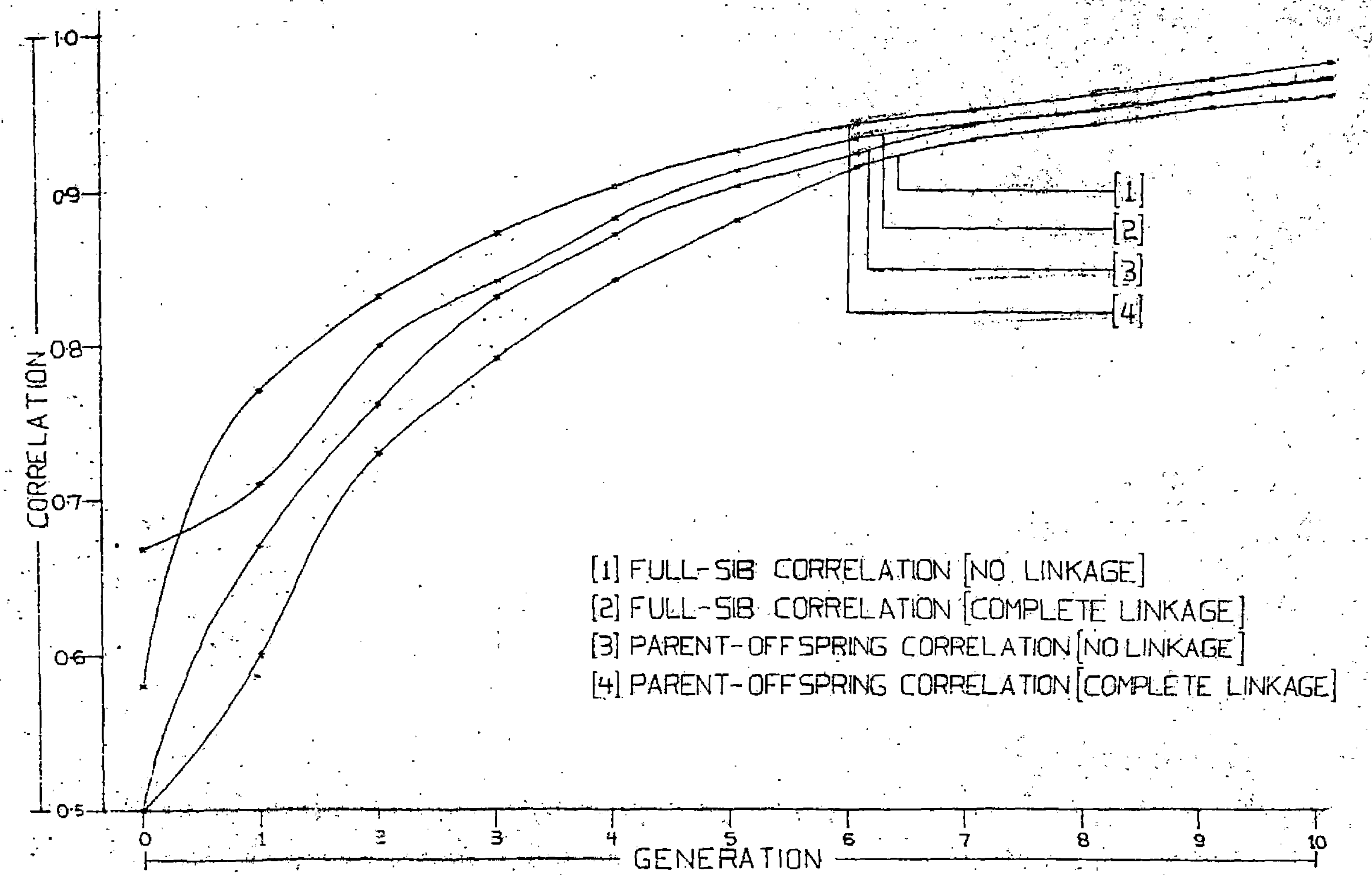

FIG:1 FULL-SIB MATING

4.2. Parent-offspring mating

There are two types of parent-offspring mating systems. In one, a fixed sire is mated repeatedly to his daughter, grand-daughter, great-grand daughter, etc., whereas in the other, each individual is mated successively with his(her) younger parent and with his(her) offspring.

Here the first type of parent-offspring mating is considered, i.e., the mating between a fixed sire and his daughter, grand-daughter, etc.

These types of matings are considered under the ten classes of phenotypic mating as follows:

I. Consider the mating of the type AB x AB

AB can be of genotypes: AABB, AABb, AaBB and AaBb.

First, the mating of AABB x AABB is considered. The former AABB denotes the older parent and the latter denotes the younger parent. The former produces gamete AB with proportion 1 and the latter produces gamete AB with proportion 1. Thus their offspring will be of genotype AABB with proportion 1. This can be shown as follows:

i)

$$
\begin{array}{ccccc}
AABB & & \times & & AABB \\
\downarrow & & & & \downarrow \\
AB & & & & AB \\
1 & & & & 1 \\
& \searrow & & \swarrow & \\
& & AABB & & \\
& & 1 & &
\end{array}
$$

Thus the parent-offspring pair will be (AABB,AABB) with proportion 1.

Second, the mating of AABB x AABb is considered. Here the former AABB is older parent and AABb is younger parent. AABB produces gamete AB with proportion 1 and AABb produces AB and Ab with proportion $\frac{1}{2}$ each. Thus their offsprings will be of genotype AABB and AABb with proportion $\frac{1}{2}$ each. This can be shown as follows:

ii) AABB x AABb

AB   AB, Ab

1   1/2   1/2

AABB   AABb

1/2   1/2

Thus the parent-offspring pairs will be (AABB,AABB) and (AABB,AABb) with proportion $\frac{1}{2}$ each.

In a similar manner the following mating types can be considered.

iii) AABb x AABB

AB, Ab   AB

1/2   1/2   1

AABB   AABb

1/2   1/2

The parent-offspring pairs with respective proportions are as follows:

(AABb,AABB) (AABb,AABb)
1/2 1/2

iv) AABB x AaBB

$\downarrow$ $\downarrow$

AB AB, aB
1 1/2 1/2

AABB AaBB
1/2 1/2

The parent-offspring pairs with respective proportions are as follows:

(AABB,AABB) (AABB,AaBB)
1/2 1/2

v) AaBB x AABB

$\downarrow$ $\downarrow$

AB, aB AB
1/2 1/2 1

AABB AaBB
1/2 1/2

The parent-offspring pairs with respective proportions are as follows:

(AaBB,AABB) (AaBB,AaBB)
1/2 1/2

vi) AABB x AaBb

$$\begin{array}{ccc} AABB & \times & AaBb \\ \downarrow & & \downarrow \\ AB & & AB,\ Ab,\ aB,\ ab \\ 1 & & 1/4\ \ 1/4\ \ 1/4\ \ 1/4 \end{array}$$

	AB(1/4)	Ab(1/4)	aB(1/4)	ab(1/4)
AB 1	AABB 1/4	AABb 1/4	AaBB 1/4	AaBb 1/4

Thus the parent-offspring pairs with respective proportions are as follows:

(AABB,AABB)	(AABB,AABb)	(AABB,AaBB)	(AABB,AaBb)
1/4	1/4	1/4	1/4

vii) AaBb x AABB

$$\begin{array}{ccc} AaBb & \times & AABB \\ \downarrow & & \downarrow \\ AB,\ Ab,\ aB,\ ab & & AB \\ 1/4\ \ 1/4\ \ 1/4\ \ 1/4 & & 1 \end{array}$$

	AB(1/4)	Ab(1/4)	aB(1/4)	ab(1/4)
AB 1	AABB 1/4	AABb 1/4	AaBB 1/4	AaBb 1/4

Thus the parent-offspring pairs with respective proportions are as follows:

(AaBb,AABB)	(AaBb,AABb)	(AaBb,AaBB)	(AaBb,AaBb)
1/4	1/4	1/4	1/4

viii) AABb x AABb

$$\begin{array}{ccc} AABb & \times & AABb \\ \downarrow & & \downarrow \\ AB,\ Ab & & AB,\ Ab \\ 1/2\ \ 1/2 & & 1/2\ \ 1/2 \end{array}$$

	AB(1/2)	Ab(1/2)
AB(1/2)	AABB 1/4	AABb 1/4
Ab(1/2)	AABb 1/4	AAbb 1/4

The offsprings are AABB, AABb and AAbb with proportions 1/4, 1/2 and 1/4 respectively.

