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rUDY OF DRYSPELLS AT KASARAGOD BY FIRST ORDER MARKOV CHAIN PROBABILITY MODEL

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The rainfall distribution at a place above a threshold value determines the success of rainfed agriculture. The limits of the threshold value can be given by variations in potential evapotranspiration at the place. The ry or wet •:'• demarcated with the rainfall deviations from such threshold values at a piace is useful for many aspects of crop production and scientific water management. The utility and application of Markov chain probability models have been brought out by many workers (Gabriel and Neumann, 1962; Caskey, 1963; Weiss, 1964; Bhargava *et al.* 1973; Bhargava ef *ai,* 1977; Victor and Sastry, 1979). In the present study the probability model has been tested with the climatological data of CPCRI Station at Kasaragod.

Materials and Methods

The :'•! order Markov chain probability model of a dry or wet day event that woutd occur on any day depends upon whether the preceding day is also wet or dry. The dry or wet day demarcation is done based on whether the rainfall is below or above the potential evapotranspiration on that day.

The parameters used in the Markov model (Gabriel and Neumann, 1962; Weiss, ıt' and Cox and Miller, 1967) are the two conditional probabilities, P(W/W) and P(W,D); where P(W/W) and P(W/D) are the probabilities of occurrence of a wet day when the preceding day is also wet or dry. These are given by

$$P(W/W) = f(W/W) / \Sigma n \quad \ldots (1)$$

$$P(W/D) = f(W/D) / \Sigma n \quad \ldots (2)$$

where f(W,,W) and f(W/D) are the frequencies of wet days when the preceding days are also wet or dry;

Σ n is the total number of days in the period for which the frequency is considered

The probability of a dryspell of length 'n days is given by

$$P(W,.D) \; [1-P(W/D)]^{n-1} \quad \ldots (3)$$

The expected length of dry and wet runs is given by

$$E(D) \; 1/P(W/D) \quad \ldots (4)$$

and $$F(W) = 1/1 - P(W/W) \quad \ldots (5)$$

The expected length of a dry day followed by a wet day or vice versa is given by

$$E(C) = E(W) + E(D) = 1/1 - P(W/W) + 1/P(W/D) \quad \ldots (6)$$

The equilibrium probabilities r (D) for dry day and (W) for wet day which are independent of the initial conditions are given by

1—P(W/W)

—

i + r i w D) — r (W/W)

The mean and variance of the wet days for a period of 30 days or longer are given by the following relationship.

$$\frac{np\ (W/D)}{1-P(W/W)+P(W/D)} \quad (9)$$

$$\frac{nP\ (W/D)\ [1-P(W/W]\ [1+P(W/W)-P(W/D)]}{[1-P\ (W/W)\ +P(W/D)]^3}$$

The dry sequence length (in days) corresponding to a return period is obtained from the following relationship (Weiss 1964);

$$\frac{[1-P(W/W)]\ +P(W/D)}{SP\ (W/D)\ [1-P(W/W)][1-P(W/D)^{l}]} \quad 11)$$

Where s is the total number of days in a month for which the sequences are counted and l length of the sequence which is being calculated.

The normal potential evapotranspiration at Kasaragod was computed using a modified Penman equation suggested by Rao *et al.* (1971) and Brown and Cocheme (1973). The dry or wet days were classified by comparing daily rainfall (1949-78) with the normal potential evapotranspiration. Statistical tests suggested by Anderson and Goodman (1957) have been applied for the data from 14 to 48 weeks to ensure whether the dry or wet day sequences constitute a first order Markov model. The parameters from equations (1) to (10) and the expected frequencies by multiplying the probabilities obtained from equation (3) with the number of days in a month were computed. The probabilities for wet ordry spells in different crop phases and the dry sequence length (in days) for a given return period were also computed.

Results and Discussions

The chi-square values for the test between independence versus first order Markov chain probability model (Anderson and Goodman, 1957) are non-significant for weeks 15, 16 and 42 at 5% level and for 17 and 43 weeks at 1 % level. Hence, the dry or wet days are strongly dependent during 18 to 48 weeks except in 42 and 43 weeks. The chi-sqaure values for the test between first order versus second order models are non-significant for 15 and 19 weeks at *5%* level and for 16 to 19, 42 and 43 weeks at 1% level. This indicates a strong dependency of dry or wet days on the preceding days condition during 20 to 41, 44 and 46 weeks. The initial and conditional probabilities for different weekly periods are shown in Fig.1.

