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ESTIMATION OF DISTRIBUTION OF CUMULATIVE RAINFALL FOR A SPECIFIC PERIOD WITHIN A *NJATTUVELA*

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Abstract. A stochastic model for determination of time distribution of cumulative rainfall for a specific period within a *njattuvela* was worked out by making use of the daily rainfall data collected from the Regional Agricultural Research Station, Kerala Agricultural University, Kumarakom. A day having a minimum of 3 mm of rainfall was regarded as a wet day. The daily rainfall depth within a *njattuvela* was characterised using an exponential distribution. Further the first and second order Markov chain models were fitted to the data. The chi-square test for adequacy of fit revealed that the first order was the best fit. Using the first and second order Markov chain probabilities, the cumulative rainfall depth of at least a reasonable amount was worked out. The study thus brought out that the significance of stochastic models for characterisation of time, distribution of rainfall depth.

INTRODUCTION

The rainfall distribution at a place above a threshold value determines the success of rainfed agriculture. Kerala experiences two spells of rainy season, viz. south-west monsoon and north-east monsoon. An year is divided into *njattuvelas* of approximately fortnightly duration and each *njattuvela* is named after a star. The *njattuvelas* that fall in the monsoon periods are of specific importance to a farmer. The total intensity of rain in a monsoon is distributed over these coincident *njattuvelas.* Even with the drastic modernisation that is taking place in the cultivation practices, the farmers even now plan the sowing, manuring and harvesting of crops depending upon the onset of the different *njattuvelas.*

Gabriel and Neumann (1962) fitted two state Markov chain model to daily rainfall occurrence. Bhargava *et al* (1972) found that a first order Markov chain model fitted well to the daily rainfall data recorded at 21 rain guage stations located at different parts in Raipur. A first order Markov chain model for explaining the occurrence of dry and wet day in Guahati was attempted by Medhi (1976). A collateral study was done by Mahajan and Rao (1981). Nguyen and Rouselle (1981) have worked out a stochastic model for the time distribution of hourly rainfall depth. The model adopted by them was a modified form of the model developed by Todorovic and Whoolhiser (1975). Rao (1984) has studied the dry spells at Kasaragod by first order Markov chain probability model. Santhosh (1987) has discussed the pattern of occurrence of rainfall and estimation of rainfall probabilities in northern districts of Kerala.

It is not only the intensity of rainfall within a *njattuvela* but also the spread over of the entire *njattuvela* that do determine the success of rainfed agriculture. Hence an attempt is made for estimation of distribution of cumulative rainfall for a specific period within each *njattuvela.*

MATERIALS AND METHODS

The approach of Nguyen and Rouselle (1981) for the estimation of time distribution of hourly rainfall depth of a place has been adopted. As an hour is too minute an interval for the study in the present context, a day is taken as the interval instead of the hour. Though a *njattuvela* need not be exactly a fortnight duration, for the sake of uniformity of intervals under consideration it has been regarded to be so. The number of consecutive rainy days as also the accumulated rainfall depth during a rainy period are random variables.

DEFINITIONS AND NOTATIONS

Definition 1

Define $m(j) = 1$, if jth day is wet
$= 0$, if jth day is dry, $j = 1, 2, \ldots 14$ (2.1)

Definition 2

M_n be defined as the number of consecutive rainy days starting from the first day of n - day period. For example, for n = 3 days.

$(M_3 = 0) = (0,0,0), (0,1,0), (0,1,1)$

$(M_3 = 1) = (1,0,0), (1,0,1)$

$(M_3 = 2) = (1,1,0)$

$(M_3 = 3) = (1,1,1)$

Definition 3

Let E_r denotes the daily rainfall depth in the r^t rainly day of the n - day period. The accumulated amount of rainfall S(n) during M_n consecutive rainy days can be defined as

$$S(n) = \overset{M_n}{S_r} = 0\, E_r \ldots\ldots (22)$$

where 'S' denotes summation
and $E_0 = 0$, $M_n = 1, 2, \ldots n$
Hence P (S(n) = 0) = P (M_n = 0) = P (m(1) = 0)
and P (S(n) > 0) = 1-P (S(n) = 0) where P( ) denotes probability.