Thus the parent-offspring pairs with respective proportions are as follows:

(AABb,AABB)	(AABb,AABb)	(AABb,AAbb)
1/4	1/2	1/4

ix)

	AABb	x	AaBB
	↓		↓
	AB, Ab		AB, aB
	1/2 1/2		1/2 1/2

	AB(1/2)	Ab(1/2)
AB(1/2)	AABB 1/4	AABb 1/4
aB(1/2)	AaBB 1/4	AaBb 1/4

The offsprings are AABB, AABb, AaBB and AaBb with proportion 1/4 each.

Thus the parent-offspring pairs with respective proportions are as follows:

(AABb,AABB)	(AABb,AABb)	(AABb,AaBB)	(AABb,AaBb)
1/4	1/4	1/4	1/4

x) AaBB x AABb

AB, aB AB, Ab

1/2 1/2 1/2 1/2

	AB(1/2)	aB(1/2)
AB(1/2)	AABB 1/4	AaBB 1/4
Ab(1/2)	AABb 1/4	AaBb 1/4

The offsprings are AABB, AaBB, AABb and AaBb with proportion 1/4 each.

Thus the parent-offspring pairs with respective proportions are as follows:

(AaBB,AABB) (AaBB,AaBB) (AaBB,AABb) (AaBB,AaBb)
1/4 1/4 1/4 1/4

xi) AABb x AaBb

AB, Ab AB, Ab, aB, ab

1/2 1/2 1/4 1/4 1/4 1/4

	AB(1/4)	Ab(1/4)	aB(1/4)	ab(1/4)
AB(1/2)	AABB 1/8	AABb 1/8	AaBB 1/8	AaBb 1/8
Ab(1/2)	AABb 1/8	AAbb 1/8	AaBb 1/8	Aabb 1/8

The offsprings are AABB, AABb, AaBB, AaBb, AAbb and Aabb with proportions 1/8, 1/4, 1/8, 1/4, 1/8 and 1/8 respectively.

Thus the parent-offspring pairs with respective proportions are as follows:

(AABb,AABB)	(AABb,AABb)	(AABb,AaBB)	(AABb,AaBb)
$1/8$	$1/4$	$1/8$	$1/4$
(AABb,AAbb)	(AABb,Aabb)		
$1/8$	$1/8$		

xii) AaBb x AABb

In this mating type, the offsprings with respective proportions are same as in the case of AABbxAaBb. Here the older parent will be AaBb and the younger parent will be AABb. Thus the parent-offspring pairs with respective proportions are as follows:

(AaBb,AABB)	(AaBb,AABb)	(AaBb,AaBB)	(AaBb,AaBb)
$1/8$	$1/4$	$1/8$	$1/4$
(AaBb,AAbb)	(AaBb,Aabb)		
$1/8$	$1/8$		

xiii)

AaBB	x	AaBB
↓		↓
AB, aB		AB, aB
$1/2$ $1/2$		$1/2$ $1/2$

	AB($1/2$)	aB($1/2$)
AB($1/2$)	AABB $1/4$	AaBB $1/4$
aB($1/2$)	AaBB $1/4$	aaBB $1/4$

The offsprings are AABB, AaBB and aaBB with proportions $1/4$, $1/2$ and $1/4$ respectively.

Thus the parent-offspring pairs with respective proportions are as follows:

(AaBB,AABB)	(AaBB,AaBB)	(AaBB,aaBB)
1/4	1/2	1/4

xiv) AaBB x AaBb

AaBB → AB, aB with 1/2, 1/2

AaBb → AB, Ab, aB, ab with 1/4, 1/4, 1/4, 1/4

	AB(1/4)	Ab(1/4)	aB(1/4)	ab(1/4)
AB(1/2)	AABB 1/8	AABb 1/8	AaBB 1/8	AaBb 1/8
aB(1/2)	AaBB 1/8	AaBb 1/8	aaBB 1/8	aaBb 1/8

The offsprings are AABB, AABb, AaBB, AaBb, aaBB and aaBb with proportions 1/8, 1/8, 1/4, 1/4, 1/8 and 1/8 respectively.

Thus the parent-offspring pairs with respective proportions are as follows:

(AaBB,AABB)	(AaBB,AABb)	(AaBB,AaBB)	(AaBB,AaBb)
1/8	1/8	1/4	1/4
(AaBB,aaBB)	(AaBB,aaBb)		
1/8	1/8		

xv) AaBb x AaBB

The offsprings with respective proportions of this mating type will be same as of above case AaBB x AaBb. In this mating type, older parent will be AaBb and younger parent will be AaBB. Thus the parent-offspring pairs with respective proportions are as follows:

(AaBb,AABB) (AaBb,AABb) (AaBb,AaBB) (AaBb,AaBb)
1/8 1/8 1/4 1/4

(AaBb,aaBB) (AaBb,aaBb)
1/8 1/8

xvi) AaBb x AaBb

AB, Ab, aB, ab AB, Ab, aB, ab
1/4 1/4 1/4 1/4 1/4 1/4 1/4 1/4

	AB(1/4)	Ab(1/4)	aB(1/4)	ab(1/4)
AB 1/4	AABB 1/16	AABb 1/16	AaBB 1/16	AaBb 1/16
Ab 1/4	AABb 1/16	AAbb 1/16	AaBb 1/16	Aabb 1/16
aB 1/4	AaBB 1/16	AaBb 1/16	aaBB 1/16	aaBb 1/16
ab 1/4	AaBb 1/16	Aabb 1/16	aaBb 1/16	aabb 1/16

The offsprings are AABB, AABb, AaBB, AaBb, AAbb, Aabb, aaBB, aaBb and aabb with proportions 1/16, 1/8, 1/8, 1/4, 1/16, 1/8, 1/16, 1/8 and 1/16 respectively.

Thus the parent-offspring pairs with respective proportions are as follows:

(AaBb,AABB) (AaBb,AABb) (AaBb,AaBB) (AaBb,AaBb) (AaBb,AAbb)
1/16 1/8 1/8 1/4 1/16

(AaBb,Aabb) (AaBb,aaBB) (AaBb,aaBb) (AaBb,aabb)
1/8 1/16 1/8 1/16

In a similar manner the other mating types under the nine classes of phenotypic mating can be considered as follows:

II. Consider the mating of the type <u>AB x Ab</u>

i) AABB x AAbb

Parent-offspring pair (AABB,AABb)

Proportion 1

ii) AAbb x AABB

Parent-offspring pair (AAbb,AABb)

Proportion 1

iii) AABB x Aabb

Parent-offspring pairs	(AABB,AABb)	(AABB,AaBb)
Proportions	1/2	1/2

iv) Aabb x AABB

Parent-offspring pairs	(Aabb,AABb)	(Aabb,AaBb)
Proportion	1/2	1/2

v) AABb x AAbb

Parent-offspring pairs	(AABb,AABb)	(AABb,AAbb)
Proportions	1/2	1/2

vi) AAbb x AABb

Parent-offspring pairs	(AAbb,AABb)	(AAbb,AAbb)
Proportions	1/2	1/2

vii) AABb x Aabb

Parent-offspring pairs	(AABb,AABb)	(AABb, AaBb)	(AABb,AAbb)
Proportions	1/4	1/4	1/4

