DESIGNS BALANCED FOR RESIDUAL EFFECTS

By

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THESIS

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DEDICATED TO THE LOVING
MEMMORY OF
LATE DR. P. U. SURENDRAN

DECLARATION

I hereby declare that this thesis entitled "DESIGNS BALANCED FOR RESIDUAL EFFECTS" is a bonafide record of research work done by me during the course of research and that the thesis has not previously formed the basis for the award to me of any degree, diploma, associateship, fellowship, or other similar title, of any other University or Society.

Place: Mannuthy

Date : 24-1-84

SATHIANANDAN, T.V.

CERTIFICATE

Certified that this thesis, entitled " DESIGNS BALANCED FOR RESIDUAL EFFECTS" is a record of research work done independently by Sri. Sathianandan, T.V. under my guidance and supervision and that it has not previously formed the basis for the award of any degree, fellowship, or associateship to him.

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I am also indebted to the staff members of the Department of Statistics, my fellow students and friends for their co-operation and sincere interest in my work.

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Introduction

INTRODUCTION

The problem of residual effects in long term experiments as well as feeding trials are of great concern to Agricultural and Animal Science research workers. Many plans have been designed in this direction by several authors such as Williams (1949), Fenney (1955), Nair (1967) to construct some designs balanced for residual effects. The recent developments in this direction are mainly due to Davis and Hall (1969), Berenblut (1970), Lawless (1971), Gray G. Koch (1972), Saha (1972), Patterson (1973) and Sharma (1982). All these workers have constructed designs balanced for residual effects. The main approach they made was through orthogonal latin squares. Many of this authors have constructed designs balanced for first order residual effects only with the assumption that the residuals will not last for a longer period. But in the case of perennial crops such as coconut, rubber, cashew, cacao etc., the residual effects due to many of the treatments especially manurial treatments will have long term residual effects when applied in different sequences. Hence it is highly essential to find suitable designs to eradicate this defect while planning the experiment. With this objective in view Atkinson (1966) and Nair (1967) constructed certain designs to suit those particuler situation. The main drawback of this layout could be seen while analysing the data - the analysis is very complicated in comparison to the designs balanced for first order residuals.

In experiments on perennial crops involving chemical fertilizers as treatments, the residual effects may not be lasting for more than one period if it is applied in sequence (sufficient gap should be given between two applications such that residuals will not effect the third application in the sequence). This case is very true in the case of Animal Science experiments mainly of feeding trials. Here the experimenter can device ways and means to adjust the sequence of treatments in such a fashion that the residuals may not last for a longer period. Cochran et al. (1941) had formulated a double change over design for dairy cattle feeding experiments in the case of animals.

From all these references mentioned above one can reasonably come to a conclusion that the residual effects of more than first order are not of very serious nature. With this objective in view the present investigation has been conducted.

The present study has been initiated with the objective to construct few designs balanced for residual effects especially balancing for the first order and also to give a simplified analysis of such layouts. In this investigation three methods have been attempted to construct designs balanced for first order residual effects. In the first method the designs are constructed, which are required for a number of treatments, following the lines of Amble (1977). In the second method the construction is based mainly on orthogonal latin squares. The third method is the construction of designs that are balanced for first order residual effects in the same line given by Nair (1967). In this method an attempt has been made to construct a design for t treatments with t sequences and (t-1)(t-2)+ 1 periods.

Finally an attempt has also been made to give a generalised and simplified analysis of designs which are balanced for first order residuals.

Review of Literature

REVIEW OF LITERATURE

In long term experiments the experimental units available will be highly hetrogenous. So each treatment is applied to each experimental unit in different periods. But here a problem comes, that is the effect of a treatment that persists for a period after the application of the treatment, which will effect the yield corresponding to the next treatment applied in succession. The effect of a treatment that persists after the application of the treatment is called residual effect of that treatment. So in long term experiments designs which adjust for residual effects and which allow estimation of both direct and residual effects should be used.

Cochran et al. (1941) described a design consisting of two latin squares in connection with feeding experiments on dairy cows, is as given below:

	Sequences					
Periods	I	II	III	IV	V	VI
I	1	2	3	1	2	3
II	2	3	1	3	1	2
III	3	1	2	2	3	1

But this type of arrangements are limited as the number of treatments are equal to the number of periods.

Williams (1949) gave designs for the situations in which each animal receives each treatment once. He has shown that if the number of treatments is even, balance can be achieved by the suitable choice of a latin square and for an odd number of treatments two such latin squares are required. He has shown that for an even number of treatments such designs can be obtained by permuting the letters occuring in the first row of the latin square in order.

Patterson (1952) gave a method of construction of balanced designs when there are residual effects of treatments. He has given seven conditions for a design to be balanced for first order residuals. For b number of units, k number of periods and v number of treatments, the method given by him consist of b/v latin rectangles each having v columns and k rows. Representing each treatment by one of the elements of an additive abelian group of order v, rectangles are formed such that the differences between successive rows of the rectangle are such that no treatment of the leading sequence are the same and each difference is an element of the additive abelian group. He has given an example for $v = 7$, $k = 3$ and $b = 21$. He has also given a method for the construction of balanced designs based on complete sets of orthogonal latin squares for $k \leqslant v$ by taking k corresponding rows from each of $(v-1)$ orthogonal latin squares of order v.

He has proved that such an arrangement is balanced. A general series for $v = 4n + 3$, a prime or a prime power, is available for n, a positive intiger and with a minimum value $\frac{1}{2}(v-1)$ for b/v and for three periods. If x is the primitive root of the equation $x^{v-1} = 1$, then the differences between rows of the required rectangle are

$$\delta_1 : x^0 \quad x^2 \quad x^4 \quad \ldots \quad x^{v-1}$$

$$\delta_2 : x^1 \quad x^3 \quad x^5 \quad \ldots \quad x^{v-2}$$

A method of construction of a series of designs for $k = \frac{1}{2}(v+1)$ and $b/v = 2$ has also been given.

Lucas (1956) has extended the usual switch back type of design for more than two treatments. Though the switch back designs result in sensitive comparison of treatments, this type of designs are limited to two treatments. He has developed such type of designs for more than two treatments by combining switch back and balanced incomplete principles. For analysis of the design he calculated $D = y_1 - 2y_2 + y_3$ based on the suggestions of Brandt (1938) where y_1, y_2 and y_3 were the performances of an individual in the three periods. He calculated the error variance as

$$S^2 = \frac{\sum_i \sum_j D_{ij}^2 - \frac{(G_1^2 + G_2^2)}{r}}{6 \times 2 \times (r - 1)}$$

with 2(r-1) d.f. where D_{ij} was the performance of the j^{th}

individual in the i^{th} sequence and G_i was the sum of D's for i^{th} sequence, $i = 1,2$; $j = 1,2, \ldots, r$. In this extended model p treatments required p(p-1) treatment sequences. If $p \geq 5$ and odd, designs with p(p-1)/2 sequences could be used. For each reduced design a complimentary design was obtained by writing the second row of the reduced design as the first row and third row of the compliment and the first row of the reduced design as the second row of the compliment. The reduced and complimentary designs together form the complete design and these were sub divided into (p-1) blocks of p sequences each. Examples for p= 3,4,5,6,7 and 9 and the method of analysis using D_{ij}'s has also been given.

Sampford (1957) gave various methods of construction and analysis of serially balanced sequences and the designs based on them. He has given a general method for the construction of Type-2 sequences with index 'k' of the sequence as 1, by an appropriate permutation of the columns of a cyclic latin square of side (t-1) in the case of t treatments. Type 1 and Type 2 sequences were first introduced by Finney and Outhwaite (1955, 1956) in which treatments were arranged in a series of complete blocks in such a way that the residual effects of any treatment occured the same number of times in conjunction with each treatment including itself (type-1 sequence) or of each other (type-2 sequences).

The type -2 sequence for t treatments given by Sampford was

1: a permutation of the numbers 2 to (t-1): 2

and one of the differences occur twice in this sequence and other differences only once. By inserting a column containing the t^{th} treatment only in between the columns which cause for the difference twice, we get the required sequence. Permutation sequences with the required property for $(t-1) > 3$ in cases when $(t-1) = 4r$, $4r+1$ and $4r+3$ were also given. He has discussed the construction of completely reversible type-2 sequences and sets of type-2 sequences with k = 1 based on completely reversible sequences and designs based on them and their selection procedure. A general method of construction of type-1 sequences with k = 1 and k = 2 from cyclic latin squares has also been discussed. Further, he has given the method of analysis of this type of designs.

In many of the designs used earlier the precision of estimation of residual effects were considerably less than that of direct effects. Lucas (1957) gave a type of extra-period latin square change over design which almost entirely overcomes this undesirable aspect of many of the designs. This is because in such designs residual effects are replicated fewer times than are direct effects. Subsequent study by him showed that an extra-period can be provided by simply repeating the last row of the latin square designs and representing periods by rows and treatment sequences by columns. In this type of designs each treatment was followed by every other treatment and itself an equal number of time and the residual effects were orthogonal to sequences.

Patterson and Lucas (1959) has considered a solution for the deficiencies of change-over designs by taking an extra-period along with the basic design. They have shown that in such designs the residual and direct effects are orthogonal and reduces estimated variances of direct and residual effects. They have given five conditions for balance in a basic change-over design. When $t = k = p$, where t is the number of treatments, k is the number of units in each block and p is the number of periods, they obtained complete balance with respect to residual effects by simply repeating the treatments of the p^{th} period in the $(p+1)^{th}$ period. The designs derived from complete sets of orthogonal latin squares require a multiple of $t(t-1)$ units. The method of analysis of extra-period designs also have been given.

Sheehe and Bross (1961) formulated a method of construction of balanced designs based on latin squares. The procedure described by them is as follows:

Write down a cyclic latin square of the order required. Interlace each row of this square with its mirror image and slice the n x 2n figure down the middle, where n is the number of treatments. The columns of each square refer to the order of presentation from left to right and the rows refer to individuals. When n is even both squares are balanced and when n is odd the two squares together gave a balanced design. They have derived a method of analysis of su designs.

Federer and Atkinson (1964) found a general method of construction of tied double change-over designs which can be used in situations wherein the effects of a treatment persists for one period after the treatment is applied. These designs allowed estimation of both direct and residual effects. Two types of constructions, one by using (t-1) orthogonal latin squares for t treatments and the other involving one square for t"even" and two squares for t "odd" have been descriced by them. The method of construction for r = tq+1 rows and c = ts columns, where q and s are positive intigers, involved repeating columns 1,2 and 3 when s is odd and repeating columns 4,5 and 6 for even s. Similarly rows 2,3 and 4 were repeated for q odd and rows 5,6 and 7 were repeated for q even. The method of construction given for t = 4 with 3 latin squares and r = 4q+1 and c = 4s involve repeating columns 1 to 4 for s = 4,7,10, ... and columns 5 to 8 for s = 6,9,12, ... repeating rows 2 to 5 in rows

14 to 17, 26 to 29, ... repeating rows 6 to 9 in rows 18 to 21, 30 to 33, ... and rows 10 to 13 were repeated in rows 22 to 25, 34 to 37, ... An analysis of the design given by them used the linear model

$$Y_{ijh} = N_{ijh}\ (\mu + \gamma_i + \beta_j + \delta_h + \sum_{p=1}^{t} N_{i(i-1)p}\,\rho_p + \epsilon_{ijh})$$

where μ was the general effect, γ_i was the effect of the i^{th} column, β_j was the effect of the j^{th} row, δ_h was the direct effect of h^{th} treatment, ρ_p was the residual effect of p^{th} treatment in the row immediately following the period, ϵ_{ijh} were independently and normally distributed with mean zero and variance σ^2 and $N_{ijh} = 1$ if h^{th} treatment appears in the i^{th} column and j^{th} row, and zero otherwise. $N_{i(i-1)p} = 1$ if p^{th} treatment appeared in the row (j-1) and in the i^{th} column, and zero otherwise.

Berenblut (1964) presented a family of designs from which direct effects and contrasts of direct effects can be estimated without loss of information by confounding and which require 2v periods and v^2 subjects for v treatments. He has generalised the design given by Quenouille (1953) for v = 2 in which the direct and residual effects were orthogonal. In this method for v treatments he represented the treatments by the letters A, B, C, ..., V and defined the following arrangements

$\alpha \equiv$	A	B	C	...	V
$\beta \equiv$	V	A	B	...	U
$\gamma \equiv$	U	V	A	...	T
. . .	. . .	. . .	. . .	. . .	. . .
$\psi \equiv$	D	E	F	...	C
$\phi \equiv$	C	D	E	...	B
$\omega \equiv$	B	C	D	...	A

Then for odd values of v, the design given by him was as given below:

Periods	Subjects (1 to v^2)			
1	α	α	...	α
2	β	γ	...	α
3	γ	γ	...	γ
4	δ	ϵ	...	γ
⋮	⋮	⋮	⋮	⋮
v-1	ϕ	ω	...	ψ
v	ω	ω	...	ω
v+1	ω	α	...	ϕ
v+2	ϕ	ϕ	...	ϕ
⋮	⋮	⋮	⋮	⋮
2 v-1	β	β	...	β
2v	α	β	...	ω

For an even value of v, the lines for periods v and v+1, for periods v-1 and v+2 etc., of the above design are interchanged. The analysis of this designs has also been given.