Determination of the distribution function of the process S(n):

Let F_n (x) denote the distribution funcion of S(n). The distribution function had been determined under the assumptions that $E_1, E_2, \ldots E_n$ are independently and identically distributed random variables with H(x) = P ($E_r \leq x$) and $E_1, E_2, \ldots E_n$ are independent of M_n.

Thus $F_n(x) = S_k = 0$
$P(X_k \leq x)\ P(M_n = K) \ldots (2.3)$
where $X_k = S_r^k = 0\ Er$

Computation of the probability $P(X_k \leq x)$:

The daily rainfall depth distribution was assumed to be roughly characterised by an exponential distribution. The probability P ($X_k \leq x$) will then be determined under the assumptions that H(x) = 1 - exp (-cx) and $E_1, E_2 \ldots E_n$ are independent.

Thus P ($X_k \leq x$) =

$$\frac{C^k}{G(k)} I_0^k\, U^{k-1} \exp(-Cu)\, du$$
for k = 1,2,......, n ..... (2.4)

Computation of the probability P (Mn = k) :

The probability has been computed under the assumption that the sequences of non-zero rainfall days follow a Markov chain. In this study first and second order Markov chains has been fitted to the data. For the first order Markov model.

$$P(Mn = n) = \overset{t+n-1}{P}(Aj = t\ m(j) = 1) = q_1(t) \ldots q_{11}(t+n-1) \ldots \quad (2.5)$$

$$P(M_n = k) = P\left(\left(A_{j=t}^{t+k-1} m(j) = 1\right) A\ m(t+k) = 0\right)$$

$$= q_1(t)q_{11}(t+1)\ldots q_{11}(t+k-1)q_{10}(t+k), k = 1,2\ldots(n-1) \quad \ldots\ldots\ldots (2.6)$$

Where A denotes intersection, $q_1(t)$ is the initial probability at the first day of the n day period, $q_{10}(t)$ and $q_{11}(t)$ are the first and second order transition probabilities at the t^t day in the n-day period.

For the second order Markov model,

$$P(M_n = n) = P\left(A_{j=t}^{t+n-1} (m(j) = 1)\right)$$

$$= q_1(t)\ q_{11}(t+1)\ q_{111}(t+2)\ldots\ldots q_{111}(t+n-1) \quad \ldots\ldots (2.7)$$

$$P(M_n = k) = P\left(A_{j=t}^{t+k-1} (m(j) = 1)\right) A\ (m(t+k) = 0))$$

$$= q_1(t)\ q_{11}(t+1)\ q_{111}(t+2)\ \ldots q_{111}(t+k-1)\ q_{110}(t+k),$$

$$k = 1,2,\ldots(n-1) \ldots\ldots (2.8)$$

$$q_1^{(t)} = \frac{N_{wt}}{N}, \quad q_{11}^{(t)} = \frac{N_{wt-1\ wt}}{N_{wt-1}},$$

$$q_{10}^{(t)} = \frac{N_{wt-1\ Dt}}{N_{wt-1}}, \quad q_{111}^{(t)} = \frac{N_{wt-2wt-1wt}}{N_{wt-2wt-1}},$$

$$q_{110}^{(t)} = \frac{N_{wt-2wt-1Dt}}{N_{wt-2wt-1}}.$$

N_{wt} - Number of fortnights that day 't' was wet

N_{wt-1wt} - Number of fortnights that days '(t-1)' and 't' were both wet.

N_{wt-1Dt} - Number of fortnights that day '(t-1)' was wet and day 't' was dry.

$N_{wt-2wt-1wt}$ - Number of fortnights that days '(t-2)', '(t-1)' and 't' were wet.

$N_{wt-2wt-1Dt}$ - Number of fortnights that days '(t-2)' and '(t-1)' were wet and t was dry.