(AABb,Aabb)
1/4

viii) Aabb x AABb

Parent-offspring pairs	(Aabb,AABb)	(Aabb,AaBb)	
Proportions	1/4	1/4	
(Aabb,AAbb)	(Aabb,Aabb)		
1/4	1/4		

ix) AaBB x AAbb

Parent-offspring pairs	(AaBB,AABb)	(AaBB,AaBb)
Proportions	1/2	1/2

x) AAbb x AaBB

Parent-offspring pairs	(AAbb,AABb)	(AAbb,AaBb)
Proportions	1/2	1/2

xi) AaBB x Aabb

Parent-offspring pairs	(AaBB,AABb)	(AaBB,AaBb)	(AaBB,aaBb)
Proportions	1/4	1/2	1/4

xii) Aabb x AaBB

Parent-offspring pairs	(Aabb,AABb)	(Aabb,AaBb)	(Aabb,aaBb)
Proportions	1/4	1/2	1/4

xiii) AaBb x AAbb

Parent-offspring pairs	(AaBb,AABb)	(AaBb,AAbb)	(AaBb,AaBb)
Proportions	1/4	1/4	1/4
(AaBb,Aabb)			
1/4			

xiv) AAbb x AaBb

Parent-offspring pairs	(AAbb,AABb)	(AAbb,AAbb)	(AAbb,AaBb)
Proportions	1/4	1/4	1/4
(AAbb,Aabb)			
1/4			

xv) AaBb x Aabb

Parent-offspring pairs	(AaBb,AABb)	(AaBb,AaBb)	(AaBb,AAbb)
Proportions	1/8	1/4	1/8

(AaBb,Aabb)	(AaBb,aaBb)	(AaBb,aabb)
1/4	1/8	1/8

xvi) Aabb x AaBb

Parent-offspring pairs	(Aabb,AABb)	(Aabb,AaBb)	(Aabb,AAbb)
Proportions	1/8	1/4	1/8

(Aabb,Aabb)	(Aabb,aaBb)	(Aabb,aabb)
1/4	1/8	1/8

III. Consider the mating of the type AB x aB

i) AABB x aaBB

Parent-offspring pairs	(AABB,AaBB)
Proportion	1

ii) aaBB x AABB

Parent-offspring pair	(aaBB,AaBB)
Proportion	1

iii) AABB x aaBb

Parent-offspring pairs	(AABB,AaBB)	(AABB,AaBb)
Proportions	1/2	1/2

iv) aaBb x AABB

Parent-offspring pairs	(aaBb,AaBB)	(aaBb,AaBb)
Proportions	1/2	1/2

v) AABb x aaBB

Parent-offspring pairs	(AABb,AaBB)	(AABb,AaBb)	
Proportions	1/2	1/2	

vi) aaBB x AABb

Parent-offspring pairs	(aaBB,AaBB)	(aaBB,AaBb)	
Proportions	1/2	1/2	

vii) AABb x aaBb

Parent-offspring pairs	(AABb,AaBB)	(AABb,AaBb)	(AABb,Aabb)
Proportions	1/4	1/2	1/4

viii) aaBb x AABb

Parent-offspring pairs	(aaBb,AaBB)	(aaBb,AaBb)	(aaBb,Aabb)
Proportions	1/4	1/2	1/4

ix) AaBB x aaBB

Parent-offspring pairs	(AaBB,AaBB)	(AaBB,aaBB)	
Proportions	1/2	1/2	

x) aaBB x AaBB

Parent-offspring pairs	(aaBB,AaBB)	(aaBB,aaBB)	
Proportions	1/2	1/2	

xi) AaBB x aaBb

Parent-offspring pairs	(AaBB,AaBB)	(AaBB,AaBb)	(AaBB,aaBB)
Proportions	1/4	1/4	1/4
(AaBB,aaBb)			
1/4			

xii) aaBb x AaBB

Parent-offspring pairs	(aaBb,AaBB)	(aaBb,AaBb)	(aaBb,aaBB)
Proportions	1/4	1/4	1/4
(aaBb,aaBb)			
1/4			

xiii) AaBb x aaBB

Parent-offspring pairs (AaBb,AaBB) (AaBb,AaBb)

Proportions 1/4 1/4

(AaBb,aaBB) (AaBb,aaBb)
1/4 1/4

xiv) aaBB x AaBb

Parent-offspring pairs (aaBB,AaBB) (aaBB,AaBb)

Proportions 1/4 1/4

(aaBB,aaBB) (aaBB,aaBb)
1/4 1/4

xv) AaBb x aaBb

Parent-offspring pairs (AaBb,AaBB) (AaBb,AaBb)

Proportions 1/8 1/4

(AaBb,Aabb) (AaBb,aaBB) (AaBb,aaBb) (AaBb,aabb)
1/8 1/8 1/4 1/8

xvi) aaBb x AaBb

Parent-offspring pairs (aaBb,AaBB) (aaBb,AaBb)

Proportions 1/8 1/4

(aaBb,Aabb) (aaBb,aaBB) (aaBb,aaBb)(aaBb,aabb)
1/8 1/8 1/4 1/8

IV. Consider the matings of the type AB x ab

i) AABB x aabb

Parent-offspring pair (AABB,AaBb)

Proportion 1

ii) aabb x AABB

Parent-offspring pair (aabb,AaBb)

Proportion 1

iii) AABb x aabb

Parent-offspring pairs (AABb,AaBb) (AABb,Aabb)

Proportions 1/2 1/2

iv) aabb x AABb

Parent-offspring pairs (aabb,AaBb) (aabb,Aabb)

Proportions 1/2 1/2

v) AaBB x aabb

Parent-offspring pairs (AaBB,AaBb) (AaBB,aaBb)

Proportions 1/2 1/2

vi) aabb x AaBB

Parent-offspring pairs (aabb,AaBb) (aabb,aaBb)

Proportions 1/2 1/2

vii) AaBb x aabb

Parent-offspring pairs (AaBb,AaBb) (AaBb,Aabb) (AaBb,aaBb)

Proportions 1/4 1/4 1/4

(AaBb,aabb)
1/4

viii) aabb x AaBb

Parent-offspring pairs (aabb,AaBb) (aabb,Aabb)

Proportions 1/4 1/4

(aabb,aaBb) (aabb,aabb)
1/4 1/4

V. Consider the matings of the type Ab x aB

i) AAbb x aaBB

Parent-offspring pair (AAbb,AaBb)

Proportion 1

ii) aaBB x AAbb

Parent-offspring pair (aaBB,AaBb)

Proportion 1

iii) AAbb x aaBb

Parent-offspring pairs	(AAbb,AaBb)	(AAbb,Aabb)
Proportions	1/2	1/2

iv) aaBb x AAbb

Parent-offspring pairs	(aaBb,AaBb)	(aaBb,Aabb)
Proportions	1/2	1/2

v) Aabb x aaBB

Parent-offspring pairs	(Aabb,AaBb)	(Aabb,aaBb)
Proportions	1/2	1/2

vi) aaBB x Aabb

Parent-offspring pairs	(aaBB,AaBb)	(aaBB,aaBb)
Proportions	1/2	1/2

vii) Aabb x aaBb

Parent-offspring pairs	(Aabb,AaBb)	(Aabb,Aabb)
Proportions	1/4	1/4

(Aabb,aaBb)	(Aabb,aabb)
1/4	1/4

viii) aaBb x Aabb

Parent-offspring pairs (aaBb,AaBb) (aaBb,Aabb)
Proportions 1/4 1/4

(aaBb,aaBb) (aaBb,aabb)
1/4 1/4

VI. Consider the matings of the type Ab x Ab

i) AAbb x AAbb

Parent-offspring pair (AAbb,AAbb)