Atkinson (1966) has given some designs in which treatments form incomplete blocks within experimental units and from which the effect of a sequence of treatments can be estimated. The method of forming designs suggested by him for t treatments and when the residual effects persists for atmost m periods, consist of applying one treatment to an individual for k periods, then applying a different treatment for a further k periods. For attaining balance t(t-1) columns were used such that each treatment is followed by each other treatment an equal number of times and no observations are made from the first (k-1) periods. In this design rows were numbered from the k^{th} row onwards as 1,2, ..., k+1 and columns were indexed by a double index (i,j) where i is the index for first treatment and j is that of the second treatment. He obtained the design with treatments in complete blocks by using the method of construction suggested by Williams. When t, the number of treatments, is even he obtained this design by repeating each row of the basic design k times and in this design each set of t(t-1) i^{th} order sequence will be blocked by the rows of the design into (t-1) subsets and the sequences will be arranged in such a way that in the first row the i^{th} treatment occurs in i^{th} column. In case when t is odd two latin squares were required and the design was obtained by repeating each row of such latin squares k times. In both the cases the first (k-1) rows were not used in the analysis.

Taylor (1967) has used orthogonal polynominals in the analysis of change-over designs with dairy cows and given four types of methods of analysis. The method of replacing observations by orthogonal polynomials of the form $F_{hi} = \sum_j \xi_{ij} Y_{hj}$, where the coefficients ξ_{ij}'s were given in the tables by Fisher and Yates (1963), were given by Patterson (1950; 1951) and Lucas (1951). They obtained the standard deviation of the error terms of F_{hi} as $\left\{\sqrt{\sum_j \xi_{ij}^2}\right\} \sigma_i$ and $\sigma_1^2 \geq \sigma_2^2 \geq \sigma_3^2$ ...., where σ_1^2 is the variance of a unit observation basis of F_{hi}. Also they obtained the estimates of parameters by least square analysis. Another method of analysis suggested by Taylor was to perform a weighted least square analysis with weights as $W_i = 1/\sigma_i^2$ and estimating the parameters by minimising

$$\sum_i W_i \sum_h \frac{(F_{hi} - \phi_{hi})^2}{\sum_j \xi_{ij}^2} .$$

where ϕ_{hi} is a linear function of τ_i, τ_k and $\rho_{(k)}$. A third method described by Cox (1958) was by fitting regression equations of appropriate degree. Patterson and Lucas (1959) performed a least square analysis by omitting the first period and they obtained $\sigma_1^2 \geqslant \sigma_2^2 = \sigma_3^2 = \ldots$, and an almost unbiased estimate of error. Taylor made a comparison of these methods.

Nair (1967) has given one method of construction of serially balanced sequences which are balanced for pairs of residual effects. He has given a method of construction of standard serially balanced sequences also. He has defined serially balanced sequences of order t with index m and balanced for pairs of residual effects as a sequence involving t distinct letters such that any three adjuscent positions are occupied by letters which are all distinct and each of the $t(t-1)(t-2)$ ordered triplets of letters occur serially exactly m times. He denoted this sequence by SBS $(t,m,2)$. According to him a standard serially balanced sequence is an SBS in which the sequence after the initial pair can be broken up into sets of t, such that each set contain distinct letters. The procedure for construction of an SBS $(t,m,2)$ given by him is as follows:

Take all possible non-zero pairs (i,j) of elements of modulus t, where $i \neq j$ and $i + j \neq 0 \bmod (t)$, and form triplets (i,j,i) where $j = i+1, i+2, \ldots, (t-i-1), (t-i+1), \ldots, (t-1)$ when $i < t/2$, t even or $i < (t+1)/2$ if t is odd; and $j = i+1, i+2, \ldots, (t-1)$ when $i \geqslant t/2$, t even or $i \geqslant (t+1)/2$, t odd; and $i = 1, 2, \ldots, (t-2)$. Now from the triplet (i,j,i)

for a fixed i, form a sequence

$i(i+1)i(i+2) \ldots i(t-i-1)i(t-i+1)i \ldots i(t-1)i$

when $i < t/2$, t even or $i < (t+1)/2$, t odd; and the sequence $i(i+1)i(i+2)i \ldots i(t-1)i$ in case when $i \geq t/2$, t even or $i \geq (t+1)/2$, t odd. Then cut off the initial i from this sequence and insert in the sequence $12131 \ldots 1(t-2)1$, just after the number i, which is to be done for all i. From the resulting sequence replace the initial 1 by the pair (1,1) and if $t \neq 4$ and $t > 3$, replace the set of $(t-3)$ numbers $2,3,\ldots,(\frac{1}{2}t-1),(\frac{1}{2}t+1),\ldots,(t-1)$ by pairs $(2,2),(3,3),\ldots,\{(t-1),(t-1)\}$ and when $t = 4$ replace 3 by (3,3) and 1 by (1,1). Denoting the resulting sequence by $\{x_j\}$, $j = 1,2,\ldots,(t-1)(t-2)+1$, construct an arrangement A of t rows and $(t-1)(t-2)+2$ columns whose $(p,q)^{th}$ element is $(p-1)+\sum_{j=0}^{q-1} x_j$; $x_0 = 0$; $1 \leq p \leq t$, $1 \leq q \leq (t-1)(t-2)+2$. Now cut off the initial pair $(t-1, 0)$ from the t^{th} row of A and insert the row so obtained in the $(t-1)^{th}$ row of A just after the pair $(t-1,0)$. Cut off the initial pair $(t-2,t-1)$ of the enlarged $(t-1)^{th}$ row of A and insert in the $(t-2)^{th}$ row just after the pair $(t-2,t-1)$ and so on until we insert the enlarged but truncated second row in the first row just after the pair (1,2). The resulting sequence is an SBS(t,1,2) and from this sequence an SBS(t,m,2), for $t > 3$ and $m > 1$, can be obtained by writing down the sequence SBS (t,1,2) after omiting the initial pair serially exactly m times and putting the last pair of the resulting sequence as its initial pair.

He has also given methods of construction of standard serially balanced sequences (SSBS), one from designs involving t treatments such that the possible t(t-1)(t-2) ordered triplets occur m times in each block having t distinct treatments and the first and last two positions of blocks contain all possible ordered pairs of treatments m times each. One method of forming SSBS (t+1,1,2) from designs with t treatments and in which all possible triplets occur once in each block, each block containing distinct treatments, the blocks can be grouped into t groups of (t-1) blocks each and each block in each group starts with distinct treatments and end in the same treatment has also been given. Another method described by him to form SSBS (t,1,2) was based on the designs given by Williams and still another method of forming SSBS (t,1,2) was based on "round table solutions". He has also been give the methods of analysis of such designs.

Berenblut (1967) gave a design for testing a quantitative factor at four equally spaced levels which is suitable for quick analysis when the presence of first order residual effect was accounted for. The arrangement of the design was such that the three degrees of freedom for the linear, quadratic and cubic components of the residual effects are mutually orthogonal. The design given by him was partially balanced for residual effects. For the purpose of analysis he has used the linear model

$$Y_{ij} = \mu + b_i + p_j + t_L \xi_1 + t_Q \xi_2 + t_C \xi_3 + r_L \eta_1 + r_Q \eta_2 + r_C \eta_3 + \epsilon_{ij},$$

where μ is the mean value b_i is the effect of i^{th} subject, p_j is the effect of j^{th} period, t_L, t_Q, t_C and r_L, r_Q, r_C are linear, quadratic and cubic components of direct and residual effects respectively and ϵ_{ij} are independently and normally distributed with mean zero and variance σ^2. He has also worked out the estimates of parameters and their variances.

Berenblut (1967) obtained the sum of squares due to residual effects adjusted for subjects in the analysis of a change-over design by using (v-1) orthogonal contrasts between the residual effects in the case of v treatments. If $\sum l_i r_i$ is one such contrast, then he obtained the sum of squares associated with the contrast as

$$\left(\sum l_i \left[B_{(i)} + 2\, vR_{(i)} \right] \right)^2 \Big/ \left[2\, v^2\, (4\, v - 2v - 1) \sum l_i^2 \right]$$

where $B_{(i)}$ was the total for subjects whose last treatment is i, $R_{(i)}$ was the total observations receiving the residual effects of treatment i and v was the number of treatments.

Abraham and Jha (1968) made an attempt to analyse critically the data of a series of experiments designed to study the direct and residual effects of phosphate. The design used by them consisted of 12 treatment sequences and treatments were alloted to different plots of a single replication successively for six years. The treatment sequences in the first three years were repeated for the next three years also in this design. In the first sequence treatment P_1 was applied in the first year and no treatments were applied in the next two years. In the second sequence the treatment P_1 was applied in the second year and the same treatment was applied in the third year in sequence 3 and no treatment applied in the rest of the periods. A similar arrangement was made for the next three sequences with P_1 replaced by P_2. In sequences 7 to 12, treatments P_1, P_2, $P_{\frac{1}{2}}$, C, C and C were repeated in the first three years respectively. Here P_1= 22.4 kg P_2O_5/ ha, P_2= 44.8 kg P_2O_5/ha and $P_{\frac{1}{2}}$ = 11.2 kg P_2O_5/ha. C is plots with only a basal dressing of nitrogen.

Berenblut (1968) has given a method of constructing change-over designs for testing treatment factors at equally spaced levels. The design given by him was consisting of n latin squares and required v periods and nv subjects for v treatments. According to him the requirement of such designs was that the degrees of freedom for the linear residual effect should be orthogonal to the linear, quadratic, cubic etc., degrees of freedom of the direct effect. Also the degrees of freedom for the linear direct x linear residual interaction should be orthogonal to each degrees of freedom of the main effects. Under these conditions he has shown that symmetrical designs exist for $v > 3$ treatments and it require (v-2) latin squares for v even and (v-3) latin squares for v odd. He has also been given the methods of construction of non-symmetrical designs. The method of analysis was also given.

Davis and Hall (1969) have shown that when in a cyclic incomplete block design blocks are considered as treatment sequences and rows are taken as periods we will get a class of change-over designs. The cyclic incomplete block design for t treatments and block size p was obtained by developing some number, b, of initial blocks mod (t). Then they obtained the required design by the cyclic development of one or more generating sequences of treatments corresponding to the initial blocks of the cyclic incomplete block design. A special feature of this design is that they may be analysed after any number of periods and further periods may be added if required. They have also given the method of analysis and efficiency of the cyclic change-over designs for different values of t and p.

Berenblut (1970) considered a type of sequences that are fully balanced for quantitative treatments at equally spaced levels and having an index 2. He has formed balanced sequences for four quantitative treatments by arranging the design for four treatments given by Berenblut (1967) in serial form and using the conversion suggested by Sampford (1957). In this design the treatment should be in ascending or descending order. He has constructed a balanced sequence for five quantitative treatments by using two pairs of latin squares to attain serial balance from the designs formed by Berenblut (1963) for five treatments. The designs formulated by him were serially balanced for linear residual effects.

Patterson (1970) has shown that though the designs described by Berenblut (1964) were suitable for the estimation of direct and residual effects, other designs are preferable for the estimation of linear direct x linear residual interaction. He has described about the construction of a large family of designs than is given by Berenblut's method. He has also been considered the efficiency of estimation of linear direct x linear residual interaction and has been shown that these designs have a factorial structure and it could be used to arrange the subjects in blocks with minimum confounding. The method given by him consisted of writing down the 16 combinations of levels in periods one and two and deriving the levels of the remaining periods by permutation of the levels in the first two periods provided by two latin squares G and H of order 4 with the first row of each square in standard order. Then the treatments of odd numbered figures i were obtained by rearranging the treatments of period 1 in the order of row ½(i+1) of G and that of the even numbered period i were obtained by rearranging the treatments of period 2 in the order of row ½ i of H. This designs compared four levels of a single quantitative factor and required only four subjects and four treatments.

