

GABRIEL'S METHOD OF GROUPING TREATMENTS IN ANALYSIS OF VARIANCE

Analysis of variance is used to test the homogeneity of more than two treatments. In case the treatments are found heterogeneous equality of any pair of treatment means is tested using least significant difference and homogeneity of any subset of treatment means, when the number of treatments is more than two, is tested more commonly by any of the multiple range tests. This paper is intended to point out the superiority of F-test for testing the homogeneity of any subset of treatment means and thereby forming groups of homogeneous treatments as suggested by Gabriel (1964) over multiple range tests.

Procedure suggested by Gabriel (1964)

Arrange the treatments by their means from the lowest to the highest, T_1, T_2, \dots, T_n and calculate least significant sum of squares $W_i = F_{i-1, edf} \times EMS \times (i-1)$, $i=2, n-1$ where edf is the error degrees of freedom.

Subdivide the means into two groups of $n-1$ means each, T_1 to T_{n-1} and T_2 to T_n . Calculate sum of squares due to treatments for these groups and compare them with W_{n-1} . If either sum of squares is less than W_{n-1} the treatments in the group are said not to differ significantly. If either sum of squares exceeds W_{n-1} the $n-1$ means are divided into two groups of $n-2$ means each and compared with W_{n-2} . The process continues until a subset of means is obtained for which the sum of squares due to treatments does not exceed the value of W_i . No subset of means is compared if the subset is included in a larger subset which has treatment sum of squares less than the value of W_i .

Example An experiment was conducted in RBD to study the difference in bunch weight of eight varieties of banana with three replications. The variety means were,

V1	V2	V3	V4	V5	V6	V7	V8
12.83	8.50	9.22	12.39	9.83	12.06	11.14	10.72

and error mean square in ANOVA 1.8217 with 14 degrees of freedom.

To facilitate comparison of treatments within various subsets in an ANOVA set up, using this test criterion, following steps can be taken systematically.

1. Arrange the varieties in the ascending order of means.

i.e.,

V2	V3	V5	V8	V7	V6	V4	V1
8.5	9.22	9.83	10.72	11.14	12.06	12.39	12.83

2. Calculate significant sums of squares W_i s

$$W_7 = F_{6, 14} \times EMS \times 6 = 2.85 \times 1.82 \times 6 = 31.1516$$

$$W_6 = F_{5, 14} \times EMS \times 5 = 27.1096$$

$$W_5 = F_{4, 14} \times EMS \times 4 = 22.7867$$

$$W_4 = F_{3, 14} \times EMS \times 3 = 18.3539$$

$$W_3 = F_{2, 14} \times EMS \times 2 = 13.7013$$

$$W_2 = F_{1, 14} \times EMS \times 1 = 8.4260$$

3. Calculate the sum of squares for the two subsets (V2 to V4 and V3 to V1) of seven means each.

Sum of squares for the subset V2 to V4 =

$$(8.5^2 + 9.22^2 + 9.83^2 + 10.72^2 + 11.14^2 + 12.06^2 + 12.39^2) \times 3 - (8.5 + 9.22 + 9.83 + 10.72 + 11.14 + 12.06 + 12.39)^2 \times 3/7 = 42.84473$$

Sum of squares for the subset V3 to V1 =

$$(9.22^2 + 9.83^2 + 10.72^2 + 11.14^2 + 12.06^2 + 12.39^2 + 12.83^2) \times 3 - (9.22 + 9.83 + 10.72 + 11.14 + 12.06 + 12.39 + 12.83)^2 \times 3/7 = 35.86206$$

4. Compare the sum of squares between treatments in each subgroup with the corresponding least significant sum of squares. The sum of squares for the subset of seven means exceeds the least significant sum of squares.

5. Therefore repeat steps 3 and 4 for next smaller subset till the largest subset with non-significant difference in treatments are arrived.

Discussion

Adopting the very same model of analysis of variance it can be proved that likelihood ratio criterion for testing the equality of any subset of treatment means leads to the test criterion.

$$\frac{\text{Between treatment mean square of the subset}}{\text{Error mean square}}$$

which follows F distribution when the null hypothesis holds with numerator degrees of freedom equal to one less than the number of treatments in the subset and denominator degrees of freedom same as the error degrees of freedom.

The least significant sum of squares in the procedure suggested are derived from this test criterion making use of the critical value of F-statistic. In other words the likelihood ratio criterion leads to the procedure discussed here. Since the likelihood ratio criterion leads to the uniformly most powerful test the procedure discussed here is superior to any other test for the purpose. The argument in favour of multiple range test would be easiness of calculation, definitely at the cost of precision. The easiness in calculation should not be taken as a criterion, particularly because of the availability of computers.

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Reference

Gabriel, K. K. 1964. A procedure for testing the homogeneity of all sets of means in analysis of variance. *Biometrics* 20: 459-477.