Proportion 1

ii) AAbb x Aabb

Parent-offspring pairs (AAbb,AAbb) (AAbb,Aabb)

Proportions 1/2 1/2

iii) Aabb x AAbb

Parent-offspring pairs (Aabb,AAbb) (Aabb,Aabb)

Proportions 1/2 1/2

iv) Aabb x Aabb

Parent-offspring pairs (Aabb,AAbb) (Aabb,Aabb)(Aabb,aabb)

Proportions 1/4 1/2 1/4

VII. Consider the matings of the type Ab x ab

i) AAbb x aabb

Parent-offspring pair (AAbb,Aabb)

Proportion 1

ii) aabb x AAbb

Parent-offspring pair (aabb,Aabb)

Proportion 1

iii) Aabb x aabb

Parent-offspring pairs (Aabb,Aabb) (Aabb,aabb)

Proportions 1/2 1/2

iv) aabb x Aabb

Parent-offspring pairs (aabb,Aabb) (aabb,aabb)

Proportions 1/2 1/2

VIII. Consider the matings of the type <u>aB x aB</u>

i) aaBB x aaBB

Parent-offspring pair (aaBB,aaBB)

Proportion 1

ii) aaBB x aaBb

Parent-offspring pairs (aaBB,aaBB) (aaBB,aaBb)

Proportions 1/2 1/2

iii) aaBb x aaBB

Parent-offspring pairs (aaBb,aaBB) (aaBb,aaBb)

Proportions 1/2 1/2

iv) aaBb x aaBb

Parent-offspring pairs (aaBb,aaBB)(aaBb,aaBb)(aaBb,aabb)

Proportions 1/4 1/2 1/4

IX. Consider the matings of the type <u>aB x ab</u>

i) aaBB x aabb

Parent-offspring pair (aaBB,aaBb)

Proportion 1

ii) aabb x aaBB

Parent-offspring pair (aabb,aaBb)

Proportion 1

iii) aaBb x aabb

Parent-offspring pairs (aaBb,aaBb) (aaBb,aabb)

Proportions 1/2 1/2

iv) aabb x aaBb

Parent-offspring pairs (aabb,aaBb) (aabb,aabb)

Proportions 1/2 1/2

X. Consider the mating of the type ab x ab

i) aabb x aabb

Parent-offspring pair (aabb,aabb)

Proportion 1

In the mating types mentioned above, all possible crosses are considered. Thus 81 types of matings are obtained. But we cannot get 81 parent-offspring pairs because some of the parent-offspring pairs such as (AABB,AAbb), (AABB,Aabb), (AaBB,AAbb), (AaBB,Aabb), (AABB,aaBB), (AABB,aaBb), (AABb,aaBB), (AABb,aaBb), (AABB,aabb), (AABb,aabb), (AaBB,aabb), (AAbb,aaBB), (AAbb,aaBb), (Aabb,aaBB), (AAbb,aabb), (aaBB,aabb) and their respective reciprocal pairs (on the whole there are 32 such pairs) cannot be obtained from any of 81 types of matings for the successive generations. Their frequencies will be always zero in all generations. Thus eliminating these

32 columns of parent-offspring pairs, the transpose of generation matrix($\underset{\sim}{A}^{*\prime}$)for the parent-offspring mating type can be obtained as shown in appendix II.

With the help of this generation matrix and the procedure given in the case of full-sib mating, the vector of frequencies after the n^{th} generation of parent-offspring mating can be calculated.

4.2.1. <u>Full-sib correlation</u>

As in the case of full-sib mating system, the parent-offspring mating system is also developed from the 81 mating types from the equilibrium random mating population.

Now the vector of frequencies, $\underline{U}^{(1)*}$, of the 81 parent-offspring mating types can be obtained as

$$\underline{U}^{(1)*} = \underset{\sim}{A}^{*} \underline{U}^{(0)} \qquad (4.2.1)$$

where $\underset{\sim}{A}^{*}$ is the generation matrix for parent-offspring mating system. Thus it can be written as,

$$U^{(1)*} = \begin{bmatrix} \frac{1}{32}(-2p^3+7p^2+8p+3) \\ \frac{1}{32}(2p^3-3p^2+1) \\ \frac{1}{32}(2p^3-5p^2+3p \\ \frac{1}{32}(2p^3-3p^2+1) \\ \frac{1}{32}(2p^3-5p^2+3p) \end{bmatrix}$$

$\frac{1}{32}(-2p^3+7p^2-8p+3)$
$\frac{1}{64}(-4p^3+10p^2-8p+3)$
$\frac{1}{8}(-p^2+p)$
$\frac{1}{32}(-2p^3+p^2+p)$
$\frac{1}{32}(-2p^3+p^2+p)$
$\frac{1}{8}(-p^2+p)$
$\frac{1}{16}(2p^2-2p+1)$
$\frac{1}{8}(-p^2+p)$
$\frac{1}{8}(-p^2+p)$
$\frac{1}{16}(2p^2-2p+1)$
$\frac{1}{8}(2p^2-2p+1)$
0
0
0
0
$\frac{1}{32}(-2p^3+p^2+p)$
$\frac{1}{32}(-2p^3+3p^2)$
$\frac{1}{32}(2p^3-5p^2+3p)$
$\frac{1}{32}(2p^3-5p^2+3p)$
0
0
0
0

$\frac{1}{64}(4p^3-2p^2+1)$
$\frac{1}{32}(2p^3+p^2)$
$\frac{1}{16}(2p^2-2p+1)$
$\frac{1}{8}(-p^2+p)$
0
0
0
0
0
0
0
0
$\frac{1}{32}(-2p^3+p^2+p)$
$\frac{1}{32}(-2p^3+3p^2)$
$\frac{1}{32}(2p^3-5p^2+3p)$
$\frac{1}{32}(2p^3-5p^2+3p)$
$\frac{1}{64}(4p^3-2p^2+1)$
$\frac{1}{32}(2p^3+p^2)$
$\frac{1}{16}(2p^2-2p+1)$
$\frac{1}{8}(-p^2+p)$
0
0
0
0
0

$$
\left|
\begin{array}{c}
0 \\
\frac{1}{64}(-4p^3+10p^2-8p+3) \\
\frac{1}{32}(-2p^3+7p^2-8p+3) \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\frac{1}{32}(-2p^3+p^2+p) \\
\frac{1}{32}(-2p^3+p^2+p) \\
\frac{1}{32}(2p^3+p^2) \\
\frac{1}{32}(-2p^3+3p^2) \\
\frac{1}{32}(-2p^3+p^2+p) \\
\frac{1}{8}(-p^2+p) \\
0 \\
0 \\
\frac{1}{32}(2p^3-5p^2+3p) \\
\frac{1}{32}(2p^3-3p^2+1) \\
\frac{1}{32}(2p^3+p^2) \\
\frac{1}{32}(-2p^3+3p^2) \\
\frac{1}{32}(-2p^3+p^2+p)
\end{array}
\right|
$$

$$\begin{bmatrix} \frac{1}{8}(-p^2+p) \\ 0 \\ 0 \\ \frac{1}{32}(2p^3-5p^2+3p) \\ \frac{1}{32}(2p^3-3p^2+1) \\ \frac{1}{32}(-2p^3+7p^2-8p+3) \end{bmatrix}$$

Now the joint distribution (correlation table) of full-sib pairs under the first generation of parent-offspring mating can be obtained by pairing the offspring in the first generation of the parent-offspring mating, within each of the vector $\underline{u}^{(1)*}$, as given in table 4.7.

The correlation coefficient $_{p-0}r^{(1)}_{F-S}$, between full-sib pairs under the first generation of parent-offspring mating is obtained directly from the correlation table by employing the method explained in full-sib mating, assuming the additive genic effect as

$$_{p-0}r^{(1)}_{F-S} = \frac{11-10p}{15-10p} \qquad (4.2.2)$$

The value of correlation coefficient is 0.733 when there is complete linkage and it is 0.6 when there is no linkage.