Lawless (1971) pointed out that the two general families of PIRD's due to Sportt (1954) yield designs which have a number of properties with respect to balance for residual effects. When v is a prime power Sportt obtained the design by adding 1,2, ..., v-1 (modulo v) to the m initial blocks $B_i(x^i, x^{i+2m}, \ldots, x^{i+4m})$, $i = 0,1,2, \ldots, (m-1)$, where x is a primitive element of GF (v). Lawless has shown that in such designs each treatment will occur next to each other treatment precisely once. When v = mk+1, is a prime power the design was having parameters v = mk+1, $b = mv$, $r = mk$, k, $\lambda = k-1$.

Hanson and Hinkelmann (1971) has been given change-over designs with additivity between direct and residual effects which were useful for testing treatments at equally spaced levels. They have also been given such designs for 2^2 and 3^2 experiments and their analysis.

Gray G. Koch (1972) used non-parametric methods in the analysis of the two period change-over design. In this design two treatments were applied to two groups of individuals in two periods and observations were taken at two periods. He has used the linear model,

$Y_{ijk} = \mu + b_{ij} + \pi_k + \phi_l + \lambda_{l'} + \epsilon_{ijk}$; $i, k, l, l' = 1, 2$ and $j = 1, 2, \ldots, n_i$. Where b_{ij} was the random effect of the j^{th} subject in the i^{th} season, π_k was the effect of the k^{th} period ϕ_l was the direct effect of l^{th} treatment, $\lambda_{l'}$ was the residual effect of l'^{th} treatment, ϵ_{ijk} were distributed independently and normally with mean zero and variance σ_e^2 and b_{ij}'s were distributed independently and normally with mean zero and variance σ_b^2. Then the non-parametric method for testing the equality of residual effects suggested by him was based on the fact that under the hypothesis $\lambda_1 = \lambda_2$, the within subject sums satisfy the same model for the subjects in the two different sequences. Hence a non-parametric statistic was obtained by ranking the sums and adding the ranks in the smaller samples. ie by using Wilcoxon test to the sums. In a similar manner for testing the equality of direct effects in the absence of residual effects, the within subject differences satisfied the same model and hence by ranking the differences and adding the ranks in the smaller samples the non-parametric Wilcoxon tests to the differences could be applied. He obtained a non-parametric statistic for testing period effects in the absence of residual effects by ranking the cross-over differences and by adding the ranks in the smaller samples. The bivariate Wilcoxon statistic for testing the equality of direct and residual effects simultaneously has also been used.

Saha (1972) defined partially balanced change-over designs as a design in which each experimental unit receives a cyclic sequence of several treatments in successive periods and which estimate direct and residual effect with varying degrees of precession. Two types of partially balanced designs which are $\lambda\beta$ designs and $\lambda\gamma$ designs has been defined by him. He described them as follows:

Suppose in a change-over design λ_{ij} is the number of times the treatment pair (i, j) occurs and β_j^i is the number of times the treatment pair (i, j) occurs in sequences with j in the last period and γ_j^i is the number of times treatment j is immediately preceeded in sequences by treatment i, i = j = 0, 1, 2, ..., (v-1).

Then $\lambda_{ij} = \lambda$, $\beta_j^i = \beta$ and $\gamma_j^i = \gamma$ for every i and j then the design become balanced. $\lambda\beta$ design is a design for which $\lambda_{ij} = \lambda$, $\beta_j^i = \beta$ and $\gamma_j^i \neq \gamma$ for every i and j, $i \neq j$ = 0, 1, ..., (v-1). $\lambda\eta$ design is a design for which $\lambda_{ij} = \lambda$, $\eta_{ij} = \eta$ and $\gamma_j^i \neq \gamma$ for every i and j where $\eta_{ij} = \beta_j^i + \beta_i^j$, $\beta_j^i \neq \beta$.

The methods of construction of two series of $\lambda\beta$ designs and one series of $\lambda\eta$ design has also been given. Also he has constructed a $\lambda\beta$ design for k periods and v sequences from the leading sequence $(a_1, a_2, \ldots, a_k)$, where a_i's are distinct elements of GF (v), by adding elements to the sequence. When the number of treatment v is of the form 4n+1, he obtained the leading sequence as

$[0, 2s, (2+4)s, \ldots, (2+4+ \ldots + v-1)s]$ and

$[0, 2as, \ldots, (2+4+\ldots+ v-1)as]$ where s is a non-zero element of GF (v) and 'a' any odd power of the primitive root 'x' of GF (v). In case when v is a prime power the leading sequences $(x^0, x^2, \ldots, x^{v-3}, 0)$ and $(x^1, x^3, \ldots, x^{v-2}, 0)$ reduced modulo v gave a design with (v+1)/2 periods and 2v sequences. For any prime or prime power v, of the form 4n+3 he obtained a $\lambda\eta$ PBCOD from the leading sequence $(x^0, x^2, \ldots, x^{v-3}, 0)$ or $(x^1, x^3, \ldots, x^{v-2}, 0)$ with (v+1)/2 periods and v sequences. A method of analysis of such designs has also been given.

Patterson (1973) has compared and extended the three serial factorial change-over designs given by Quenouille (1953). He has also been considered about the general methods of construction of such designs for even and odd number of treatments separately. These designs required 2t periods and t^2 subjects for t treatments. The method of analysis of such designs has also been given.

Dey and Balachandran (1976) considered switch-over designs with the number of periods less than the number of treatments. The construction given by them has been based on a special class of BIBD and the designs so obtained were totally balanced and they exist for any number of treatments v of the form mk+1, which is a prime or prime power. This designs were having k periods and mv sequences where $k < v$. Sportt (1954) has been shown that for any prime or prime power v, of the form mk+1, a BIBD can be constructed with $v = mk+1$, $b = mv$, $r = mk$ and $\lambda = k-1$ based on the initial block $(x^i, x^{i+m}, x^{i+2m}, \ldots, x^{i+\overline{k-1}m})$ where x is the primitive element of GF(v). They obtained the required design from such BIB designs by taking blocks of the BIBD as columns and by placing the last treatment in each column in a period preceeding the first period. This were having (k+1) periods, mv sequences and (mk+1) treatments. For the purpose of analysis observations from the first period were omited. This design was totally balanced for first residuals. Method of analysis and efficiency factors also have been worked out by them.

Sharma (1982) has been given a method of construction and analysis of extra-period balanced change-over designs for t treatments in 2nt periods using t subjects. He denoted the treatments by 0,1,2, ..., (t-1) and formed two initial sequences of 2t elements each by interlacing the elements of the sequence 0,1,2, ..., t-1 with the elements of the reverse sequence t-1, t-2, ..., 1,0 as

[0, t-1, 1, t-2, ..., t-2, 1, t-1, 0] and
[t-1, 0, t-2, 1, ..., 1, t-2, 0, t-1.]

By developing either of this sequences he obtained an arrangement with t rows and 2t columns. Then by repeating the 2t columns in the same order n times he obtained the required sequence. In this design each treatment occured once in each period and 2n times in each sequence and each treatment was preceeded by every other treatment 2n times and itself by (2n-1) times. He added an extra period at the end by repeating the last period treatments to make the direct and residual effects orthogonal. This design is called extra-period change-over design and was having (2nt+1) periods for t treatments. The methods of analysis both by retaining as well as omiting first period observations has also been given.

Materials and Methods

In the present investigation construction of designs that are balanced for residual effects are being undertaken. This has been done through different approaches.

The first method of construction of balanced design is based on cyclic latin squares. The method is in the line of Amble (1977) as given in the following steps:

1. Write down a cyclic latin square of the order required.
2. Write the latin square which is the mirror image of the cyclic latin square.
3. Interlace the two squares by writing one column of one square and one column of the other square successively and slice in half to get two latin squares.
4. Write the two latin squares with rows as columns and vice versa.

The second method of construction of designs balanced for residual effects is based on orthogonal latin squares. The regular method of construction of orthogonal latin squares is as follows:

Let 0, 1, α, α^2, ..., α^{s-2} be the elements of a Galois field, GF($p^n = s$) where p is a prime number and n a positive intiger and α is the primitive element of the Galois field. If we denote by u_i, the i^{th} element of the Galois field, then the element in the x^{th} row and y^{th} column of the i^{th} orthogonal latin square is given by the expression $u_i\ u_x + u_y$, $i = 1, 2, \ldots, (s-1)$, $x, y = 0, 1, 2, \ldots, (s-1)$ and $u_0 = 0$, $u_1 = 1$, $u_2 = \alpha$, $u_3 = \alpha^2$, ..., $u_{s-1} = \alpha^{s-2}$. There are (s-1) such orthogonal latin squares of order s. When the first latin square (key latin square) is obtained, the rest of them can be obtained by suitable permutation of the rows. If we denote the squares by L_1, L_2, ..., L_{s-1} and the rows of the first square by 0, 1, ..., (s-1), then L_2 can be obtained from L_1 as follows:

From L_1, cut off the first row and add as the last row to obtain L_2. By the same procedure L_3 can be obtained from L_2. Continuing this procedure we can obtain all the (s-1) orthogonal latin squares of order s.

Now for obtaining the first latin square we have the expression $u_1\ u_x + u_y$ which is same as $u_x + u_y$ as $u_1 = 1$. But $u_x + u_y$ is nothing but the addition table of the Galois field after arranging the elements in the order $u_0 = 0$, $u_1 = 1$, $u_2 = \alpha$, $u_3 = \alpha^2$, ..., $u_{s-1} = \alpha^{s-2}$. Now by replacing each element in the addition table by its suffix we will obtain the first latin square. Hence we can obtain all the (s-1) orthogonal latin squares in this way.

A third method of construction of designs which are balanced for first order residual effects which is going to be discussed is based on the construction of serially balanced sequences given by Nair (1967). The method given by him for constructing an SBS $(t,1,2)$ is as follows:

Denote the t treatments by the elements of a residue class mod (t). Consider all possible pairs of non-zero elements (i,j), $i \neq j$, of the residue class such that $i+j \neq 0$ mod (t). From the ordered pairs of types (i,j) and (j,i) form triplets (i,j,i), $j = i+1, i+2, \ldots, (t-i-1), (t-i+1), \ldots, (t-1)$ when $i < t/2$, t even or $i < (t+1)/2$, t odd; $j = i+1, i+2, \ldots, (t-1)$, when $i \geq t/2$, t even or when $i \geq (t+1)/2$, t odd; where $i = 1,2, \ldots, (t-2)$. When t is odd, there are $\frac{1}{2}(t-1)(t-3)$ such triplets and when t is even there are $\frac{1}{2}(t-2)^2$ such pairs. From triplets (i,j,i), for a fixed i, form a sequence $i(i+1)i(i+2)i \ldots i(t-i-1)i(t-i+1)i \ldots i(t-1)i$, when $i < t/2$, t even or when $i < (t+1)/2$, t odd; and the sequence $i(i+1)i(i+2)i \ldots i(t-1)i$, when $i \geq t/2$, t even or when $i \geq (t+1)/2$ t odd. We get $(t-2)$ such sequences when i takes the values $1,2,\ldots,(t-2)$. Now cut off the initial i from the sequence $i(i+1)i(i+2)i \ldots i(t-1)i$ and insert this sequence just after the number i in the sequence $12131 \ldots 1(t-2)1$, which is to be done for all $i = 2,3,\ldots,(t-2)$. In the single sequence so obtained, replace the initial 1 by a pair (1,1). In addition if $t \neq 4$ is even, replace a set of (t-3) numbers 2, 3, ..., $(\frac{1}{2}t-1)$, $(\frac{1}{2}t+1)$, ..., (t-1) in this sequence by pairs (2,2), (3,3), ..., (t-1, t-1). When t = 4, replace a 3 in the sequence by the pair (3,3) in addition to replacing 1 in the sequence by (1,1). Denote the sequence so obtained by $\{x_j\}$, j = 1, 2, ..., (t-1)(t-2)+1. From this basic sequence construct an arrangement A of t rows and (t-1)(t-2)+2 columns such that the $(p,q)^{th}$ element in A is given by $(p-1) + \sum_{j=0}^{q-1} x_j$, where $x_0 = 0$; $1 \leq p \leq t$, $1 \leq q \leq (t-1)(t-2)+2$. Now cut off the initial pair (t-1, 0) from the t^{th} row of A and insert the row so obtained into the $(t-1)^{th}$ row just after the pair (t-1, 0). Then cut off the initial pair (t-2, t-1) from the so enlarged $(t-1)^{th}$ row of A and insert this truncated but enlarged $(t-1)^{th}$ row into the $(t-2)^{th}$ row of A just after the pair (t-2, t-1) in it and so on until we insert the enlarged but truncated second row obtained after cutting off its initial pair (1,2) in it. The resulting sequence is an SBS (t,1,2).

Analysis of the designs using latin squares are being discussed taking into account the first order residual effects.

Results

RESULTS

In the present investigation three different methods of construction of designs which are balanced for first order residual effects are being discussed. A general simplified method of analysis of such designs based on an intuitive method is also tried in this investigation.