With an annual normal rainfall of 3478 mm, Kasaragod experiences an assu- wet spell from June to September and a dry spell from December to April. Dry spells during the intermittent periods have been studied with the computed parameters of Markov chain probability model (Table. 1) If the preceding day conditions were taken, the probability for occurrence of a wet day at the place increases from April to May and decreases from September to November. Simple probability for occurrence of dry day decreases from April to May and increases from September to November. The expected length of dry spell varies from 18.86 days (April) to 6.85 days (October) whereas the length of wet run ranges from 1.0 to 1.4 days for all the months. The expected number of dry days varies from 16.74 (September) to 28.14 (April) whereas the number of wet days ranges from 1.86 (April) to 13.26 (September) respectively. The values of standard deviation of dry or wet days shows that the dispersion of dry or wet days is maximum in September. The equilibrium probabilities for dry days are high (0.831 to 0.949) for attaining the stationary position independent of the initial conditions.

The observed and theoretical frequencies of dry spells greater than 4 days length with chi-square statistics for Kasaragod are presented in Tabfe 2. The chi-square values for April, May, October and November are non-significant at both the 6% and 1 % levels, whereas the same is significant for the month September at both the levels. Hence the model is useful for prediction of longer dry sequences in the rainfall transition periods of April, May, October and November.

Table 1

Parameters and properties of Markov chain probability mode!

	April	May	September	October	November
Conditional Probabilities					
P(W/W)	0.016	0.136	0.293	0.127	0.064
P (W/D)	0.053	0.122	0.144	0.146	0.077
Initial Probability					
P(D)	0.938	0.712	0.558	0.694	0.872
Expected length of runs					
Dry	18.86	8.19	6.94	6.85	12.98
Wet	1.02	1.16	1.41	1.15	1.07
Dry wet cycle	19.88	9.35	8.35	8.00	14.06
Expected number of days					
Dry	28.14	22.07	16,74	21.51	26.16
Wet	1.86	8.93	13.26	9,49	3.84
Standard deviation of dry and wet days	1.162	1.859	2.387	1.914	1.433
Equilibrium probabilities					
II (D)	0.949	0.876	0.831	0.856	0.924
II (W)	0.051	0.124	0.169	0.143	0.076

Table 2

Expected and observed frequency of dryspells

Runlength (Days)	April Obser-ved	April Expect-ed	May Obser-wed	May Expect-ed	September Obser-ved	September Expect-ed	October Obser-ved	October Expect-ed	November Obser-wed	November Ex-pected
5	52	36	47	48	37	39	48	50	48	44
6	47	34	41	42	29	33	40	43	42	40
7	44	32	36	37	21	29	30	37	36	37
8	41	31	29	32	18	24	28	31	33	34
9	39	29	27	29	17	21	25	27	31	32
10	36	27	25	25	13	18	20	23	29	29
11	31	26	22	22	10	15	15	19	27	27
12	29	25	18	19	8	13	12	17	25	25
13	28	23	14	17	5	11	10	14	23	23
14	25	22	10	15	4	10	8	12	22	21
15	23	21	9	13	4	8	5	10	22	20
16	21	20	8	12	2	7	4	9	21	18
17	19	19	7	10	1	6	2	8	16	17
18	17	18	7	9	1	5	2	6	15	18
19	17	17	7	8	1	4	—	6	13	14
20	16	16	6	7	1	4	—	5	12	13
21	16	15	5	6	1	3	—	4	10	12
22	15	14	5	5	1	3	—	3	10	11
23	12	14	5	5	1	2	—	3	8	10
24	12	13	4	4	—	2	—	3	8	10
25	11	12	4	4	—	2	—	2	8	9
26	10	12	4	3	—	2	5=3	2	8	8
27	8	11	2	3	—	1		2	5	8
28	8	10	2	2	—	1	—	1	5	7
29	7	10	2	2	—	1	—	1	5	6
30	6	9	1	2	—	1	—	1	5	6
38	—	—	1	1	—	—	—	1	—	—
Chisquare	330		7.8		37.0 **		19.9		4.7	
df	25		25		18		13		. 25	

*** Significant at 5%**

**** Significant at 1% and 5%**

From the Table 2 it can also be seen that the CPCR1 Station experiences dry spells of 30 days length on six times in April, five times in November and once in May in a period in of 30 years. The observed maximum dry sequence at the place in September is of 23 days length and 18 days length in October, The frequency of dry day sequence decreases with an increase in the sequence length.