The adequacy of goodness of fit of the Markov models was tested using the chi-square statistic developed by Anderson and Goodman (1957).

RESULTS AND DISCUSSION

The daily rainfall data collected from the meteorological records maintained at the Regional Agricultural Research Station (KAU), Kumarakom for the year 1965 to 1988 were used for the study. A day with minimum 3 mm of rainfall was regarded as a wet day. The daily rainfall data were fused into biweekly totals. A biweek was taken in such a way that it coincided with a particular *njattuvela*. Also it could be reasonably assumed that all *njattuvelas* are of uniform fortnightly duration. Numerous illustrations are available in literature where the daily rainfall depth could be reasonably approximated by an exponential distribution (Eagleson 1972; Howard 1976; Santhosh 1987). The overall mean daily rainfall was found to be 7 mm. The parameter C of the exponential distribution was estimated as C = 1/7.

The first and second order Markov chain models were fitted to the dat, taking n = 4, k = 4 in (2.6) and (2.8) for the sake of illustration. The chi-square test for adequacy of fit (Anderson and Goodman, 1957) revealed that the first order Markov chain had a better fit to the data compared to the second order for all *njattuvelas*. But the serial dependence of wet days could

Table 1. Probability of occurrence of a cumulative rainfall depth of at least 10 mm within a span of 4 days

Njattuvelas	Probability	
	First order Markov model	Second order Markov model
Jan 11-23	0.0000	0.0000
Jan 24-6 Feb	0.0781	0.0781
Feb 7-19	0.0000	0.0000
Feb 20-3 March	0.7810	0.0781
March 4-16	0.0714	0.0714
March 17-30	0.0417	0.0417
March 31-12 April	0.1752	0.1753
Aprill3-26	0.1885	0.1882
April 27-10 May	0.0801	0.0806
May 11-24	0.2635	0.2630
May 25-7 June	0.4876	0.3506
June 8-20	0.5546	0.5590
June 21-4 July	0.6571	0.6602
July 5-18	0.5239	0.5298
July 19-1 August	0.5111	0.5375
August 2-15	0.7321	0.7336
August 16-29	0.4577	0.4717
August 30-12 Sept	0.4091	0.4120
Sept 13-26	0.2870	0.2891
Sept 27-9 Oct	0.4466	0.4545
Oct 10-23	0.2512	0.2562
Oct 24-5 Nov	0.1610	0.1624
Nov 6-18	0.4007	0.4043
Nov 19-1 Dec	0.2586	0.2593
Dec 2-14	0.0820	0.0820
Dec 15-28	0.2292	0.1198
Dec 29-10 Jan	0.2292	0.1198

noy be ruled out.

Using the first and second order Markov chain probabilities, the probability of occurrence of a cumulative rainfall depth of 10 mm or more within a span of 4 days was worked out taking x = 10 in (2.3) for each *njattuvela*. The probability distributions are presented in Table 1. The figures n = 4 and x = 10 were taken for illustrative purpose only. Using equations (2.3), (2.6) and (2.8) any probability distribution for different values of n (1 to 15) and x (0 to any meaningful limit) could be worked out. But as the same were quite elaborative, no attempt was made forall the permutations and combinations of 'n' and 'x' values.

From Table 1, it could clearly read, not only about the onset of south-west and north-east monsoons but also about the quantum of rain depth that a farmer could reasonably expect during the relevant *njattvelas*. The probability of occurrence for a cumulative rainfall depth of 10 mm within a span of four days during April 13-26 is 0.1885 which is higher compared to the preceeding and succeeding *njattuvelas*. The *njattuvela* coincident with the period April 13-26 is of extreme importance to a farmer who usually transplants seedlings. This *njattuvela* typifies the beginning of the virippu season (first crop), in which the crop is purely rainfed. Similar inferences could be drawn for all other seasons. A highly economical crop planning could very well be formulated by exploiting the probability of occurrence of various rainfall depth by adopting the criteria discussed.

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