In a similar manner the column vector, $\underline{u}^{(2)*}$, of the frequencies of the 81 parent-offspring mating types in the second generation of parent-offspring mating can be obtained as,

$$\underset{\sim}{u}^{(2)*} = \underset{\sim}{A}^{*}\,\underset{\sim}{u}^{(1)*} \qquad (4.2.3)$$

Hence $\underset{\sim}{u}^{(2)*}$ can be obtained,

$$\underset{\sim}{u}^{(2)*} = \begin{bmatrix} \frac{1}{128}(-2p^3+23p^2-40p+19) \\ \frac{1}{128}(2p^3+p^2-8p+5) \\ \frac{1}{128}(2p^3-15p^2+13p) \\ \frac{1}{128}(2p^3+p^2-8p+5) \\ \frac{1}{128}(2p^3-15p^2+13p) \\ \frac{1}{128}(-2p^3+7p^2-8p+3) \\ \frac{1}{256}(-4p^3+22p^2-20p+9) \\ \frac{1}{16}(-3p^2+3p) \\ \frac{1}{128}(-2p^3-p^2+3p) \\ \frac{1}{128}(-2p^3-p^2+3p) \\ \frac{1}{16}(-p^2+p) \\ \frac{1}{16}(2p^2-2p+1) \\ \frac{1}{16}(-3p^2+3p) \\ \frac{1}{16}(-p^2+p) \\ \frac{1}{16}(2p^2-2p+1) \\ \frac{1}{8}(2p^2-2p+1) \\ 0 \\ 0 \end{bmatrix}$$

0
0
$\frac{1}{128}(-2p^3-9p^2+11p)$
$\frac{1}{128}(-2p^3+7p^2)$
$\frac{1}{128}(2p^3-7p^2+5p)$
$\frac{1}{128}(2p^3-7p^2+5p)$
0
0
0
0
$\frac{1}{256}(4p^3+10p^2-12p+7)$
$\frac{1}{128}(2p^3+p^2)$
$\frac{1}{16}(2p^2-2p+1)$
$\frac{1}{16}(-p^2+p)$
0
0
0
0
0
0
0
0
$\frac{1}{128}(-2p^3-9p^2+11p)$

$$\frac{1}{128}\,(-2p^3+7p^2)$$

$$\frac{1}{128}\,(2p^3-7p^2+5p)$$

$$\frac{1}{128}\,(2p^3-7p^2+5p)$$

$$\frac{1}{256}\,(4p^3+10p^2-12p+7)$$

$$\frac{1}{128}\,(2p^3+p^2)$$

$$\frac{1}{16}\,(2p^2-2p+1)$$

$$\frac{1}{16}\,(-p^2+p)$$

$$0$$

$$0$$

$$0$$

$$0$$

$$0$$

$$0$$

$$\frac{1}{256}\,(-4p^3+22p^2-20p+9)$$

$$\frac{1}{128}\,(-2p^3+7p^2-8p+3)$$

$$0$$

$$0$$

$$0$$

$$0$$

$$0$$

$$0$$

$$\frac{1}{128}\,(-2p^3-p^2+3p)$$

$$\frac{1}{128}\,(-2p^3-p^2+3p)$$

$$\left[\begin{array}{l}
\frac{1}{128}\,(2p^3+17p^2) \\
\frac{1}{128}\,(-2p^3+7p^2) \\
\frac{1}{128}\,(-2p^3-9p^2+11p) \\
\frac{1}{16}\,(-3p^2+3p) \\
\qquad 0 \\
\qquad 0 \\
\frac{1}{128}\,(2p^3-15p^2+13p) \\
\frac{1}{128}\,(2p^3+p^2-8p+5) \\
\frac{1}{128}\,(2p^3+17p^2) \\
\frac{1}{128}\,(-2p^3+7p^2) \\
\frac{1}{128}\,(-2p^3-9p^2+11p) \\
\frac{1}{16}(-3p^2+3p) \\
\qquad 0 \\
\qquad 0 \\
\frac{1}{128}\,(2p^3-15p^2+13p) \\
\frac{1}{128}\,(2p^3+p^2-8p+5) \\
\frac{1}{128}\,(-2p^3+23p^2-40p+19)
\end{array}\right]$$

Now as in the previous case, by pairing and pooling the frequencies of respective full-sib pairs from the offspring of the second generation of parent-offspring mating, the joint distribution of full-sib pairs in the second generation of parent-offspring mating can be obtained as in table 4.8.

The correlation coefficient $p\text{-}O^{F}F\text{-}S^{(2)}$, between the full-sib pairs after the second generation of parent-offspring mating is obtained directly by assuming additive genic effect as

$$p\text{-}O^{F}F\text{-}S^{(2)} = \frac{55-50p}{69-50p} \qquad (4.2.4)$$

The value of correlation coefficient is 0.7971 when there is complete linkage and it is 0.6818 when there is no linkage.

Similarly, the joint distribution and correlation coefficients for full-sib pairs, in the 3rd, 4th, ... etc., generations of parent-offspring mating can be worked out. The correlation coefficients between full-sib pairs upto ten generations of parent-offspring, in general and in case of complete linkage as well as in case of no linkage are worked out and are given in table 4.9.

The above correlations have been graphically represented and are shown in Fig.2, curve (1) and (2).

4.2.2. <u>Parent-offspring correlation</u>
<u>Parent-offspring correlation</u>

In section 4.2.1, we have seen that the column vector of frequencies of the parent-offspring pairs from an equilibrium random mating population was given by $\underline{U}^{(1)*}$. Again the column vector of frequencies of the parent-offspring mating type in the first generation of parent-offspring mating, $\underline{U}^{(2)*}$, has been given in (4.2.3).

Table 4.9

Correlation coefficients between full-sib pairs in ten generations of parent-offspring mating

Generation	Correlation coefficient	Value of correlation coefficient	
		Complete linkage ($p=0$)	No linkage ($p=1/2$)
0	$\frac{1-p}{\sqrt{(1-p)(3-2p)}}$	0.5773	0.5
1	$\frac{11-10p}{15-10p}$	0.7333	0.6
2	$\frac{55-50p}{69-50p}$	0.7971	0.6818
3	$\frac{245-226p}{297-226p}$	0.8249	0.7174
4	$\frac{1033-962p}{1233-962p}$	0.8378	0.7340
5	$\frac{4241-3970p}{5025-3970p}$	0.8440	0.7421
6	$\frac{17185-16130p}{20289-16130p}$	0.8470	0.7461
7	$\frac{69185-65026p}{81537-65026p}$	0.8485	0.7480
8	$\frac{277633-261122p}{326913-261122p}$	0.8493	0.7490
9	$\frac{1112321-1046530p}{1309185-1046530p}$	0.8496	0.7495
10	$\frac{4453865-4190210p}{5239809-4190210p}$	0.8498	0.7498

By using this vector of frequencies, $\underline{U}^{(2)*}$, shown in section 4.2.1, the correlation table of parent-offspring pairs in the first generation of parent-offspring mating can be worked out as in table 4.10.

The correlation coefficient ${}_{p-0}r^{(1)}_{p-0}$ can be worked out directly from the above table and it is given as

$$ {}_{p-0}r^{(1)}_{p-0} = \frac{3(1-p)}{\sqrt{(1-p)(15-10p)}} \qquad (4.2.5) $$

The value of correlation coefficient is 0.7746 when there is complete linkage and it is 0.6708 when there is no linkage.

Similarly, the correlation coefficients of parent-offspring pairs under parent-offspring mating for 2nd, 3rd, ..., etc., can be worked out from correlation tables by assuming additive genic effects.