The different methods of construction discussed here are (1) method of construction in the line of Amble (1977), (2) method of construction based on orthogonal latin squares and (3) method of construction based on the method given by Nair (1967).

Method 1.

The design required for a number of treatments can be obtained by this method in the following steps.

Step 1. Write down a cyclic latin square of the order required.

Step 2. Write the latin square which is the mirror image of the cyclic latin square.

Step 3. Interlace the two squares by writing one column of one square and one column of the other square successively and slice in half to get two latin squares.

Step 4. Write the two squares with rows as columns and vice versa.

If the number of treatments is even each one of these two latin squares will be balanced and when the number of treatments is odd both of these latin squares together give a balanced arrangement.

By definition, a design is said to be balanced if every letter follows every other letter equally frequently.

A general proof of how this arrangement becomes balanced can be given as follows:

Let there be p treatments. Take a cyclic latin square of order p in numbers 1 to p. We shall assume that the first row is 1,2,3, ..., p. The latin square is symmetric in the numbers. We shall denote this latin square by C and its columns by $C_1, C_2, \ldots, C_p$. Now define another latin square R obtained by placing the columns of C in the reverse order; that is $R = (C_p, C_{p-1}, \ldots, C_2, C_1)$. We shall call R, the mirror reflection of C. Thus for example for p = 4 and 5

$$C = \begin{matrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{matrix} \quad \text{and} \quad R = \begin{matrix} 4 & 3 & 2 & 1 \\ 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \end{matrix}$$

$$C = \begin{matrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 & 4 \end{matrix} \quad \text{and } R = \begin{matrix} 5 & 4 & 3 & 2 & 1 \\ 1 & 5 & 4 & 3 & 2 \\ 2 & 1 & 5 & 4 & 3 \\ 3 & 2 & 1 & 5 & 4 \\ 4 & 3 & 2 & 1 & 5 \end{matrix}$$

We shall now determine the position of 1 in C. If (i,j) denote the i^{th} row j^{th} column position, 1 occurs in C in positions $(1,1)$, $(2,p)$, $(3, p-1)$, $(4,p-2)$, ..., $\{i, p-(i-2)\}$, ..., $(p-1,3)$, $(p,2)$.

The elements in the corresponding positions of R are obtained by remembering the fact that the $(i,j)^{th}$ element of R is $\{i, p-(j-1)\}^{th}$ element of C. Hence these elements are p, $2,4,6$, ..., $2(i-1)$, ..., $(p-4)$, $(p-2)$, where when $2(i-1)$ exceeds p, it will have to be replaced by the remainder after division by p.

In R, the element 1 occurs in positions $(1,p)$, $(2,1)$, $(3,2)$, $(4,3)$, ..., $(i, i-1)$, ..., $(p-1, p-2)$, $(p,p-1)$ and the elements in the corresponding positions of C are, $p,2,4,6$, ..., $2(i-1)$, ... $(p-4),(p-2)$ where, when $2(i-1)$ exceeds p it will have to be replaced by the remainder after division by p.

Further we note that the element in the $(i,j)^{th}$ position of C is $i+j-1$, when this exceeds by p it will have to be replaced by the remainder after division by p.

Now we differentiate the two cases namely $p = 2k$ and $p = 2k+1$.

<u>Case (i)</u> $p = 2k$.

We shall denote the columns of C by $C_1, C_2, \ldots, C_{2k}$ and the same of R by $R_1, R_2, \ldots, R_{2k}$. We now consider the arrangement

$$C_1 R_1 C_2 R_2 \ldots C_k R_k \qquad \text{------} \qquad (1)$$

so that it has 2k rows and 2k columns each containing the 2k numbers exactly because (1) is nothing but the 2k columns of C in some order.

In C_1R_1 the element 1 occurs in R_1 in the second row and the element in the corresponding position of C_1 is 2 and hence the ordered pair (2,1) is obtained. As 1 occurs in the first row and the corresponding position of R_1 is occupied by 2k, the ordered pair (1,2k) also is obtained.

Consider the arrangement $C_1R_1C_2$. The element 1 occurs in C_2 in the $2k^{th}$ row and the element in the corresponding position of R_1 is (2k-1). Hence we get the ordered pair (2k-1, 1). Since 1 occurs in R_1 in the second row and the element in the corresponding position of C_2 is 3, the ordered pair (1,3) is obtained. Thus from $C_1R_1C_2$ we get the ordered pairs (2,1), (2k-1,1), (1,2k), (1,3) each occuring once and these are the only ordered pairs in which 1 can be an element.

Consider the arrangement $C_1R_1C_2R_2$. The element 1 occurs in R_2 in the third row. The element in the corresponding position of C_2 is 4 and hence we get the ordered pair (4,1). Since the element in the $2k^{th}$ row of R_2 is $2k-2$, the ordered pair $(1, 2k-2)$ is also obtained.

It is now clear that the ordered pair in which the element 1 is the second number can arise from $R_{i-1}\ C_i$, $i=2, \ldots, k$ and also from C_iR_i, $i=1,2, \ldots, k$.

We have already seen that the element 1 occurs in C in the position $\{i,\ p-(i-2)\}$. Putting $j = p-(i-2)$ we get $i = p-j+2$. Thus in the i^{th} column of C the element 1 occurs in the $(p-i+2)^{th}$ row. As already observed the $(i,j)^{th}$ element of R is the $\{i,\ p-(j-1)\}^{th}$ element of C. The element in the $(p-i+2)^{th}$ row of R_{i-1} is $(p-i+2,\ i-1)^{th}$ element of R and this is $\{p-i+2,\ p-(i-2)\} = (p-i+2, p-i+2)^{th}$ element of C and this is $(p-i+2)+(p-i+2)-1 = 2p-2i+3$ and this should be replaced by the remainder after division by p when it is greater than p. Since we have taken $p=2k$, $R_{i-1}\ C_i$ gives rise to the ordered pairs $(4k-2i+3,\ 1)$, $i=2,3, \ldots,k$.

Putting $i = 2,3, \ldots, k$, we get the ordered pairs

$(2k-1),1),(2k-3,1),(2k-5,1), \ldots,(5,1),(3,1)$ — — — — (2)

In R_i, the element 1 occur in the $(i+1)^{th}$ row. The element in the $(i+1)^{th}$ row of C_i is $i+(i+1)-1 = 2i$.

Thus C_iR_i, $i=1,2,\ldots,k$ gives the ordered pairs

$$(2,1),(4,1),(6,1),\ \ldots,\ (2k,1) \quad ------ \quad (3)$$

Combining (2) and (3) the element 1 follows every other element in its row once. Hence if $D = C_1R_1C_2R_2 \ldots C_kR_k$ in the columns of D' every element is followed by 1 exactly once. Since this is symmetric in all the elements it follows that every number follows every other number exactly once.

In C_i the element 1 occurs in the $(p-i+2)^{th}$ row and the element in the corresponding position of R_i is same as the element in the $\{p-i+2,\ p-(i-1)\}^{th}$ position of C and this is $p-i+2+p-(i-1)-1 = 2p-2i+2 = 4k-2i+2$ because $p = 2k$. Hence C_iR_i, gives the ordered pair $(1,4k-2i+2)$, $i=1,2,\ldots,k$. That is C_iR_i gives the ordered pairs

$$(1,2k),(1,2k-2),(1,2k-4),\ \ldots,\ (1,2) \quad ------ \quad (4)$$

In R_{i-1}, the element 1 occurs in the i^{th} row. The element in the i^{th} row of C_i is $2i-1$. Hence $R_{i-1}C_i$, $i=2,\ \ldots,\ k$ gives the ordered pair $(1,2i-1)$, $i= 2,3,\ldots,\ k$.
Thus $R_{i-1}C_i$, $i=2,\ \ldots,\ k$ gives the ordered pairs

$$(1,3),(1,5),(1,7),\ldots,\ (1,2k-1) \quad ------ \quad (5)$$

Combining (4) and (5) we see that 1 preceeds every number in the row of D exactly once. This is same as saying that 1 preceeds every number in the columns of D' exactly once.

Putting together all the results of (2),(3),(4) and (5) we see that 1 follows every other number exactly once and it also is followed by each of the other numbers exactly once in the columns of D'. Since the arrangement is symmetric in all the numbers, what is true of 1 is true of other numbers also.

Now define

$$D = R_{k+1}\ C_{k+1}\ R_{k+2}\ C_{k+2}\ \cdots\ R_{2k}\ C_{2k}$$
$$= C_k\ R_k\ C_{k-1}\ R_{k-1}\ \cdots\ C_1 R_1$$

This is D written in the reverse order. In D' every number is followed by every other number. The same property will be true for D' also. This completes the proof for $p = 2k$.

<u>Case (ii)</u> $p = 2k+1$.

Consider the arrangement $C_1 R_1 C_2 R_2 \cdots C_k R_k\ C_{k+1} = D_1$. The ordered pairs involving 1 can arise from pairs of columns of the types $C_i R_i$ or $R_i\ C_{i+1}$.

Take the pairs of the type $C_i R_i$. In R_i, the element 1 occurs in the $(i+1)^{th}$ row. The element in the $(i+1)^{th}$ row of C_i is $(i+1)+(i-1) = 2i$; $i = 1, 2, \ldots, k$. Thus $C_i R_i$ gives rise to pairs of the type

$$(2,1), (4,1), (6,1), \ldots, (2k,1) \quad - - - - - - \qquad (6)$$

Turning to pairs of the type $R_i C_{i+1}$, $i = 1, 2, \ldots, k$. We note that in C_{i+1} the element 1 occurs in the $(p-i+1)^{th}$ row.

The corresponding element in R_i is the $(p-i+1, i)^{th}$ element of R which is the $\{p-i+1, p-(i-1)\}^{th}$ element of C. This element is therefore equal to $(p-i+1)+ p-(i-1)-1 = 2p-2i+1 = 2(2k+1)- 2i+1 = 4k-2i+3$; $i= 1, 2, \ldots, k$.

Thus we get the ordered pairs

$$(2k,1),(2k-2,1),(2k-4,1), \ldots, (4,1),(2,1) \quad - - - - - \quad (7)$$

We note that (6) and (7) are identical. Thus in the rows of D_1, that is in the columns of D_1', each of the pairs in (7) occurs twice.

Now define $E_1 = R_{k+1} C_{k+2} R_{k+2} C_{k+3} \ldots C_{2k+1} R_{2k+1}$

Here also the ordered pairs involving 1 arises from pairs of columns of the type $R_i C_{i+1}$, $i = k+1, k+2, \ldots, 2k$ and of another type $C_i R_i$, $i = k+2, \ldots, 2k+1$.

Take pairs of the type $R_i C_{i+1}$, $i = k+1, \ldots, 2k$. In C_{i+1}, the element 1 occurs in $(p-i+1)^{th}$ row. The corresponding element of R_i is the $(p-i+1, i)^{th}$ of R, which is the $\{p-i+1, p-(i-1)\}^{th}$ element of C and this is $(p-i+1)+(p-i+1)-1 = 2p-2i+1$, $i= k+1, \ldots, 2k$. This gives rise to the ordered pairs

$$(2k+1,1)(2k-1,1),(2k-3,1), \ldots (5,1),(3,1) \quad - - - - \quad (8)$$

Taking the pairs $C_i R_i$, $i = k+2, \ldots, 2k+1$, the element 1 occurs in R_i in the $(i+1)^{th}$ row and the element in the corresponding position of C_1 is $2i$, $i = k+2, k+3, \ldots, 2k+1$. This gives rise to the ordered pairs

$$(3,1),(5,1),(7,1), \ldots, (2k-1,1),(2k+1,1) \qquad ---- (9)$$

Thus in the rows of E_1, that is in the columns of E_1', each of the ordered pairs in (9) occurs twice.

Taking (6),(7),(8) and (9) together we see that the columns of D_1' and E_1' give all the paired differences involving 1 exactly twice.

We further note that when p is odd, the columns of D_1' and E_1' together alone will give all the paired differences. Since the cyclic latin square is symmetric in all the elements what is true of 1 is true for any other number. Hence the proof for p is odd.

<u>Examples</u>:-

(1) p = 5, ie an odd number.

