## B.Tech (Agrl.Engg.) 2016 Admission

## II Semester Final Examination-August-2017

Engineering Mathematics-II (2+1)

Marks: 50
Time: 2 hours
$(10 \times 1=10)$
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## Answer the following

1 The $p$-series $\frac{1}{1^{9}}+\frac{1}{2^{p}}+\frac{1}{3^{p}}+\ldots$. Converges if
2
$\lim _{n \rightarrow \infty}\left(1+\frac{1}{n}\right)^{n}=$ $\qquad$
$3 f(x)=\tan x$ is an even function (T/F)
4 Fourier expansion of an odd function has only terms.
5 The complementary function of $\left(D^{2}-4 D D^{1}+4 D^{1^{2}}\right) z=x+y$ is
$6\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)=0$ is quasi-linear (T/F)
7 Define analytic function.
$8 \sin x, \cos x$ are periodic functions of period
9 State Cauchy-Riemann equation in Cartesian co-ordinates.
10 What is essential singularity?

## Write short notes on any FIVE

1 Define absolutely convergent series. Test $\sum_{2}^{\infty} \frac{(-1)^{n}}{n(\log n)^{2}}$ for convergence and absolute convergence.
2 Test for convergence the series $\sum \frac{4.7 \ldots(3 n+1)}{1.2 \ldots n} x^{n}$
Expand $f(x)=f(x)=\left\{\begin{array}{l}\frac{1}{4}-x, \text { if } 0<x<\frac{1}{2} \\ x-\frac{3}{4}, \text { if } \frac{1}{2}<x<1\end{array}\right.$ as the Fourier series of sine terms.
4 Form a partial differential equation from $f\left(x^{2}+y^{2}, z-x y\right)=0$
5
Solve $\frac{\partial^{2} z}{\partial x^{2}}-\frac{\partial^{2} z}{\partial x \partial y}=\cos x \cos 2 y$
6 What type of singularity have the function $(z+1) \sin \left(\frac{1}{z-2}\right)$
7 Evaluate $\int_{c}\left(z^{2}+3 z+2\right) d z$ where $c$ is the arc of the cycloid $x=$ $a(\theta+\sin \theta), y=a(1-\cos \theta)$ between the points $(0,0)$ and $(\pi a, 2 a)$

## Answer any FIVE

( $5 \times 4=20$ )
1 Show that the series is divergent.
2 Find the Fourier cosine integral of
3 Find the radius of convergence of the series
4 Show that $u(x, t)=f(x+c t)+g(x-c t)$ is a solution of $u_{t t}=c^{2} u_{x x}$
5 a) State Cauchy's integral formula.
b) Evaluate $\int_{c} \frac{\cos \pi z}{z^{2}-1} d z$ around a rectangle with vertices $2 \mp i,-2 \mp i$

Find the Fourier transform of $f(x)=\left\{\begin{array}{c}1-x^{2},|x| \leq 1 \\ 0,|x|>1\end{array}\right.$. Hence evaluate $\int_{0}^{\infty} \frac{x \cos x-\sin x}{x^{2}} \cos \left(\frac{x}{2}\right) d x$
Solve $\frac{\partial^{2} z}{\partial x^{2}}+\frac{\partial^{2} z}{\partial x \partial y}-6 \frac{\partial^{2} z}{\partial y^{2}}=\cos (2 \dot{x}+y)$

1 Evaluate $\int_{-\infty}^{\infty} \frac{e^{a x}}{e^{x}+1} d x$
2 a) Find the half range cosine series of the function

$$
\begin{gathered}
f(x)=4,0<x<\pi / 2 \\
=0, \pi / 2<x<\pi
\end{gathered}
$$

b) Find the bilinear transformation which maps $z=l, i,-1$ respectively onto $w=i, 0,-i$. For this transformation find the image of $|z| \leq 1$
c) Find the Taylor's series expansion of the function $f(z)=\frac{1}{(z-1)(z-3)}$ about the point $z=2$