The correlation coefficients between parent and offspring upto the ten generations of parent-offspring mating in general and in case of complete linkage as well as in case of no linkage are worked out and are given in table 4.11.

The above correlations have been graphically represented and are shown in Fig. 2, curve (3) and (4).

Table 4.11

Correlation coefficients between parent and offspring in ten generation of parent-offspring mating

Generation	Correlation coefficient	Value of correlation coefficient: Complete linkage ($p=0$)	Value of correlation coefficient: No linkage ($p=1/2$)
0	$\frac{1-p}{\sqrt{(1-p)(3-2p)}}$	0.5773	0.5
1	$\frac{3(1-p)}{\sqrt{(1-p)(15-10p)}}$	0.7746	0.6708
2	$\frac{7(1-p)}{\sqrt{(1-p)(69-50p)}}$	0.8427	0.7462
3	$\frac{15(1-p)}{\sqrt{(1-p)(297-226p)}}$	0.8704	0.7819
4	$\frac{31(1-p)}{\sqrt{(1-p)(1233-962p)}}$	0.8828	0.7994
5	$\frac{63(1-p)}{\sqrt{(1-p)(5025-3970p)}}$	0.8887	0.8079
6	$\frac{127(1-p)}{\sqrt{(1-p)(20289-16130p)}}$	0.8916	0.8122
7	$\frac{255(1-p)}{\sqrt{(1-p)(81537-65026p)}}$	0.8930	0.8144
8	$\frac{511(1-p)}{\sqrt{(1-p)(326913-261122p)}}$	0.8937	0.8154
9	$\frac{1023(1-p)}{\sqrt{(1-p)(1309185-1046530p)}}$	0.8942	0.8160
10	$\frac{2047(1-p)}{\sqrt{(1-p)(5239809-4190210p)}}$	0.8943	0.8162

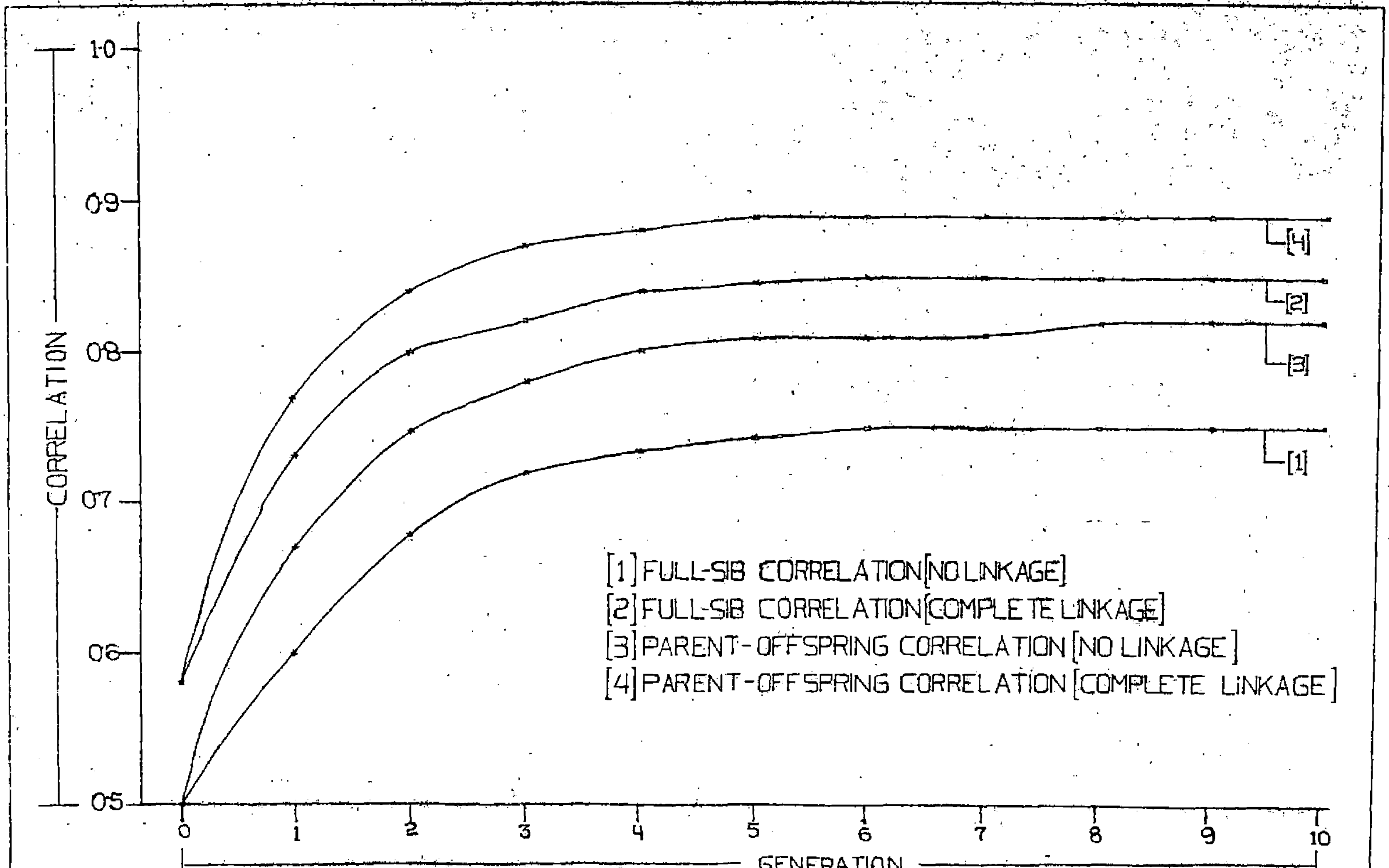

FIG: 2 PARENT-OFFSPRING MATING

DISCUSSION

CHAPTER V

DISCUSSION

In the present investigation an attempt was made to study the genetic correlations of full-sib pairs and parent-offspring pairs under full-sib mating system and parent-offspring mating system in case of two loci with two alleles at each locus. The study of correlations of full-sib pairs and parent-offspring pairs under full-sib mating system were carried out in section 4.1.1 and section 4.1.2. Similarly the study of correlations of full-sib pairs and parent-offspring pairs under parent-offspring mating system were carried out in section 4.2.1 and section 4.2.2.

In section 4.1.1, the joint distribution of full-sib pairs in the first generation of full-sib mating was derived using generation matrix and the corresponding correlation was worked out therefrom. Further, a series of full-sib correlation coefficients for first ten generations of full-sib mating in general and in case of complete linkage as well as in case of no linkage was obtained as given in table (4.3). It could be observed from this table that when there was no linkage, the correlation coefficient steadily increased from 0.5 under random mating to 0.9627, and when there was complete linkage it increased from 0.6667 under random mating to 0.9705, in tenth generation of full-sib mating. This was fully in agreement with the principles of inbreeding.

It was also observed that (i) even under random mating the value of correlation coefficient was 0.5 when there was no linkage, (ii) it was 0.6667 when there was complete linkage and (iii) the values of correlation coefficient in complete linkage were greater than the values of the correlation when there was no linkage in all generations.

Section (4.1.2) concerned with the derivation of joint distribution of parent-offspring pairs and the calculation of parent-offspring correlation under full-sib mating, adopting generation matrix approach. The correlation coefficients of parent-offspring pairs in the first ten generations of full-sib mating in general and both cases of complete linkage and no linkage were worked out in table (4.6). The table indicated that when there was no linkage, the correlation increased from 0.5 under random mating to 0.9697 and when there was complete linkage, it increased from 0.5773 under random mating to 0.9957 in the tenth generation of full-sib mating. These findings were also in full agreement with the phenomena of inbreeding. Here also all the values of correlations in complete linkage were greater than the correlation coefficients in case of no linkage.