Denote treatments by 1,2,3,4 and 5. Cyclic latin square C required and its mirror image R are

$$C = \begin{matrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 & 4 \end{matrix} \quad \text{and } R = \begin{matrix} 5 & 4 & 3 & 2 & 1 \\ 1 & 5 & 4 & 3 & 2 \\ 2 & 1 & 5 & 4 & 3 \\ 3 & 2 & 1 & 5 & 4 \\ 4 & 3 & 2 & 1 & 5 \end{matrix}$$

Interlacing columns of C and R and slicing in half and writing columns as rows of each latin square we will get the following design:

	Square I					(Sequences)	Square II			
Periods	I	II	III	IV	V	VI	VII	VIII	IX	X
I	1	2	3	4	5	3	4	5	1	2
II	5	1	2	3	4	4	5	1	2	3
III	2	3	4	5	1	2	3	4	5	1
IV	4	5	1	2	3	5	1	2	3	4
V	3	4	5	1	2	1	2	3	4	5

The above arrangement as a whole will give a balanced design with 5 treatments, 10 sequences and in 5 periods.

(2) p = 6 which is even and so two separate designs are possible, which can be obtained by a similar procedure and it is given below:

Treatments are denoted by 1,2,3,4,5, and 6.

(i)

	Sequences					
Periods	I	II	III	IV	V	VI
I	1	2	3	4	5	6
II	6	1	2	3	4	5
III	2	3	4	5	6	1
IV	5	6	1	2	3	4
V	3	4	5	6	1	2
VI	4	5	6	1	2	3

(ii)

Periods	Sequences I	II	III	IV	V	VI
I	4	5	6	1	2	3
II	3	4	5	6	1	2
III	5	6	1	2	3	4
IV	2	3	4	5	6	1
V	6	1	2	3	4	5
VI	1	2	3	4	5	6

(3) p = 7.

If we denote the treatments by 1,2,3,4,5,6 and 7, the required design with 7 periods and 14 treatment sequences is

	Square I							Square II (Sequences)						
Periods	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV
I	1	2	3	4	5	6	7	4	5	6	7	1	2	3
II	7	1	2	3	4	5	6	5	6	7	1	2	3	4
III	2	3	4	5	6	7	1	3	4	5	6	7	1	2
IV	6	7	1	2	3	4	5	6	7	1	2	3	4	5
V	3	4	5	6	7	1	2	2	3	4	5	6	7	1
VI	5	6	7	1	2	3	4	7	1	2	3	4	5	6
VII	4	5	6	7	1	2	3	1	2	3	4	5	6	7

(4) p = 8. Treatments are denoted by 1,2,...,8. Since p is even there are two squares each of which will give balanced design with 8 treatments, 8 sequences and 8 periods.

(i) Sequences

Periods	I	II	III	IV	V	VI	VII	VIII
I	1	2	3	4	5	6	7	8
II	8	1	2	3	4	5	6	7
III	2	3	4	5	6	7	8	1
IV	7	8	1	2	3	4	5	6
V	3	4	5	6	7	8	1	2
VI	6	7	8	1	2	3	4	5
VII	4	5	6	7	8	1	2	3
VIII	5	6	7	8	1	2	3	4

(ii) Sequences

Periods	I	II	III	IV	V	VI	VII	VIII
I	5	6	7	8	1	2	3	4
II	4	5	6	7	8	1	2	3
III	6	7	8	1	2	3	4	5
IV	3	4	5	6	7	8	1	2
V	7	8	1	2	3	4	5	6
VI	2	3	4	5	6	7	8	1
VII	8	1	2	3	4	5	6	7
VIII	1	2	3	4	5	6	7	8

Method 2.

This method of construction is by using orthogonal latin squares. This can be stated in the form of a theorem.

Theorem:- In a set of (s-1) orthogonal latin squares of order s x s, each treatment follows each other treatment exactly (s-1) times.

For the proof of this theorem we require the following preliminary ideas and lemma.

Let us consider the orthogonal latin squares of order 3 and 4. They are

0	1	2		0	1	2
1	2	0		2	0	1
2	0	1		1	2	0

and

0	1	2	3		0	1	2	3		0	1	2	3
1	0	3	2		2	3	0	1		3	2	1	0
2	3	0	1		3	2	1	0		1	0	3	2
3	2	1	0		1	0	3	2		2	3	0	1

Let $A_i^{(3)}$ be the sum of arrangements showing the numbers preceeding i in 3 x 3 latin square and $A_i^{(4)}$ similar sum in the case of 4 x 4 orthogonal latin squares. Then

$$A_0^{(3)} = \begin{bmatrix} - & - & 2 \\ - & 2 & - \end{bmatrix} + \begin{bmatrix} - & 1 & - \\ - & - & 1 \end{bmatrix} = \begin{bmatrix} - & 1 & 2 \\ - & 2 & 1 \end{bmatrix} \quad \text{(by matrix addition)}$$

$$A_1^{(3)} = \begin{bmatrix} 0 & - & - \\ - & - & 0 \end{bmatrix} + \begin{bmatrix} - & - & 2 \\ 2 & - & - \end{bmatrix} = \begin{bmatrix} 0 & - & 2 \\ 2 & - & 0 \end{bmatrix} \quad \text{(by matrix addition)}$$

$$A_2^{(3)} = \begin{bmatrix} - & 1 & - \\ 1 & - & - \end{bmatrix} + \begin{bmatrix} 0 & - & - \\ - & 0 & - \end{bmatrix} = \begin{bmatrix} 0 & 1 & - \\ 1 & 0 & - \end{bmatrix} \quad \text{-do-}$$

$$A_0^{(4)} = \begin{bmatrix} - & 1 & - & - \\ - & - & 3 & - \\ - & - & - & 1 \end{bmatrix} + \begin{bmatrix} - & - & 2 & - \\ - & - & - & 1 \\ - & 2 & - & - \end{bmatrix} + \begin{bmatrix} - & - & - & 3 \\ - & 2 & - & - \\ - & - & 3 & - \end{bmatrix} = \begin{bmatrix} - & 1 & 2 & 3 \\ - & 2 & 3 & 1 \\ - & 2 & 3 & 1 \end{bmatrix}$$

$$A_1^{(4)} = \begin{bmatrix} 0 & - & - & - \\ - & - & - & 2 \\ - & - & 0 & - \end{bmatrix} + \begin{bmatrix} - & - & - & 3 \\ - & - & 0 & - \\ 3 & - & - & - \end{bmatrix} + \begin{bmatrix} - & - & 2 & - \\ 3 & - & - & - \\ - & - & - & 2 \end{bmatrix} = \begin{bmatrix} 0 & - & 2 & 3 \\ 3 & - & 0 & 2 \\ 3 & - & 0 & 2 \end{bmatrix}$$

$$A_2^{(4)} = \begin{bmatrix} - & - & - & 3 \\ 1 & - & - & - \\ - & 3 & - & - \end{bmatrix} + \begin{bmatrix} 0 & - & - & - \\ - & 3 & - & - \\ - & - & - & 0 \end{bmatrix} + \begin{bmatrix} - & 1 & - & - \\ - & - & - & 0 \\ 1 & - & - & - \end{bmatrix} = \begin{bmatrix} 0 & 1 & - & 3 \\ 1 & 3 & - & 0 \\ 1 & 3 & - & 0 \end{bmatrix}$$

$$A_3^{(4)} = \begin{bmatrix} - & - & 2 & - \\ - & 0 & - & - \\ 2 & - & - & - \end{bmatrix} + \begin{bmatrix} - & 1 & - & - \\ 2 & - & - & - \\ - & - & 1 & - \end{bmatrix} + \begin{bmatrix} 0 & - & - & - \\ - & - & 1 & - \\ - & 0 & - & - \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 & - \\ 2 & 0 & 1 & - \\ 2 & 0 & 1 & - \end{bmatrix}$$

In general if (s-1) orthogonal latin squares of order s are considered, assuming that the first row of every latin square is 1,2,....,s , $A_j^{(s)}$ will be a (s-1)x s arrangement in which j^{th} column is blank and every row will contain 1,2,....,s (except j) exactly once.

<u>Lemma - 1</u>:- Let there be (s-1) orthogonal latin squares of order s x s in numbers 1 to s. The first row of each latin

square has numbers 1,2, ...,s in that order in the s columns. Take any (s-1) numbers except j, one in each of the (s-1) columns other than the j^{th} column of one of the orthogonal latin squares P_i, say. If these numbers are distinct a set of elements which fall on these numbers when the (s-1) orthogonal latin squares are superimposed on P_i will contain each of the (s-1) numbers (other than j) (s-2) times.

<u>Proof</u>:- Let $a_1, a_2, \ldots, a_{j-1}, a_{j+1}, \ldots, a_s$ be the numbers in the first, ..., $(j-1)^{th}$, $(j+1)^{th}$, ..., s^{th} columns of P_i. Assume that these are distinct. When (s-1) orthogonal latin squares are superimposed on P_i, the numbers other than j which fall on

a_1, are a_1 and other (s-2) numbers other than j;
a_2, are a_2 and other (s-2) numbers other than j;
etc.

In the set so obtained each of the numbers other than j occur exactly (s-2) times.

<u>Note - 1</u>:- If we take any one row without j the lemma will be satisfied.

<u>Lemma - 2</u>:- Let there be (s-1) orthogonal latin squares of order s x s in the numbers 1,2,...., s. The first row of each latin square has numbers 1,2,....,s in that order in the s columns. Take any one of the orthogonal latin squares,

say P_i, and take (s-1) numbers other than j in (s-1) columns excluding the j^{th} column. If the set of elements which fall on these numbers, when the (s-1) orthogonal latin squares are superimposed on P_i, contain each of the numbers other than j equally frequently, the (s-1) numbers taken in P_i are all distinct.

Proof:- We shall establish the result by obtaining a contradiction. If possible let there be two identical numbers, say a_1, a_1 among the (s-1) numbers taken in P_i in (s-1) columns other than the j^{th}. Let us assume, without loss of generality that (a_1, a_1) occurs in the first two columns. It then follows that when (s-1) orthogonal latin squares are superimposed on P_i the numbers which fall on a_1

in the first column are a_1 and (s-2) numbers other than j

in the second column are a_1 and (s-2) numbers other than j.

In the first of these 1 will not be present and in the second 2 will not be present. Thus in the overall set of elements a_1 will be present (s-1) times and each of the other numbers other than j atmost (s-2) times. Hence, if the set is to contain all numbers other than j, equally frequently, the orthogonal set from P_i should contain (s-1) distinct numbers.

Note:- If a multiple of (s-1) numbers are taken in P_i, lemma - 2 will be true if each of the (s-1) distinct numbers occurs equally frequently.

Now take one of the orthogonal latin squares P_i and replace all members other than those immediately preceding j by zero. Denote the square so obtained by D_i, i=1,2,...,(s-1). Add D_i, i=1,2, ...,(s-1) as if they are matrices of order s x s. Let $D = D_1 + \ldots + D_{s-1}$. Then D is a square with j^{th} column and last row containing zero only. In other places we get elements which precede j. Superimpose the (s-1) orthogonal latin squares over the square and obtain the set of numbers, excluding j, which fall on the non-zero element of D. By lemma-1 each of the numbers other than j will occur equally frequently in this set and therefore by lemma-2 the basic set consisting of the non-zero elements of D will contain each of the numbers 1,2, ..., s excepting j equally frequently.

Proof of the theorem stated:-

We shall denote the elements of the Galois Field GF $(s=p^n)$ where p is a prime by

$$u_0 = 0,\ u_1 = 1,\ u_2 = \alpha,\ u_3 = \alpha^2,\ \ldots,\ u_{s-1} = \alpha^{s-2}$$

where α is a primitive element of GF(s). Then if we put j, where j is defined by $u_i u_x + u_y = u_j$, in the x^{th} row and y^{th} column of i^{th} latin square, i=1,2,..., s-1, we get (s-1) orthogonal latin squares of order s. In all the (s-1) squares j occur in the $(j+1)^{st}$ column of the first row. Excepting this, for any given i

$$u_i u_x + u_y = u_j \qquad ------ \qquad (1)$$

has (s-1) solutions. The element preceeding j in the same column when (1) holds true is j' given by

$$u_i u_{x-1} + u_y = u_{j'} \qquad ------ \qquad (2)$$

Hence taking the difference between (1) and (2) we get

$$u_i \; (u_x - u_{x-1}) = (u_j - u_{j'}) \qquad ------ \qquad (3)$$

Since $u_x \neq u_{x-1}$ and $u_j \neq u_{j'}$, the equation (3) has a non-zero solution. The equation (2) has exactly (s-1) solutions for a fixed i. As this is true for all i = 1,2, ..., (s-1), in the (s-1) orthogonal latin squares (s-1)(s-1) numbers immediately preceed j. We have already seen that in this set each of the (s-1) numbers other than j occurs equally frequently. Thus each number preceeds j exactly (s-1) times in the (s-1) orthogonal latin squares.

<u>Corollary - 1</u>:- In (s-1) orthogonal latin squares of order s each number will precede other numbers exactly (s-1) times. The result follows immediately from the proof of the theorem.

<u>Corollary- 2</u>:- Since every pair of treatments occur the same number of times every sequence will occur the same number of times.

