



Marks: 50
Time: 2 hours
(10x1=10)

I Fill in the blanks:

- 1 $\lim_{x \rightarrow 0} \frac{a^x - 1}{x}$ is
- 2 If $f(x, y) = xy^2 - \cos y$, then the value of the partial derivative $\frac{\partial f}{\partial x}$ is.....
- 3 The degree of the homogeneous function $f(x, y) = \frac{x^4 + y^4}{x + y}$ is
- 4 If two functions are functionally dependent then their Jacobian is
- 5 $\iint_R dx dy$ represent of the region R.
- 6 The degree of the differential equation $\frac{d^2y}{dx^2} - 3 \left(\frac{dy}{dx}\right)^2 + y = 0$ is
- 7 The solution of the differential equation $(D^2 - 4)y = 0$ is
- 8 The Particular integral of the differential equation $(D + 5)y = e^{2x}$ is
- 9 If $\varphi(x, y, z) = x^2 + y^2 + z^2$, then the gradient $\nabla\varphi$ is
- 10 If F is a vector field with $\text{curl } F = 0$, then the vector F is said to be

II Write Short notes on any FIVE of the following

(5x2=10)

- 1 Find the Maclaurin's series of $f(x) = e^x$
- 2 If $u = x^2 - y$, $v = x + y$, then find the Jacobian of u and v with respect to x and y .
- 3 Show that $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy$ is an exact differential equation.
- 4 If $u = x^2 + y^2$, with $x = a \cos t$, $y = b \sin t$ find $\frac{du}{dt}$.
- 5 Evaluate $\int_0^1 \int_0^2 (x + 3) dx dy$
- 6 State Stoke's Theorem.
- 7 If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then evaluate $\text{div } \vec{r}$.

III Answer any FIVE of the following.

(5x4=20)

- 1 If $u = e^{x^3 + y^3}$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3u \log u$
- 2 Find the maxima and minima of $x^3 + y^3 + 3xy$.
- 3 Change the order of integration and hence evaluate $\int_0^1 \int_x^1 \frac{x}{x^2 + y^2} dy dx$

Use triple integrals to find the volume bounded by the cylinder $x^2 + y^2 = 9$, the planes $z = 1$ and $x + z = 5$.

5 Solve the Bernoulli's differential equation: $x \frac{dy}{dx} + y = xy^3$.

6 Solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ using the method of variation of parameters.

7 Use Green's Theorem to evaluate $\oint_C (x - 2y)dx + (3x - y)dy$, where C is the boundary of a unit square

IV Write an essay on any one of the following

(1x10=10)

1 Solve the Legendre's linear equation:

$$(3x + 2)^2 \frac{d^2y}{dx^2} + 3(3x + 2) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$$

2 Verify Gauss divergence theorem for $\vec{F} = (x + y)\hat{i} + x\hat{j} + x\hat{k}$ taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.