These four correlations, viz., full-sib correlation (no linkage), full-sib correlation (complete linkage), parent-offspring correlation (no linkage) and parent-offspring correlation (complete linkage) were exhibited graphically in Fig.1 by curve (1), curve (2), curve (3) and curve (4) respectively. From these curves, it could be observed that initially the increase was in linear fashion and after third generation it increased with decreasing rate in all cases. From the investigation of the correlation curves, eventhough the full-sib correlation coefficient at the presence of linkage was greater than the parent-offspring correlation in initial generation (random mating), the latter increased more rapidly than the former from the initial generation to first generation. It was revealed by the facts that (i) slope of curve (4) was steeper than that of curve (2), (ii) curve (4) crossed the curve (2) between initial and first generation. Nevertheless, from the second generation onwards, the rate of increase in both of curves of correlation coefficients were nearly the same upto tenth generation. From the fourth generation onwards, correlation coefficients became nearly equal to each other and tend to unity in infinite number of generations.

A comparative study of these four curves revealed that the parent-offspring correlations were of comparatively higher order than the full-sib correlations in case of no linkage as well as in case of complete linkage.

In section 4.2.1, the joint distribution of full-sib pairs under the first generation of parent-offspring mating was derived using generation matrix theory and the corresponding correlation was worked out therefrom. Proceeding from this, full-sib correlation coefficients for first ten generations of parent-offspring mating in general and in case of complete linkage as well as in case of no linkage were obtained as given in table 4.9. From this table, it could be seen that, the correlation increased as the number of generation increased in both cases of complete linkage and no linkage. When there was no linkage the correlation increased from 0.5 under random mating to 0.7498 and when there was complete linkage it increased from 0.5773 under random mating to 0.8498 in the tenth generation of parent-offspring mating. This finding was in full agreement with the principles of inbreeding. In this case also the values of correlation coefficients were more in complete linkage in all generations and it was found that even under random mating it was 0.5773.

Further the derivation of joint distribution of parent-offspring pairs and the calculation of parent-offspring correlation under parent-offspring mating system were carried out under the section 4.2.2. The correlation coefficients of parent-offspring pairs in first ten generations of parent-offspring mating in general and in both cases of complete linkage and no linkage were worked out in table 4.11. The table indicated that the correlation increased as the number of generation increased. It was also in full agreement with the phenomena of inbreeding. The graph exhibiting the trends of full-sib correlation (no linkage), full-sib correlation (complete linkage), parent-offspring correlation (no linkage) and parent-offspring correlation (complete linkage) under parent-offspring mating was given in Fig.2 by curve (1), curve (2), curve (3) and curve (4) respectively. From that figure it was seen that the correlation increased as the number of generation increased, but the rate of increase gradually reduced as the number of generation increased.

On comparing the parent-offspring correlation with the full-sib correlation, it was seen that the trend in both cases remain the same, but the value of parent-offspring correlation was always greater than that of the full-sib correlation in case of complete linkage as well as in case of no linkage.

In comparison of all these correlations, one could easily observe that the correlations increased as the number of generation increased and ultimately reached the limit unity when the number of generations increased indefinitely large. It was interesting to note that the parent-offspring correlation was higher in magnitude than that of full-sib correlation under both systems of mating in case of no linkage as well as in case of complete linkage. It was also observed that in case of complete linkage the magnitude of correlation was more than in case of no linkage even under the same system of mating. When there was complete linkage, the parent-offspring correlation under parent-offspring mating increased at a rapid rate than the parent-offspring correlation under full-sib mating upto third generation, but from fourth generation onwards the latter increased at a more rapid rate than the former and became almost unity in the tenth generation. When there was no linkage, eventhough the parent-offspring correlations under both systems of mating were the same in both initial generation (random mating) and the first generation, the parent-offspring correlation under full-sib mating system increased at a rapid rate than the parent-offspring correlation under parent-offspring mating system from second generation onwards and became nearly unity at the tenth generation.

Eventhough in the presence of linkage the values of full-sib correlations under full-sib mating was greater than that of full-sib correlation under parent-offspring mating in the initial generation (random mating), it was found that the latter became greater than the former in the first generation. However, from the second generation onwards the full-sib correlation under full-sib mating increased more rapidly than the full-sib correlation under parent-offspring mating, and tend to unity as the number of generation became indefinitely large.

It was also interesting to note that eventhough the above two correlations in the absence of linkage remain the same in initial and first generation, from the second generation onwards the full-sib correlation under full-sib mating increased more rapidly than the full-sib correlation under parent-offspring mating and tend to unity as the number of generation became indefinitely large.

SUMMARY

CHAPTER VI

SUMMARY

The objectives of the present investigation entitled "A Study of Genetic Correlations under Full-sib Mating System (two loci case)" were mainly four-fold, viz., (i) to derive the joint distribution-correlation table and to find the correlation between full-sib pairs under full-sib mating system in the case of two loci, when there is no linkage as well as when there is complete linkage, (ii) to derive the joint distribution-correlation table and to find the correlation between parent-offspring pairs under full-sib mating system in the case of two loci, when there is no linkage as well as, when there is complete linkage, (iii) to derive the joint distribution-correlation table and to find the correlation between full-sib pairs under parent-offspring mating system in the case of two loci, when there is no linkage as well as, when there is complete linkage and (iv) to derive the joint distribution-correlation table and to find the correlation between parent-offspring pairs under parent-offspring mating system in the case of two loci, when there is no linkage as well as, when there is complete linkage.

The study of correlation between relatives under different inbred systems so far had been made only in the case of single locus with two alleles. Hence an attempt was made in this investigation to study the genetic correlations of full-sib pairs and parent-offspring pairs under full-sib and parent-offspring mating system in the case of two loci with two alleles at each hours, when there is no linkage as well as when there is complete linkage.

To find out these correlations, the generation matrix $\underset{\sim}{A}$ for full-sib mating with dimension (45 x 45) was obtained from 45 genotypic mating types under ten classes of phenotypic mating types. Similarly the generation matrix $\underset{\sim}{A}^*$ for parent-offspring mating with dimension (81 x 49) was obtained from 81 genotypic mating types under ten classes of phenotypic mating.

Denoting the vector of frequencies for n^{th} generation under full-sib mating system by $\underline{U}^{(n)}$, the vector of frequencies for the successive generations under full-sib mating system was computed by the recurrence relation given by

$$\underline{U}^{(n)} = \underset{\sim}{A}\,\underline{U}^{(n-1)}$$

From this recurrence relation, the correlation tables for full-sib pairs for any generation of full-sib mating system could be easily worked out and the correlations were calculated therefrom. The correlations between full-sib pairs were calculated for the first ten generations of full-sib mating in both the cases of complete linkage and no linkage. When there was no linkage, the correlation between full-sib pairs under full-sib mating system ranged from 0.5 in initial generation to 0.9627 in tenth generation. When there was complete linkage, it ranged from 0.6667 in initial generation to 0.9705 in tenth generation.

The correlation tables for parent-offspring pairs under any generation of full-sib mating system was also developed from the recurrence relation

$$\underline{U}'^{(n)*} = \underset{\sim}{A}^{*} \underline{U}^{(n-1)}$$

From this recurrence relation, the correlation tables for parent-offspring pairs for the first ten generations of full-sib mating were worked out and the correlations were calculated therefrom in both the cases of complete linkage and no linkage. In case of no linkage the correlation between parent-offspring pairs under full-sib mating system ranged from 0.5 in initial generation to 0.9700 in tenth generation. In case of complete linkage it ranged from 0.5773 in initial generation to 0.9760 in tenth generation. When there was no linkage, the values of correlation coefficients for full-sib pairs and parent-offspring pairs were found to be 0.5 in initial generation. However, when there was complete linkage the values of these correlations were found to be 0.6667 and 0.5773 respectively.