<u>Examples</u>:-

(1) s = 5. The orthogonal latin squares in which the treatments are denoted by 0,1,2,3 and 4 are given below which as a whole will give the required balanced design for 5 number of

treatments, with 20 treatment sequences and 5 periods.

(i)

0	1	2	3	4
1	2	4	0	3
2	4	3	1	0
3	0	1	4	2
4	3	0	2	1

(ii)

0	1	2	3	4
2	4	3	1	0
3	0	1	4	2
4	3	0	2	1
1	2	4	0	3

(iii)

0	1	2	3	4
3	0	1	4	2
4	3	0	2	1
1	2	4	0	3
2	4	3	1	0

and (iv)

0	1	2	3	4
4	3	0	2	1
1	2	4	0	3
2	4	3	1	0
3	0	1	4	2

(2) s = 7. elements of GF (7) are 0,1,3,2,6,4,5 and the 6 orthogonal latin squares of order 7 are:

(i)

0	1	2	3	4	5	6
1	3	5	2	0	6	4
2	5	4	6	3	0	1
3	2	6	5	1	4	0
4	0	3	1	6	2	5
5	6	0	4	2	1	3
6	4	1	0	5	3	2

(ii)

0	1	2	3	4	5	6
2	5	4	6	3	0	1
3	2	6	5	1	4	0
4	0	3	1	6	2	5
5	6	0	4	2	1	3
6	4	1	0	5	3	2
1	3	5	2	0	6	4

(iii)

0	1	2	3	4	5	6
3	2	6	5	1	4	0
4	0	3	1	6	2	5
5	6	0	4	2	1	3
6	4	1	0	5	3	2
1	3	5	2	0	6	4
2	5	4	6	3	0	1

(iv)

0	1	2	3	4	5	6
4	0	3	1	6	2	5
5	6	0	4	2	1	3
6	4	1	0	5	3	2
1	3	5	2	0	6	4
2	5	4	6	3	0	1
3	2	6	5	1	4	0

(v)

0	1	2	3	4	5	6
5	6	0	4	2	1	3
6	4	1	0	5	3	2
1	3	5	2	0	6	4
2	5	4	6	3	0	1
3	2	6	5	1	4	0
4	0	3	1	6	2	5

and (vi)

0	1	2	3	4	5	6
6	4	1	0	5	3	2
1	3	5	2	0	6	4
2	5	4	6	3	0	1
3	2	6	5	1	4	0
4	0	3	1	6	2	5
5	6	0	4	2	1	3

Method - 3:-

This method of construction of designs that are balanced for first order residual effects for t treatments with t sequences and $(t-1)(t-2)+1$ periods is as follows:

Denote the treatments by $0, 1, 2, \ldots, (t-1)$. Form pairs of the form (i, j) where i and j are non-zero elements of the residue class modulus t, $i \neq j$ and $i+j \neq 0 \bmod (t)$. From pairs of the type (i,j) and (j,i) form triplets (i,j,i) for $j = i+1, i+2, \ldots, (t-i-1), (t-i+1), \ldots, (t-1)$ when $i < t/2$ for t even or $i < (t+1)/2$, for t odd and $j = (i+1), (i+2), \ldots, (t-1)$ when $i \geq t/2$ for t even or $i \geq (t+1)/2$, t odd. Now for triplets (i,j,i) for a fixed i, form a sequence $i(i+1)i(i+2)i \ldots i(t-i-1)i(t-i+1)\ldots i(t-1)i$ when $i < t/2$, t even or $i < (t+1)/2$, t odd; and the sequence $i(i+1)i\ldots i(t-1)i$ when $i \geq t/2$ t even or $i \geq (t+1)/2$, t odd. This is to be found for $i = 1,2,\ldots,(t-2)$. Now cut off the initial i from the sequence $i(i+1)i\ldots i(t-1)i$ and insert in the sequence $12131\ldots1(t-2)1$ just after the number i. This is to be done for $i = 2,3,\ldots,(t-2)$. Then replace the initial 1 in the sequence by the pair (1,1) and when $t \neq 4$ is even, replace the $(t-3)$ numbers $2,3,\ldots,(\frac{1}{2}t-1),(\frac{1}{2}t+1),\ldots,(t-2)$ by the pairs $(2,2)(3,3)\ldots,(t-2,t-2)$ respectively. When $t = 4$, replace 3 by (3,3) and 1 by (1,1). Denote the sequence so obtained by $\{x_j\}$, $j = 1,2,\ldots,(t-1)(t-2)+1$. Now form a matrix $A = (a_{pq})$, whose element in the p^{th} row and q^{th} column is given by $a_{pq} = (p-1) + \sum_{j=0}^{q-1} x_j$, where $x_0 = 0$, $1 \leq p \leq t$, $1 \leq q \leq (t-1)(t-2)+1$ Then A will be of order $t \times \{(t-1)(t-2)+1\}$. If now the rows of A' are taken as periods and columns of A' are taken as sequences we will get a balanced design which will be balanced for the first order residual effects, with t treatments, t sequences and $(t-1)(t-2)+1$ periods.

Examples:-

(1) t = 4.

Denote treatments by 0,1,2 and 3. Possible pairs of the form(i,j), $i \neq j$, $i+j \neq 0$, $i,j, \neq 0$ are (1,2), (2,1), (2,3) and (3,2) and possible triplets are (1,2,1) and (2,3,2) and corresponding sequences are 121 and 232. Inserting this sequences in the basic sequence 121 we get the sequence 12321. Replacing 1 by (1,1) and 3 by (3,3) we will get the sequence $\{x_j\}$, as 1123321. Now forming the matrix $A = (a_{pq})$ where $a_{pq} = (p-1) + \sum_{j=0}^{q-1} x_j$, $x_0 = 0$, $1 \leq p \leq 4$, $1 \leq q \leq 7$, we get

$$A = \begin{bmatrix} 0 & 1 & 2 & 0 & 3 & 2 & 0 \\ 1 & 2 & 3 & 1 & 0 & 3 & 1 \\ 2 & 3 & 0 & 2 & 1 & 0 & 2 \\ 3 & 0 & 1 & 3 & 2 & 1 & 3 \end{bmatrix}$$

Hence the required design is

	Sequences			
Periods	I	II	III	IV
I	0	1	2	3
II	1	2	3	0
III	2	3	0	1
IV	0	1	2	3
V	3	0	1	2
VI	2	3	0	1
VII	0	1	2	3

(2) $t = 5$. Treatments are denoted by 0,1,2,3 and 4. Possible pairs satisfying the required conditions are (1,2), (1,3), (2,4), (2,1), (3,1), (3,4), (4,2) and (4,3). Triplets formed from these pairs are (1,2,1), (1,3,1), (2,4,2) and (3,4,3). Sequences based on these triplets are 12131, 242 and 343. Inserting the last two sequences in the basic sequence we get the sequence 124213431. Replacing 1 by (1,1), 2 by (2,2), 3 by (3,3) and 4 by (4,4) we get the sequence 1124221334431. Then the matrix $A' = (a_{pq})'$ where $a_{pq} = (p-1) + \sum_{j=0}^{q-1} x_j$ is

$$A' = \begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \\ 2 & 3 & 4 & 0 & 1 \\ 4 & 0 & 1 & 2 & 3 \\ 3 & 4 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 0 & 1 \\ 3 & 4 & 0 & 1 & 2 \\ 1 & 2 & 3 & 4 & 0 \\ 4 & 0 & 1 & 2 & 3 \\ 3 & 4 & 0 & 1 & 2 \\ 2 & 3 & 4 & 0 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{matrix}$$

In the above arrangement rows represent periods and columns represent treatment sequences.

(3) $t = 6$. Treatments are 0,1,2,3,4 and 5 and possible pairs are (1,2), (1,3), (1,4), (2,1), (2,3), (2,5), (4,1), (4,3), (4,5), (3,1), (3,2), (3,4), (3,5), (5,2), (5,3) and (5,4).

Triplets are (1,2,1), (1,3,1), (1,4,1), (2,3,2), (2,5,2), (3,4,3), (3,5,3) and (4,5,4). Sequences generated by these triplets are 1213141; 23252; 34353 and 454. Inserting the last three sequences in the sequence 1213141 we get the sequence 12325213435314541. Now replacing the elements 1,2,4 and 5 respectively by pairs we get the sequence $\{x_j\}$ as 112322552134435314541. Now define $A = (a_{pq})$, where $a_{pq} = (p-1) + \sum_{j=1}^{q-1} x_j$, we can get A' as

$$A' = \begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 & 0 \\
2 & 3 & 4 & 5 & 0 & 1 \\
4 & 5 & 0 & 1 & 2 & 3 \\
1 & 2 & 3 & 4 & 5 & 0 \\
3 & 4 & 5 & 0 & 1 & 2 \\
5 & 0 & 1 & 2 & 3 & 4 \\
4 & 5 & 0 & 1 & 2 & 3 \\
3 & 4 & 5 & 0 & 1 & 2 \\
5 & 0 & 1 & 2 & 3 & 4 \\
0 & 1 & 2 & 3 & 4 & 5 \\
3 & 4 & 5 & 0 & 1 & 2 \\
1 & 2 & 3 & 4 & 5 & 0 \\
5 & 0 & 1 & 2 & 3 & 4 \\
2 & 3 & 4 & 5 & 0 & 1 \\
1 & 2 & 3 & 4 & 5 & 0 \\
4 & 5 & 0 & 1 & 2 & 3 \\
5 & 0 & 1 & 2 & 3 & 4 \\
3 & 4 & 5 & 0 & 1 & 2 \\
2 & 3 & 4 & 5 & 0 & 1 \\
0 & 1 & 2 & 3 & 4 & 5
\end{array}$$

(4) t = 7. Treatments are denoted by 0,1,2,3,4,5 and 6. Then by a similar procedure as in the above cases we can get the required design as follows:-

0	1	2	3	4	5	6
1	2	3	4	5	6	0
2	3	4	5	6	0	1
4	5	6	0	1	2	3
0	1	2	3	4	5	6
2	3	4	5	6	0	1
4	5	6	0	1	2	3
1	2	3	4	5	6	0
5	6	0	1	2	3	4
0	1	2	3	4	5	6
6	0	1	2	3	4	5
5	6	0	1	2	3	4
0	1	2	3	4	5	6
1	2	3	4	5	6	0
4	5	6	0	1	2	3
0	1	[illegible]	3	4	5	6
5	6	0	1	2	3	4
3	4	5	6	0	1	2
6	0	1	2	3	4	5
5	6	0	1	2	3	4
1	2	3	4	5	6	0
2	3	4	5	6	0	1
6	0	1	2	3	4	5
4	5	6	0	1	2	3
1	2	3	4	5	6	0
0	1	2	3	4	5	6
4	5	6	0	1	2	3
5	6	0	1	2	3	4
3	4	5	6	0	1	2
2	3	4	5	6	0	1
0	1	2	3	4	5	6

(5) t = 8. Denoting treatments by 0,1,2,...,7 and obtaining the matrix A we can get the required design as given below:

0	1	2	3	4	5	6	7
1	2	3	4	5	6	7	0
2	3	4	5	6	7	0	1
4	5	6	7	0	1	2	3
7	0	1	2	3	4	5	6
1	2	3	4	5	6	7	0
3	4	5	6	7	0	1	2
7	0	1	2	3	4	5	6
1	2	3	4	5	6	7	0
6	7	0	1	2	3	4	5
3	4	5	6	7	0	1	2
5	6	7	0	1	2	3	4
4	5	6	7	0	1	2	3
3	4	5	6	7	0	1	2
5	6	7	0	1	2	3	4
6	7	0	1	2	3	4	5
1	2	3	4	5	6	7	0
4	5	6	7	0	1	2	3
0	1	2	3	4	5	6	7
3	4	5	6	7	0	1	2
1	2	3	4	5	6	7	0
7	0	1	2	3	4	5	6
2	3	4	5	6	7	0	1
1	2	3	4	5	6	7	0
4	5	6	7	0	1	2	3
5	6	7	0	1	2	3	4
1	2	3	4	5	6	7	0
6	7	0	1	2	3	4	5
2	3	4	5	6	7	0	1
0	1	2	3	4	5	6	7
4	5	6	7	0	1	2	3
3	4	5	6	7	0	1	2
7	0	1	2	3	4	5	6
0	1	2	3	4	5	6	7
5	6	7	0	1	2	3	4
3	4	5	6	7	0	1	2
0	1	2	3	4	5	6	7
7	0	1	2	3	4	5	6
4	5	6	7	0	1	2	3
5	6	7	0	1	2	3	4
3	4	5	6	7	0	1	2
2	3	4	5	6	7	0	1
0	1	2	3	4	5	6	7

(6) $t = 9$. The design obtained in this case is as follows:

S e q u e n c e s

I	II	III	IV	V	VI	VII	VIII	IX
0	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	0
2	3	4	5	6	7	8	0	1
4	5	6	7	8	0	1	2	3
7	8	0	1	2	3	4	5	6
0	1	2	3	4	5	6	7	8
2	3	4	5	6	7	8	0	1
6	7	8	0	1	2	3	4	5
1	2	3	4	5	6	7	8	0
3	4	5	6	7	8	0	1	2
8	0	1	2	3	4	5	6	7
4	5	6	7	8	0	1	2	3
6	7	8	0	1	2	3	4	5
3	4	5	6	7	8	0	1	2
0	1	2	3	4	5	6	7	8
2	3	4	5	6	7	8	0	1
1	2	3	4	5	6	7	8	0
0	1	2	3	4	5	6	7	8
2	3	4	5	6	7	8	0	1
3	4	5	6	7	8	0	1	2
6	7	8	0	1	2	3	4	5
0	1	2	3	4	5	6	7	8
4	5	6	7	8	0	1	2	3
7	8	0	1	2	3	4	5	6
3	4	5	6	7	8	0	1	2
6	7	8	0	1	2	3	4	5
4	5	6	7	8	0	1	2	3
2	3	4	5	6	7	8	0	1
5	6	7	8	0	1	2	3	4

(contd....)