The simple and conditional probabilities of dry and wetspells in crop phases for some crops grown in the region are also presented in Table 3. The simple

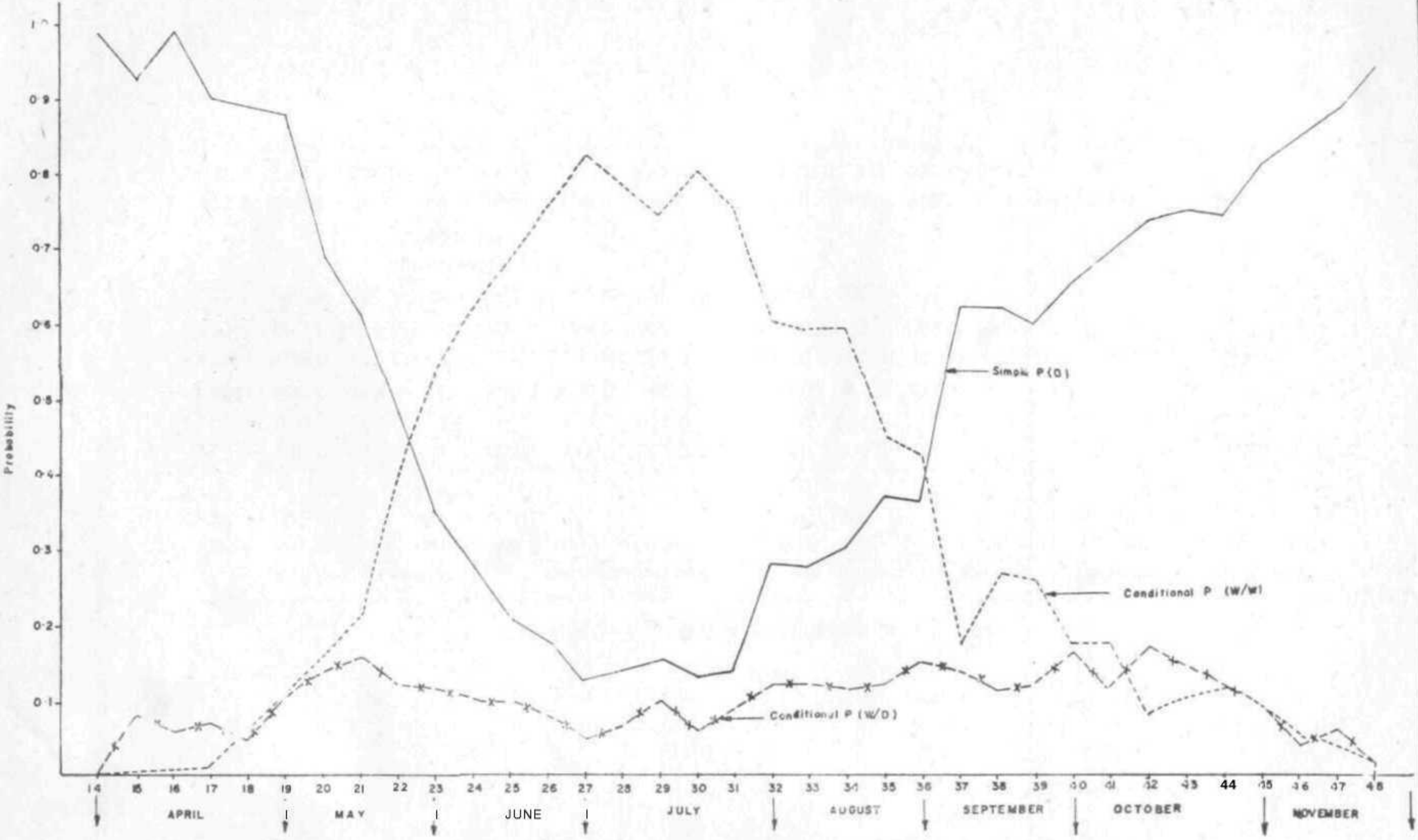

FIG. I INfTIAL AND CONDITIONAL RAINFALL PROBABILITIES OF KASARGOD

Table 3

Dry and wet spell probabilities in the cropping seasons

Name of the Crop	Planting time- Istweek of	Sowing and germination				Vegetative phase				Flowering and Maturity			
		Duration in weeks	P(D)	P(W/W)	P(W/D)	Duration in weeks	P(D)	P(W/W)	P(W/D)	Duration in weeks	P(D)	P(W/W)	P(W/D)
1 Paddy	June	1	0.352	0.533	0.105	10	0192	0.716	0.091	6	0.490	0.360	0.131
	October	1	0.667	0.171	0.162	9	0.709	0.076		6	0.999	0.002	0,001
	December	1	0.999	0.001	0.003	3	0.991	0.009	0.049	5	0.991	0.009	0.049
2 Pulses	May	1	0.881	0.001	0.114	6	0.439	0.437	0.122	5	0.147	0.781	0,073
	January	1	0.999	0.001	0.001	5	0,999	0,001	0.001	5	0.998	0.002	0.001
3 Groundnut	May	1	0.881	0.001	0.114	11	0,307	0.597	0.009	6 (Flowering & Pegging)	0.305	0.569	0.122
4 Sesamum	December	1	0.999	0.001	0.001	5	0.999	0.001	0.001	6	0.999	0.001	0.001
	June	1	0.352	0.533	0.105	5	0.189	0.734	0,079	7	0.243	0.647	0.107

probabilities of dryspells thus given for different crop phases also express the probabilities for irrigation in those cropping periods. The probable frequency at a given length of dry day sequence that can occur in different cropping periods can be obtained by substituting the P (W/D) values of that cropprng period in the equation (3), For example, an annual crop sown in the first week of May at Kasaragod will suffer with 9 per cent chances of a dry day sequence of 3 days length in the sowing and germination period.

Dry sequence lengths (in days) corresponding to a given return period (years) for rainfall greater than normal potential evapotranspiration at the place are presented in Table 4. It can be seen that on an average, every alternate year, Kasaragod experiences a dry sequence of 13 to 20 days during April, May, September, October and November. The possible return period for occurring dry sequences of greater length at the place can be read from the Table. This return period analysis will be useful for irrigation planning in the area.

