KERALA AGRICULTURAL UNIVERSITY

B.Tech (Food.Engg.) 2015 Admission
IIIrd Semester Final Examination-February-2017

Marks:50.00

Time: 2 hours

Cat. No: Base.2108.

Title: Engineering Mathematics- III (2+1)

(10x1=10)

I Fill in the blanks/Define/ State true or False

1. A vector with zero divergence is called a ----------- vector.
2. A point where the function ceases to be analytic is called a ----------- point.
3. A function $f(t) = \int_{0}^{\infty} A(\omega) \cos \omega t \, d\omega$ is a ----------- integral representation of $f(t)$.
4. A transformation of the form $w = \frac{az+b}{cz+d}$ is called a ----------- transformation.
5. A series of the form $a_0 + a_1(z-a) + a_2(z-a)^2 + \dots + a_n(z-a)^n + \dots$ is called a

   ----------- series.
6. A pole of order one is also called a ----------- pole.
7. Define conjugate functions of an analytic function.
8. What is a unit step function?
9. Write Cauchy-Riemann equations.
10. State true or false. Any solution of the Laplace's equation is called a harmonic function.

(5x2=10)

II Write short notes/answers on any FIVE of the following

1. Given $r = \sin t\, i + \cos t\, j + tk$, find $\frac{dr}{dt}$ and $\frac{d^2r}{dt^2}$.
2. Prove that $div\ r = 3$ and $curl\ r = 0$, where $r = xi + yj + zk$.
3. Show that the transformation $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic.
4. Distinguish between isogonal transformation and conformal transformation.
5. Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ onto the

   straight line $4u + 3 = 0$.
6. Expand $\frac{1}{z^2-3z+2}$ in the region $|z| < 1$
7. Write notes on singularities and zeros.

(5x4=20)

III Write short answers on any FIVE

1. Show that $f = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational and hence find its

   scalar potential.
2. Evaluate $\int_{0}^{1} \int_{x}^{\sqrt{x}} (x^2 + y^2) dy dx$
3. Find the Fourier series expansion of the periodic function $f(x) = x^2, \ -\pi \leq x \leq \pi$ of

   period 2π.
4. Determine a, b, c, d so that the function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$

   is analytic.
5. Under the transformation $w = \frac{1}{z}$, find the image of $|z - 2i| = 2$.

6. Evaluate $\int_{0}^{1+i} (x^2 - iy)dz$ along the paths a) $y = x$ b) $y = x^2$

7. Determine the poles of the function $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$ and the residue at each pole.

IV Write essay on any ONE   (1x10=10)

1. Verify Greens theorem in the plane for $\oint_{C}(3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$.

2. Evaluate $\oint_{C} \dfrac{z-3}{z^2+2z+5}dz$ where C is the circle
   i. $|z| = 1$
   ii. $|z + 1 - i| = 2$
   iii. $|z + 1 + i| = 2$

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