Sequences

I	II	III	IV	V	VI	VII	VIII	IX
4	5	6	7	8	0	1	2	3
7	8	0	1	2	3	4	5	6
8	0	1	2	3	4	5	6	7
3	4	5	6	7	8	0	1	2
0	1	2	3	4	5	6	7	8
4	5	6	7	8	0	1	2	3
2	3	4	5	6	7	8	0	1
6	7	8	0	1	2	3	4	5
5	6	7	8	0	1	2	3	4
0	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	0
6	7	8	0	1	2	3	4	5
3	4	5	6	7	8	0	1	2
8	0	1	2	3	4	5	6	7
6	7	8	0	1	2	3	4	5
2	3	4	5	6	7	8	0	1
1	2	3	4	5	6	7	8	0
6	7	8	0	1	2	3	4	5
7	8	0	1	2	3	4	5	6
4	5	6	7	8	0	1	2	3
2	3	4	5	6	7	8	0	1
8	0	1	2	3	4	5	6	7
7	8	0	1	2	3	4	5	6
4	5	6	7	8	0	1	2	3
5	6	7	8	0	1	2	3	4
3	4	5	6	7	8	0	1	2
2	3	4	5	6	7	8	0	1
0	1	2	3	4	5	6	7	8

Here also rows represent periods and columns represent treatment sequences.

ANALYSIS

A general simple method of analysis by using intuitive method can be given as follows:-

Let there be v treatments applied in v periods and rv sequences in r squares of order v by using a balanced design for first order residual effects. Here we assume that the direct and residual effects are additive and the residual effects last only upto the next period.

If Y_{ijk} is the yield of the i^{th} treatment in the j^{th} period in the k^{th} sequence, then the linear model is

$$Y_{ijk} = \mu + t_i + r_j + s_k + \epsilon_{ijk}$$

where μ is the general effect, t_i is the effect of the i^{th} treatment, r_j is the residual effect of the j^{th} treatment, s_k is the effect of the k^{th} sequence and ϵ_{ijk} are distributed as independently and normally with mean zero and variance σ^2, $i = 1,2, \ldots, v$; $j = 1,2, \ldots, v$; $k = 1,2,\ldots, rv$.
If T_i is the total of all observations for the i^{th} treatment, then $T_i = rv\mu + rvt_i + \lambda \sum_{j \neq i = 1}^{v} r_j + v \sum_k s_k$ where is the number of times a treatment followed by other treatments.

ie $T_i = vr\,\mu + rvt_i - \lambda r_i$ as $\sum_j r_j = 0$ and $\sum_k s_k = 0$.

If R_i denote the sum of all observations for the treatments just succeeded by i^{th} treatment, then

$$T_i = r(v-1)\,\mu + r(v-1)\,r_i + \lambda\Big(\sum_{j\neq i} t_j\Big) + \sum_{k'(i)} s_{k'}$$

where $\sum_{k'(i)} s_{k'}$ is the summation taken over all sequence effects except for the sequences ending with the treatment i.

$$R_i = r(v-1)\,\mu + r(v-1)\,r_i - \lambda t_i - \sum_{(i)} s_{(i)} \quad \text{where} \quad \sum_{(i)} s_{(i)}$$

denotes the sum of the effects of sequences in which the last treatment is i. If S_i denote the sum of totals of columns which ends with the treatment i, then

$$S_i = rv\,\mu + v\sum_{(i)} s_{(i)} + r\sum_{j\neq i} r_j = rv\,\mu + v\sum_{(i)} s_{(i)} - r\,r_i$$

Now $T_i + vR_i + S_i = rv(v+1)\,\mu \quad v(r-\lambda)t_i + \left[rv(v-1)-(\lambda+r)\right]\; r_i$

But since the treatment i can proceed any of the other (v-1) treatments in the r(v-1) sequences we get

$$r(v-1) = \lambda(v-1) \text{ ie } r = \lambda$$

Hence $T_i + vR_i + S_i = rv\;(v+1)\,\mu + (v^2-v-2)r\;r_i$

Now if G is the grand total of $r\,v^2$ observations then μ can be estimated as $\frac{G}{rv^2}$. Hence we can estimate r_i's

Now $(v^2-v-1)\;T_i = vr(v^2-v-1)\;\mu - vr(v^2-v-1)t_i - r(v^2-v-1)r_i$

$$vR_i = v(v-1)r\;\mu + rv(v-1)r_i - vrt_i - v\sum_{(i)} s_{(i)}$$

$$S_i = vr\;\mu + v\sum_{(i)} s_{(i)} - r\,r_i.$$

$\therefore\; (v^2-v-1)\;T_i + vR_i + S_i = v(v^2-1)r\;\mu + v(v^2-v-2)r\;t_i$

Hence t_i can also be estimated.

Now $$\frac{T_i + vR_i + S_i}{(v^2-v-2)r} = \frac{v(v-1)}{(v^2-v-2)}\mu + r_i$$

$$\therefore \quad r_i - r_j = \frac{Q_i - Q_j}{(v-2)(v+1)r} \quad \text{where } Q_i = T_i + vR_i + S_i$$

$$V(r_i - r_j) = \frac{V(Q_i) - 2\,\mathrm{Cov}\,(Q_i, Q_j) + V(Q_j)}{(v-2)^2\,(v+1)^2\,r^2} \qquad - - - - - \quad (1)$$

Since T_i is the total of all observations for the treatment it is a sum of rv independent observations. So we get

$$V(T_i) = rv\,\sigma^2.$$

Since R_i is the total of observations in periods just succeeded by the treatment i, it is a sum of r(v-1) independent observations. Hence

$$V(R_i) = r(v-1)\,\sigma^2.$$

Since S_i is the sum of observations of sequences ending with treatment i it is a sum of rv independent observations. So we get

$$V(S_i) = rv\,\sigma^2.$$

$\mathrm{Cov}(T_i, R_i) = 0$ because there is no observation common to them. Similarly

$$\mathrm{Cov}(T_i, S_i) = r\,\sigma^2; \quad \mathrm{Cov}(R_i, S_i) = 0.$$

Hence

$$V(Q_i) = V(T_i - vR_i - S_i) = [2r(v+1) + v^2 r(v-1)]\,\sigma^2 - - - - \quad (2)$$

$$\mathrm{Cov}(Q_i, Q_j) = \mathrm{Cov}(T_i + vR_i + S_i,\ T_j + vR_j + S_j)$$
$$= (4\,vr + 2r)\,\sigma^2 \qquad - - - - - \qquad (3)$$

Substituting (2) and (3) in (1) and simplifying we can get

$$V(r_i - r_j) = \frac{2v}{r(v^2-v-2)}\,\sigma^2$$

We have

$$(v^2-v-1)T_i + vR_i + S_i = v(v^2-1)r\,\mu + v(v^2-v-2)r\,t_i$$

$$\therefore \quad \frac{(v^2-v-1)T_i + vR_i + S_i}{vr(v^2-v-2)} = \frac{(v^2-1)}{(v^2-v-2)}\,\mu + t_i$$

$$\therefore \quad t_i - t_j = \frac{L_i - L_j}{vr(v^2-v-2)} \text{ where } L_i = (v^2-v-1)T_i + vR_i + S_i$$

$$V(t_i - t_j) = \frac{V(L_i) + V(L_j) - 2\mathrm{Cov}(L_i, L_j)}{v^2 r^2\,(v^2-v-2)^2} \qquad - - - - \qquad (4)$$

Now $V(L_i) = V\left[(v^2-v-1)\,T_i + vR_i + S_i\right]$

$$= \left[(v^2-v-1)^2 vr + v^2(v-1)r + vr + 2(v^2-v-1)r\right]\sigma^2 - (5)$$

Similarly

$$V(L_j) = \left[(v^2-v-1)^2 vr + v^2(v-1)r + vr + 2(v^2-v-1)r\right]\sigma^2 - - - (6)$$

$$\mathrm{Cov}(L_i, L_j) = \left[2v^2 r(v-1) + 2r(v^2-v-1)\right]\sigma^2 \qquad - - - - \qquad (7)$$

Substituting (5), (6) and (7) in (4) and simplifying we can get

$$V(t_i - t_j) = \frac{2(v^2-v-1)}{vr(v^2-v-2)}\,\sigma^2$$

Here the sum of squares due to residual effects adjusted for direct effects is

$$= \frac{\sum_i (T_i + vR_i + S_i)^2 - \frac{\left[\sum_i (T_i + vR_i + S_i)\right]^2}{v}}{rv\,(v^2-v-2)}$$

and sum of squares due to direct effects adjusted for residual effects is

$$= \frac{\sum_i \left[(v^2-v-1)\; T_i + vR_i + S_i\right]^2 - \frac{\left[\sum_i \{(v^2-v-1)T_i + vR_i + S_i\}\right]^2}{v}}{rv(v^2-v-1)\;(v^2-v-2)}$$

The error sum of squares can be obtained as

Error SS = Total SS – SS between squares – SS between periods

– SS due to period x square interaction

– SS between animals within squares

– SS due to residual effects (adjusted)

– SS due to direct effects (unadjusted)

Here the different sum of squares like sum of squares between squares, sum of squares between periods, sum of squares due to period x square interaction, sum of squares between animals within squares and sum of squares due to direct effects (unadjusted) can be obtained as usual.

The analysis of variance table will take the following form.

A N O V A

Source	df	
Between squares	(r-1)	
Between periods	(v-1)	
Period x square	(r-1)(v-1)	
Between animals within squares	r(v-1)	
Residuals (adjusted)	(v-1)	Residuals (unadjusted)
	or	
Direct (unadjusted)	(v-1)	Direct (adjusted)
Error		
Total	$r\ v^2-1$	

Illustrative Examples:

(1) The following are the data obtained from six Sindhi cows in an experiment conducted by Krishnankutty (1969) at the District Livestock Farm, Mannuthy. The experiment was conducted to study the effect of certain commercial compounded feeds on milk production in cattle. The animals were divided into two sets of three each, each animal in a set being subjected at random to one of the three feeds to

start with. The total duration of the experiment with each feed was 63 days divided into three equal periods of 21 days. The experimental design adopted in this case was the "switch-over design" as given below:-

	Set I			Cows	Set II		
Periods	1	2	3		4	5	6
I	A	B	C		A	B	C
II	B	C	A		C	A	B
III	C	A	B		B	C	A

Here A = Al brand cattle feed; B = Hindlever Cattle feed and C = Pushti Cattle feed.

Milk yields in kg of the individual cows for 3 x 3 weeks are given below:

	Set I			Set II		
Animal No.	853	815	747	827	767	849
Period I	A 84	B 77	C 83	A 85	B 104	C 105
Period II	B 92	C 62	A 96	C 92	A 100	B 119
Period III	C 79	A 60	B 90	B 97	C 115	A 108

Analysis:

Totals for squares : Q_1 = 723; Q_2 = 925

Totals for sequences : S_{ij}: 255 199 269 274 319 332

Grand Total : G = 1648.