Table 4

Dry sequence length (days) corresponding to a given return period (years)

Td	April	May	September	October	November
25	—		29	29	
15	—	30	26	26	—
10	—	27	23	23	
8	—	25	22	22	
5	—	22	19	19	29
3	28	18	15	16	23
2	20	15	13	13	18

Thus the analysis shows that the model is useful for prediction of dry day sequences in the months of April. May, October and November and provides a comprehensive information on the frequency and distribution of dryspells in Kasaragod.

Summary

The sequential one day dryspells at Kasaragod during weekly and monthly periods have been analysed by fitting a first order Markov chain probability model to the daily rainfall data in relation to potential evapotranspiration at the place. The parameters and properties of the model, dry sequence length corresponding to a return period and probabilities of dry and wet days in different crop phases are presented and disussed. The mode! is useful for prediction of dry day sequences of various length at the place during rainfall transition periods of April, May, October and November.

സംഗ്രഹം

കാസർകോട് പ്രതിദിന വർഷപാതത്തെ ബാഷ്പീകരണ സ്വേദന സാദ്ധ്യതകളുമായി ബന്ധപ്പെടുത്തി ഫസ്റ്റ് ഓഡർ മാർക്കോവ് ചെയിൻ പ്രോബബിലിറ്റി മാതൃക ഉപയോഗിച്ച് പ്രസ്തുത പ്രദേശത്ത് ആഴ്ചയിലും മാസത്തിലും അനുക്രമമായി ഓരോ ദിവസമുണ്ടായ വരണ്ട സമയദൈർഘ്യത്തെപ്പറ്റി അപഗ്രഥനം നടത്തുകയുണ്ടായി. പ്രസ്തുത മാതൃകയുടെ ഘടകങ്ങളുടെ ഗുണവും, അനുക്രമമായുണ്ടാകുന്ന വരണ്ട കാലദൈർഘ്യവും വിവിധ വിളകളിൽ ഉണ്ടാകാൻ സാദ്ധ്യതയുള്ള വരണ്ടതും ഈർപ്പമയമായതുമായ ദിവസങ്ങളെക്കുറിച്ച് പ്രതിപാദിക്കുന്നു. പ്രസ്തുത മാതൃക ഏപ്രിൽ, മെയ്, ഒക്ടോബർ, നവംബർ തുടങ്ങിയ വർഷപാതരഹിതമായ കാലയളവിൽ അനുക്രമമായുണ്ടാകുന്ന വരണ്ട കാലദൈർഘ്യത്തെ മുൻകൂട്ടി നിർണ്ണയിക്കാൻ സഹായകമാണ്,

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