Denoting the vector of frequencies for n^{th} generation under parent-offspring mating system by $\underline{u}^{(n)*}$, the vector of frequencies for full-sib pairs for n^{th} generation under parent-offspring mating system could be obtained from the recurrence relation given by

$$\underline{u}^{(n)} = \underset{\sim}{A}\, \underline{u}^{(n-1)*}$$

From the above relation, correlation tables for full-sib pairs for the first ten generations of parent-offspring mating were worked out and the correlations were calculated therefrom in both cases of no linkage and complete linkage. When there was no linkage, the correlation between full-sib pairs under parent-offspring mating system ranged from 0.5 in initial generation to 0.7498 in tenth generation. When there was complete linkage it ranged from 0.5773 in initial generation to 0.8498 in tenth generation.

Similarly, the correlation tables for parent-offspring pairs under any generation of parent-offspring mating system could be obtained from the recurrence relation given by

$$\underline{u}^{(n)*} = \underset{\sim}{A}^{*} \underline{u}^{(n-1)*}$$

From this recurrence relation, the correlation tables for parent-offspring pairs for the first ten generations of parent-offspring mating were worked out and the correlations were calculated therefrom in both cases of complete linkage and no linkage. In case of no linkage the correlation between parent-offspring pairs under parent-offspring mating system ranged from 0.5 in initial generation to 0.8162 in tenth generation. In case of complete linkage it ranged from 0.5773 in initial generation to 0.8943 in tenth generation.

Under parent-offspring mating system also the values of correlation coefficients for full-sib pairs and parent-offspring pairs were found to be 0.5 in initial generation when there was no linkage. However, when there was complete linkage the values of these correlations were found to be 0.5773 each.

The correlations such as full-sib correlation (no linkage), full-sib correlation (complete linkage), parent-offspring correlation (no linkage) and parent-offspring correlation (complete linkage) were graphically represented against the first ten generations of full-sib mating in figure 1. Similarly, the same types of correlations under parent-offspring mating were graphically represented against the first ten generations in figure 2.

It was found that the correlations in complete linkage were always greater than that of correlations in no linkage in both the system of mating. The values of correlations for full-sib pairs and parent-offspring pairs under parent-offspring mating were lower order than that of correlations under full-sib mating system in both cases of no linkage, and complete linkage. However, all these correlations increased as the number of generation increased and reached the limit unity when the number of generation increased indefinitely large.

REFERENCES

REFERENCES

*Fisher, R.A. (1918). Correlation between relatives on the supposition of Mendelian inheritance. Trans. Roy. Soc. Edinb. 52: 399-433.

Fisher, R.A. (1949). Theory of inbreeding. 2nd edn. 1965 Oliver and Boyd, London.

George, K.C. (1974). On correlation between relatives in inbred population. Unpublished Ph.D. thesis, submitted to IARI, New Delhi.

George, K.C. and Narain, P. (1975). Parent offspring correlation under full-sib and parent offspring mating systems. Jour. Indian Soc. Agric. Statist. 27(2): 51-70.

George, K.C. (1979). Parent-offspring correlation under half-sib mating system. Jour. Indian Soc. Agric. Statist. 30(2): 51-70.

George, K.C. (1983a). Generation matrix method of studying inbreeding systems-I. Jour. Indian Soc. Agric. Statist. 35(1): 35-48.

George, K.C. (1983b). Generation matrix method of studying inbreeding systems II. Jour. Indian Soc. Agric. Statist. 35(3): 42-56.

*Haldane, J.B.S. (1937). Some theoretical results of continued brother-sister mating. Jour. Genet. 34: 265-274.

*Haldane, J.B.S. (1955). The complete matrices for brother-sister and alternate parent-offspring mating involving one locus. Jour. Genet. 53: 315-324.

*Horner, T.W. (1956). Parent-offspring and full-sib correlations under parent-offspring mating system. Genetics 41: 460-468.

*Kampthorne, O. (1954). The correlation between relatives in a random mating population. Proc. Roy Soc. B., 143: 103-113.

*Kempthorne, O. (1955). The theoretical values of correlation between relatives in random mating populations. Genetics. 40: 153-167.

*Kempthorne, O. (1955a). The correlation between relatives in random mating populations: Cold Spring Harbor Symp. Quant. Biol. 20: 60-75.

*Kempthorne, O. (1955c). The correlation between relatives in inbred populations. Genetics. 40: 681-691.

Kempthorne, O. (1957). An introduction to genetic statistics. John Wiley & Sons Inc., New York.

*Korde, V.T. (1960). Correlation between relatives for a sex-linked character under inbreeding. Heredity. 14: 401-409.

Li, C.C. (1955). Population genetics. The University of Chicago Press Ltt., London.

Li, C.C. and Sacks, L. (1954). The derivation of joint distribution and correlation between relatives by the use of stochastic matrices. Biometrics. 10: 347-360.

Wright, S. (1921). Systems of mating II. The effects of inbreeding on the genetic composition of a population. Genetics. 6: 124-143.

* Original not seen

APPENDIX

A STUDY OF GENETIC CORRELATIONS UNDER FULL-SIB MATING SYSTEM (TWO LOCI CASE)

By

KHIN MOE MOE

ABSTRACT OF A THESIS

submitted in partial fulfilment of the
requirements for the degree of

MASTER OF SCIENCE (AGRICULTURAL STATISTICS)

Faculty of Agriculture
Kerala Agricultural University

Department of Statistics
COLLEGE OF VETERINARY AND ANIMAL SCIENCES
Mannuthy - Trichur

1985

ABSTRACT

A purely theoretical investigation entitled "A Study of Genetic Correlations under Full-sib Mating System (two loci case)" was carried out with the following objectives.

i) to derive the joint distribution (correlation table) and to find the correlation between full-sib pairs under full-sib mating system in the case of two loci when there is no linkage as well as when there is complete linkage.

ii) to derive the joint distribution (correlation table) and to find the correlation between parent-offspring pairs under full-sib mating system in the case of two loci when there is no linkage as well as when there is complete linkage.

iii) to derive the joint distribution (correlation table) and to find the correlation between full-sib pairs under parent-offspring mating system in the case of two loci when there is no linkage as well as when there is complete linkage.

iv) to derive the joint distribution (correlation table) and to find the correlation between parent-offspring pairs under parent-offspring mating system in the case of two loci when there is no linkage as well as when there is complete linkage.

The joint distributions of full-sib pairs and parent-offspring pairs under full-sib mating system were derived with the help of generation matrix technique and the correlations were worked out therefrom, assuming additive genic effects and using the product-moment correlation coefficient formula. The correlations were worked out for the first ten generations of full-sib mating in both cases of no linkage and complete linkage.

A comparative study of full-sib correlations and parent-offspring correlations, conducted both numerically and graphically, revealed that (i) eventhough full-sib correlation was greater than parent-offspring correlation in initial generation (random mating) when there was complete linkage, the latter increased more rapidly than the former from initial generation to first generation and (ii) from the second generation onwards, the rate of increase in both of correlations were nearly the same upto tenth generation. It was interesting to note that the parent-offspring correlations were of comparatively higher order than the full-sib correlations in both cases of complete linkage and no linkage.

Similarly, the joint distributions (correlation tables) for full-sib pairs and parent-offspring pairs under parent-offspring mating system were derived employing generation matrix approach and the correlations for the first ten generations of parent-offspring mating in both cases of no linkage and complete linkage were worked out therefrom. A comparative study of these correlations was carried out both numerically and graphically. It was found that the trend in both correlation curves remain the same, but the value of parent-offspring correlation was always greater than that of full-sib correlation in case of no linkage as well as in case of complete linkage.

In comparison of all these correlations, it was found that the correlations increased as the number of generation increased and ultimately reached the limit unity when the number of generations increased indefinitely large. It was also observed that the magnitude of correlation in case of complete linkage was more than that of correlation in case of no linkage even under the same system of mating.