Treatment No (i)	T_i	R_i	S_i	$T_i+vR_i+S_i$	$(v^2-v-1)T_i+vR_i+S_i$
1	533	389	531	2231	4363
2	579	349	543	2169	4495
3	536	372	574	2226	4370

CF = 150883.55

Residuall SS (adjusted) = 93.8613

Direct SS (adjusted) = 78.2167

Direct SS (unadjusted) = 220.7833

SS due to animals (within squares) = 1532.2715

SS between periods = 44.1166

Square x periods SS = 150.1056

SS due to squares = 2266.8944

Total SS = 4444.4445

Error SS = 131.4618

A N O V A

Source	df	SS	MS	F
Between squares	1	2366.8944	2366.8944	68.97**
Between periods	2	44.1166	22.0583	0.67
Period x Square	2	150.1056	75.0528	2.28
Between animals within squares	4	1532.2215	383.0554	11.65*
Direct (adjusted)	2	78.2167	39.1084	1.19
Residual (adjusted)	2	98.8613	49.4307	1.50
Error	4	131.4618	32.8655	
Total	17	4444.4445		

$V(r_i - r_j) = 24.6491$; $V(t_i - t_j) = 13.6940$.

Inference:- The above analysis reveals that there is no significant difference between direct effects of treatments as also for residual effects of treatments. But significant difference between squares and significant difference between animals within squares are found.

(2). The following are the data collected in four years from an experiment conducted at CPCRI, Kannara in four periods to study the effects of Inter and mixed cropping of banana in Arecanut garden.

Average weight of fruits (kg)

Periods	Replication I				Replication II			
75-76	A 7.33	B 6.28	C 9.76	D 10.03	C 13.13	D 8.01	A 12.83	B 14.47
76-77	D 13.14	A 16.56	B 13.86	C 12.04	B 17.40	C 17.23	D 13.47	A 13.55
77-78	B 7.49	C 13.39	D 9.14	A 13.38	D 8.01	A 10.44	B 8.05	C 12.06
78-79	C 8.50	D 12.09	A 10.34	B 9.40	A 9.20	B 11.34	C 12.61	D 9.57

Analysis:

Total for squares : Q_1 = 171.63 ; Q_2 = 191.37

Total for sequences:

S_{ij} = 36.36; 48.32; 42.1; 44.85; 47.74; 47.02; 46.96; 49.65

Grand Total : G = 363

Treatment No (i)	T_i	R_i	S_i	$T_i+vR_i+S_i$	$(v^2-v-1)T_i+vR_i+S_i$
1	93.53	72.80	89.84	474.57	1409.87
2	98.29	68.37	91.87	453.64	1336.54
3	97.72	76.74	83.32	498.00	1465.20
4	83.46	64.35	97.97	438.83	1273.43

SS due to sequences (within squares) = 20.3307

SS due to squares = 12.1772

SS between periods = 117.3756

SS between period x square interaction = 28.6028

Total SS = 269.3284

Residual SS (adjusted) = 17.8544

Direct SS (adjusted) = 23.9678

Direct SS (unadjusted) = 14.4382

Error SS = 58.5495

A N O V A

Source	df	SS	MS	F
Between squares	1	12.1772	12.1772	2.49
Between periods	3	117.3756	39.1252	8.02
Period x square	3	28.6028	9.5343	1.95
Between plants within squares	6	20.3307	3.3885	0.69
Direct (adjusted)	3	23.9678	7.9893	1.64
Residual (adjusted)	3	17.8544	5.9515	1.22
Error	12	58.5495	4.8791	
Total	31	269.3284		

$V(r_i - r_j) = 1.9516$; $V(t_i - t_j) = 1.3418$

Inference:- From the above analysis of variance it is found that the treatments are homogenous with respect to direct effects and residual effects. But there is significant difference between period effects.

Discussion

DISCUSSION

Williams (1949) gave a special method of construction of balanced change-over designs balanced for first order residuals by the method of module differences. In the present investigation similar designs have been constructed based on cyclic latin squares in the line of Amble (1977). It was found that the construction based on cyclic latin squares also showed balance for first order residual effects. In this particular study, as was done by Williams, two cases were considered, using even and odd number of treatments. While comparing the two methods viz., Williams method and cyclic latin square method, it was found that a design was said to be balanced for first order residual effects if

(i) each treatment is preceeded by each other treatment equally frequently; and

(ii) each treatment shall occur equally frequently at each position in order of application to the sites.

Since every treatment should occur in all sites it followed that the number of sites shall be a multiple of the number of treatments. We should assume that there were n treatments and we represent them by residue class modulo n. If, now, we arrange the first row of a square in such a way that the (n-1) adjacent differences are 1, 2, ..., (n-1), itself with (n-1) rows obtained by adding 1 to the previous row would be a balanced design because in the rows every difference would occur equally frequently, thus showing that every treatment would be preceeded by every other treatment.

Now consider a cyclic latin square in 0, 1, 2, ..., 2m-1. ie. when n = 2m. Then

$$C = \begin{matrix} 0 & 1 & 2 & \cdots & 2m-2 & 2m-1 \\ 1 & 2 & 3 & \cdots & 2m-1 & 0 \\ 2 & 3 & 4 & \cdots & 0 & 1 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 2m-1 & 0 & 1 & \cdots & 2m-3 & 2m-2 \end{matrix}$$

$$\text{and } R = \begin{matrix} 2m-1 & 2m-2 & \cdots & 2 & 1 & 0 \\ 0 & 2m-1 & \cdots & 3 & 2 & 1 \\ 1 & 0 & \cdots & 4 & 3 & 2 \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdot \\ 2m-2 & 2m-3 & \cdots & 1 & 0 & 2m-1 \end{matrix}$$

Then the initial row of $D = C_0 R_0 C_1 R_1 \ldots C_{m-1} R_{m-1}$ is

0, 2m-1, 1, 2m-2, 2, 2m-3, ..., m+1, m-1, m

The differences arising from neighbouring pairs are

2m-1, 2, 2m-3, 4, 2m-5, ..., 2m-2, 1

and these are all the (2m-1) difference in the module.

Further from the construction of C it follows that every row of D is obtained by adding 1 to the previous row. We know that in the rows of D every ordered pair occur exactly once. Hence the method of module difference is same as the construction by cyclic latin squares when n is even.

Let us now take n, an odd integer, say n = 2m+1. The residue class mod (2m+1) consist of the elements 0,1,2,...,2m and the cyclic latin square in these elements is

$$C = \begin{matrix} 0 & 1 & 2 & 3 & \cdots & 2m-1 & 2m \\ 1 & 2 & 3 & 4 & \cdots & 2m & 0 \\ 2 & 3 & 4 & 5 & \cdots & 0 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 2m & 0 & 1 & 2 & \cdots & 2m-2 & 2m-1 \end{matrix}$$

Therefore the reflection R of C is

$$R = \begin{matrix} 2m & 2m-1 & \cdots & 3 & 2 & 1 & 0 \\ 0 & 2m & \cdots & 4 & 3 & 2 & 1 \\ 1 & 0 & \cdots & 5 & 4 & 3 & 2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 2m-1 & 2m-2 & \cdots & 2 & 1 & 0 & 2m \end{matrix}$$

The first row of $D_1 = C_0R_0C_1R_1 \ldots C_{m-1}R_{m-1}C_m$ is

$$0,\ 2m,\ 1,\ 2m-1,\ 2,\ 2m-2,\ \ldots,\ m-1,\ m+1,\ m$$

The differences arising from the neighbouring pairs are

$$2m,\ 2,\ 2m-2,\ 4,\ 2m-4,\ \ldots,\ 2,\ 2m$$

That is all even elements of the module occuring exactly twice.

Consider E_1 where

$$E_1 = R_m\ C_{m+1}\ R_{m+1}\ C_{m+2}\ \cdots\ R_{2m-1}\ C_{2m}\ R_{2m}$$

Its first row is

$$m,\ m+1,\ m-1,\ m+2,\ \ldots,\ 1,\ 2m,\ 0$$

The differences arising from the neighbouring pairs are

$$1,\ 2m-1,\ 3,\ \ldots,\ 2m-1,\ 1$$

that is all odd elements of the module occuring exactly twice.

We also know that each row of E_1 is obtained by adding 1 to the previous row. Further we know that the columns of D_1' and E_1' give a balanced design for the residual effects with each treatment following each other twice. Hence the method of module differences is same as the construction by cyclic latin squares when n is odd also.

Hence the method of module differences given by Williams and the method suggested in the present study through cyclic latin squares were leading to the same result for both n is even and odd. But it could be seen that the cyclic latin square method explained in the present investigation was much more easier then the method of differences. Both these designs account for first order residuals and require n experimental units and n periods when n is even and it require 2n experimental units and n periods when n is odd.

The second method explained in the present study was based on (n-1) orthogonal latin squares of order n. This method of construction could be easily made use of when n is a prime or prime power. Here each treatment followed each other treatment exactly (n-1) times. Hence in the present design the residual effects could be more efficiently estimated than the previous designs discussed. Thus when the residual effects were equally important as that of the direct effects this design could be more appropriate.

The third method attempted in the present investigation was in the line of Nair (1967). In his method Nair had visualised the estimation of the residual effects upto second order. The analysis suggested by him was also very much complicated. In our present investigation we were interested only in the adjustment of first order residuals. This was achieved by making a deviation of Nair's method after forming the matrix A whose $(p,q)^{th}$ element was given by the expression

$$(p-1)+\sum_{j=0}^{q-1} x_j;\ x_0=0;\ 1\leqslant p\leqslant t,\ 1\leqslant q\leqslant (t-1)(t-2)+1$$

where $\{x_j\}$ was obtained in the same manner as explained by Nair for the construction of designs balanced for pairs of residual effects. By using this devise a generalised design requiring t experimental units and $(t-1)(t-2)+1$ periods in the case of t treatments was constructed. This design would be balanced for the first order residuals. This construction seemed to be quite simple and could be easily understood. This type of design could be easily adopted when the total duration of the experiment could be divided into large number of periods of shorter length.

The method of analysis attempted in this present investigation was of very general nature which was purely an intuitive method. But in the usual analysis suggested by different authors were based on the method of fitting constants which was very cumbersome to put into practice. In the present method of analysis one could easily calculate sum of squares due to direct effects adjusted for residual effects and sum of squares due to residual effects adjusted for direct effects by using the relationship

$$\left.\begin{array}{c}\text{SS due to direct effects}\\ \text{(unadjusted)}\\ +\\ \text{SS due to residual effects}\\ \text{(adjusted)}\end{array}\right\} = \left\{\begin{array}{c}\text{SS due to direct effects}\\ \text{(adjusted)}\\ +\\ \text{SS due to residual effects}\\ \text{(unadjusted)}\end{array}\right.$$

From all the above three methods of investigation in the present study it could be easily seen that each one was superior to the corresponding existing designs in the light of the present objectives. The analysis explained in this investigation was also simple and easy to adopt in comparison to the existing analysis.

Summary

SUMMARY

A general method of construction of designs that are balanced for first order residual effects based on cyclic latin squares in the line of Amble (1977) has been derived. Examples of layout of this design have been worked out in different cases when the number of treatments are 5,6,7 and 8.

A second method of construction of designs that are balanced for first order residual effects, when the number of treatments is a prime number or power of a prime number, has been explained by the rule "In a set of (s-1) orthogonal latin squares of order s x s, each treatment follows each other treatment exactly (s-1) times". Examples of layout of this type of designs have also been worked out for values of s = 3,4,5 and 7 (where s being the number of treatments).

A third method of construction of designs balanced for first order residual effects with more number of periods have been established based on the procedure given by Nair (1967). Layout of such designs have been worked out for the number of treatments t = 4,5,6,7,8 and 9. This designs constructed by this method are found to be balanced for first order residual effects.

All the above methods have been compared with the

corresponding existing methods given by different authors. The method of module differences by Williams (1949) has been found to be similar to the method of cyclic latin squares constructed in this investigation.

A general intuitive and easy method of analysis has been devised. By this method of analysis residual and direct effects of treatments can also be easily estimated. Illustrative examples, one each from agriculture and animal sicence sector, have also been worked out.

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DESIGNS BALANCED FOR RESIDUAL EFFECTS

By

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ABSTRACT OF A THESIS

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ABSTRACT

The usual problem in long term experiments is that due to residual effects of treatments. The effect of a treatment that persists for a period after the application of the treatment is referred to as residual effect of that treatment. In the present study an attempt is made to construct designs which will balance for first order residual effects to suit the above mentioned situations. By definition a design is said to be balanced if every treatment follows every other treatment equally frequently.

We have established three different methods of construction of such type of designs. The first method of construction is by using cyclic latin squares as in the line of Amble (197[illegible]) and we have shown that such an arrangement is balanced for first order residual effects.

The second method of construction is based on the set of (v-1) orthogonal latin squares of order v in the case of v treatments.

The third method of construction of designs balanced for first order residual effects is also given. This is based on the procedure given by Nair (1967) for the construction of designs balanced for pairs of residual effects.

A general intuitive procedure of analysis is also given.